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Analysis of Ride Quality of Tractor Semi-Trailers

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ANALYSIS OF RIDE QUALITY OF TRACTOR SEMI-TRAILERS

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ABSTRACT

This thesis develops parameter variation techniques for calculating the set of vehicle parameters that result in the best ride comfort for the driver. The model is a fifteen degree-of-freedom (15 DOF) tractor semi-trailer vertical dynamic ride model. The modeling and simulation techniques used in this thesis are extensions of the research performed by Trangsrud [1] and Vaduri [3]. Features of the model include suspension characteristics for (a) each of the five axles (tractor steer axle, two tractor drive axles, and two trailer axles), (b) tires, (c) a flexible engine mount, (d) the tractor cab, (e) the driver's seat, and (f) a fifth wheel suspension system. Also taken into consideration are the beaming effects of the tractor and trailer frame. The simulation of the model is conducted using MATLAB. The input to the system is a user-defined power spectral density (PSD) function of the vertical road irregularities. Other user inputs include the beaming frequencies of the tractor and trailer frame, tire types, cab suspension configurations, seat suspension configurations, and fifth wheel suspension configurations. Outputs from the simulation include root mean square (RMS) accelerations experienced at the driver's seat and at the center of gravity (CG) of the trailer, static axle loads and deflections, various transfer functions of response variables, and surface plots of the RMS combined weighted acceleration at the driver's seat and the RMS vertical weighted acceleration at the trailer CG as different parameters of the vehicle are varied. In addition, the RMS acceleration spectra of the driver are plotted together with the ISO 2631 [5,7] comfort curves. Results from the case

studies explored in this thesis suggested lowering the stiffness values for the axle suspensions and tires and raising the corresponding damping values. Also, beaming frequencies of the tractor and trailer frames should be kept above 20 Hz to avoid large accelerations caused by coupling with other modes. Finally, the implementation of an idealized vertically-oriented fifth wheel suspension system did not lower accelerations experienced at the driver's seat in the nominal vehicle, but was shown to have beneficial effects when coupled with a full cab suspension system.

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TABLE OF CONTENTS

Page

LIST OF TABLES

List of Tables (Continued)

Page

List of Tables (Continued)

Page

LIST OF FIGURES

Page

CHAPTER 1

INTRODUCTION

Introduction

The focus of this thesis is the development and simulation of a ride comfort model for a cab-over style tractor semi-trailer, and parameter variation programs that can provide the user with the best set of parameters based on the combined ISO weighted acceleration of the driver and the vertical ISO weighted acceleration of the trailer CG. Also factored in are constraints caused by factors such as axle load limits, vehicle ride height, and stroke across the fifth wheel. Previous simulations have studied the dynamic response of the tractor semi-trailer and the effect that certain parameters have on the response, but this simulation is unique in that it shows how the dynamic response changes in response to the variation of multiple parameters over a wide range of values. The model has 15 degrees-of-freedom (DOFs) and focuses on the vertical dynamic response. Among the outputs given by the program are the RMS accelerations that are present at the driver's seat and the trailer CG, transfer functions for various response variables, and static loads for all axles.

Chapter 1 provides information about previous related research on this topic and why this particular model was developed. The details of the model and the derivation of the governing equations can be found in Chapter 2. Chapter 3 discusses the details of the simulation program and parameter variation programs.

The results from the simulation program and case studies from the parameter variation programs are presented and discussed in Chapter 4. Finally, Chapter 5 gives a summarization of the completed research and provides topics for possible future research on this topic. Additional information, data, and the MATLAB codes are available in the Appendices.

Research Motivation and Problem Statement

A revolutionary new tire design has been conceived and manufactured by the Michelin Tire Corporation of North America. This new design is aimed at replacing the dual tires which are in wide use in the trucking industry with a single, wide-base tire. However, the different types of trucks which can be outfitted with this tire are virtually limitless in their configurations of the tractor as well as the load. The research and data presented with this thesis can provide Michelin with a way to discover the set of parameters that provide the best ride quality for the driver, the lowest accelerations experienced at the trailer CG, and how the parameters of the complete system can be chosen to achieve this. The computer simulation of the tractor semi-trailer allows the parameters to be varied and the response to be studied with multiple types of loading conditions, suspension configurations, road conditions, tire types, and speeds.

Literature Review of Tractor Semi-Trailer Ride Comfort

Driver ride comfort in medium and heavy duty trucks has been an area of great interest for truck manufacturers and their operators for many years.

Excessive driver discomfort and fatigue can have a direct impact on productivity and safety. Improved ride comfort would not only allow the driver to remain more alert at the wheel, but would also allow the driver to safely operate the tractor semi-trailer for longer periods of time.

Studies have been conducted to address various problems using a wide range of vehicle models which focus on individual issues. The model developed in this thesis serves as a continuation of the work performed by Trangsrud [1] and the 14 degree-of-freedom (DOF) tractor semi-trailer model he developed. Trangrud's model and simulation investigated the effects on the ride comfort of the driver of the new wide-base tires developed by Michelin. Also, his model included the possibility to motions of the engine with respect to the tractor chassis and beaming of the semi-trailer. Trangsrud studied the effects of tire non-uniformities and friction in the suspension system and their effect on the dynamic response of the tractor semi-trailer. Finally, he studied the effects on dynamic response caused by random variations in tire pressures, tire non-uniformities, and axle spring stiffnesses.

Much of Trangsrud's work, like the work presented in this thesis, was based on work done by or parallel to that of many others. Vaduri [3] investigated the effects of cab and seat suspension on the isolation of the driver from the road inputs. His model included the effects of tractor frame beaming and the presence of tire radial stiffness non-uniformities. LeFerve [8] performed a broad study concerning the effects of different parameters on the tractor and trailer ride dynamics. Among the parameters he investigated were the cab and seat suspensions, fifth wheel location, frame bending vibrations, tire and wheel non-uniformities, and trailer pitching motions.

A literature survey was presented by Jiang et al [9] in which seven different tractor semi-trailer models were discussed as well as five different driver-seat models. The different tractor semi-trailer models include a simple six DOF pitch and vertical heave model developed by Dhir et al [11] to study the effects of dry or coulomb friction in the axle suspension. Also included is a 21 DOF pitch, heave, and roll model developed by Cole and Cebon [12] that included detailed suspension models to study the connection between heavy vehicle design and the dynamic pavement loading.

The effects of cab and seat suspension have been an area of particular interest because of the considerable ride comfort improvements they provide. Studies were conducted by Foster [13] and Flower [14] to analyze the effects of various cab suspensions. Both concluded that they were a very effective method for improving the driver ride comfort. The greatest improvements in acceleration were found to be in the frequency range in which the human body is the most sensitive, 1.0 to 20 Hz [5,7]. Foster's study examined a front and rear cab suspension with the addition of a suspension system for the driver's seat with low natural frequencies. This element provided the necessary isolation of the driver from the accelerations in the tractor frame and cab in the key frequency range. Flower conducted research which examined the effects of front-only and rear-only cab suspension. Both configurations provided significant improvement to the driver ride quality, but the front suspension proved to be difficult to implement and service.

The 14 DOF model developed by Trangsrud [1] provided a comparison of the wide-base tires and conventional dual tire assemblies in the frequency domain. This allowed a good assessment of the vehicle ride quality. Frequency response methods allow the results to be easily compared to ride quality standards set forth by the International Standard Organization, ISO 2631 [5,7]. Of course, different body types of drivers, seating positions, etc. make it nearly impossible to determine an exact comfort limit for every driver. However, the ISO 2631 standard is still regarded as a leading standard for quantifying ride quality. The ride quality standards exist as upper boundaries of the RMS vertical and longitudinal accelerations measured at the driver's seat (Figure 1.1) over the frequency range from 0.1 to 50 Hz. The boundaries represent the amount of time the driver can sustain that particular acceleration before becoming uncomfortable. As one would expect, lower acceleration magnitudes can be tolerated as the driver operates the vehicle for longer periods of time.

It should be noted that in recent years, the comfort dependence on time in the ISO 2631 standards has been dropped [7]. This was due to research results that indicated that the dependence of comfort on duration was questionable, particularly during short time intervals. However, the time dependence was retained in the health evaluation. Since this research concerns vehicles which generally operate for long periods of time, and the comfort boundaries provide a good reference point for which to compare ride comfort criteria. Also, the frequency information provided by comparing results against the ISO 2631 ride comfort standards is quite useful in making design decisions. Due to these factors, the time dependent comfort criteria will be retained.

This thesis extends the work of Trangsrud and Vaduri [1,3] to describe ways of predicting the dynamic ride response and the comparisons of different parameters and configurations of the vehicle. Methods are presented in this thesis that will aid in the determination of the set of parameters that result in improved ride comfort performance of the vehicle. Also, this thesis explores the possibility of adding a vertically oriented fifth wheel vertical suspension system, and determining the set of parameters for that system that will result in the most desirable ride response. A picture depicting a common fifth wheel connection may be found in Appendix C labeled Figure C.3. Finally, the trade-off between ride comfort and rollover characteristics is briefly examined.

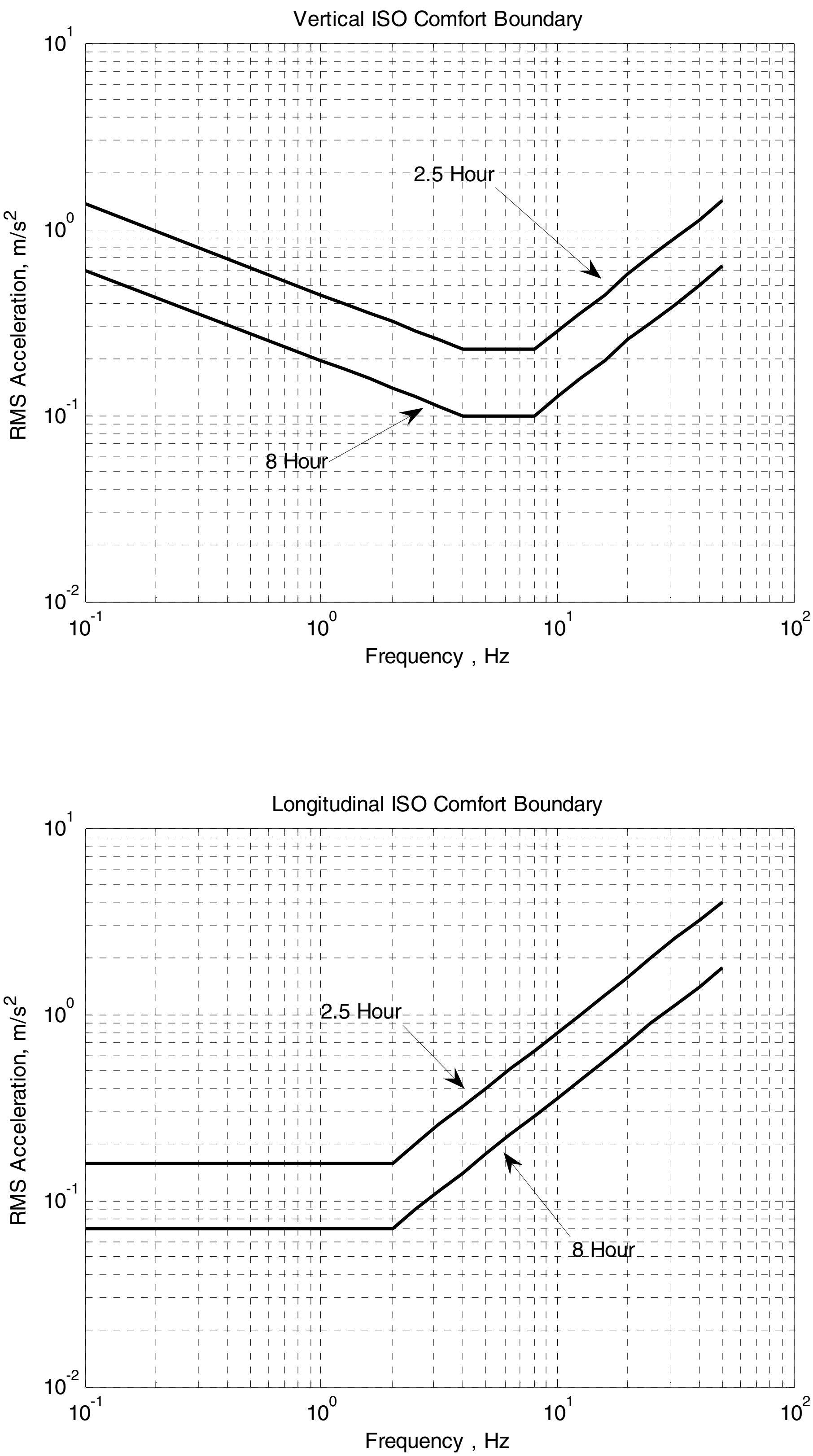


Figure 1.1: ISO Whole Body Acceleration Comfort Limits [5]

CHAPTER 2

MODEL DERIVATION

Introduction

The tractor semi-trailer under study in this thesis is a cab-over type tractor with a basic box semi-trailer and was modeled as having a 15 degree-of-freedom system (DOF), with ten DOFs for the tractor and five DOFs for the semi-trailer (Figure 2.1). The model is based on work by Trangsrud [1] with the addition of a fifth wheel suspension system, which allows for heave of the trailer frame relative to the tractor. The degrees of freedom describing the tractor are the driver seat heave, cab pitch and heave, engine heave, tractor frame pitch and heave, tractor frame beaming, and heave of each of three axles (one steer axle and two drive axles). Describing the trailer are the pitch and heave of the trailer frame, the beaming of the trailer frame, and the heave of each of the two trailer axles. The governing equations were derived using the Lagrangian approach [15] which uses the kinetic and potential energies of each of the tractor semi-trailer elements.

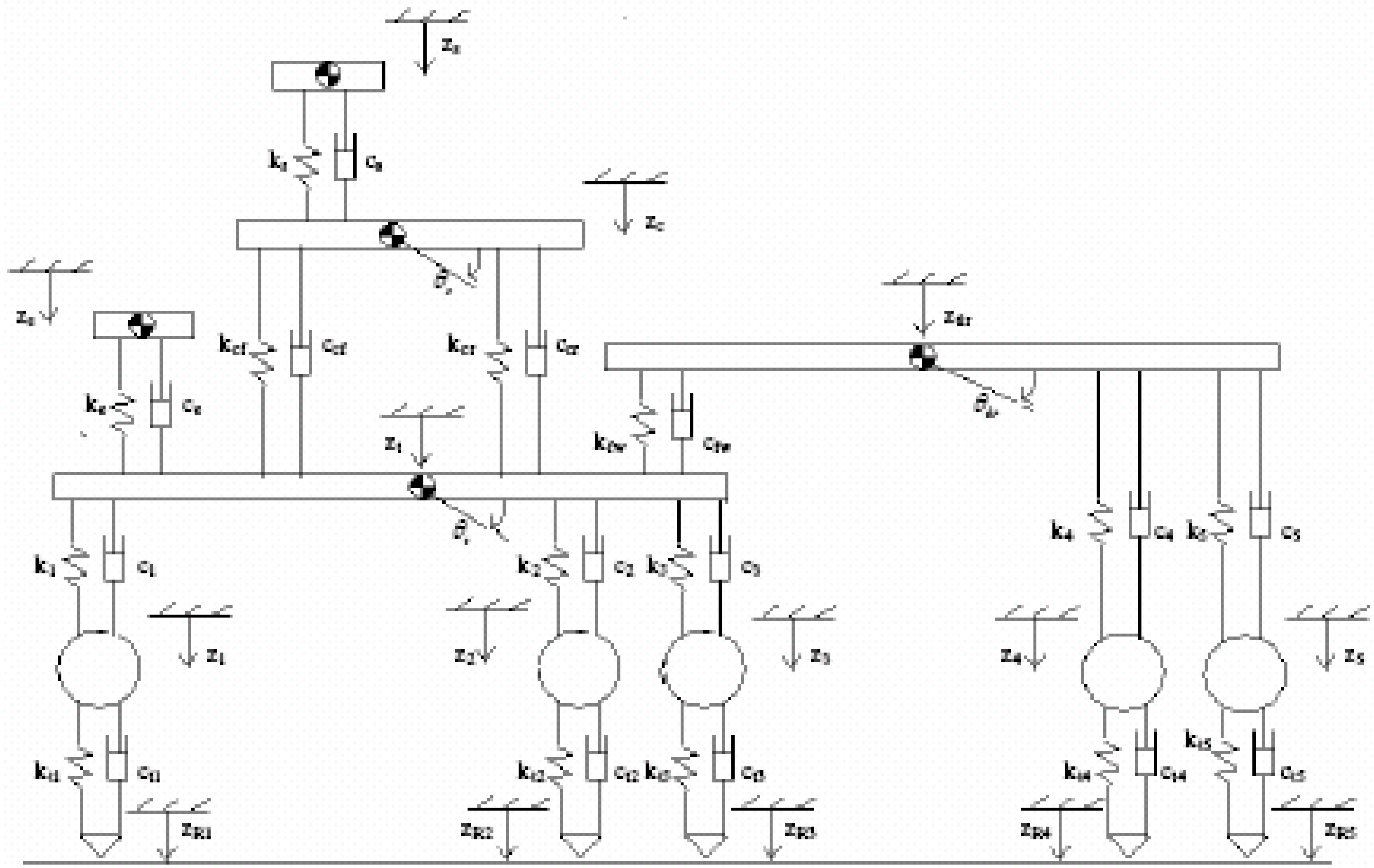


Figure 2.1: Fifteen Degree-of-Freedom System Model

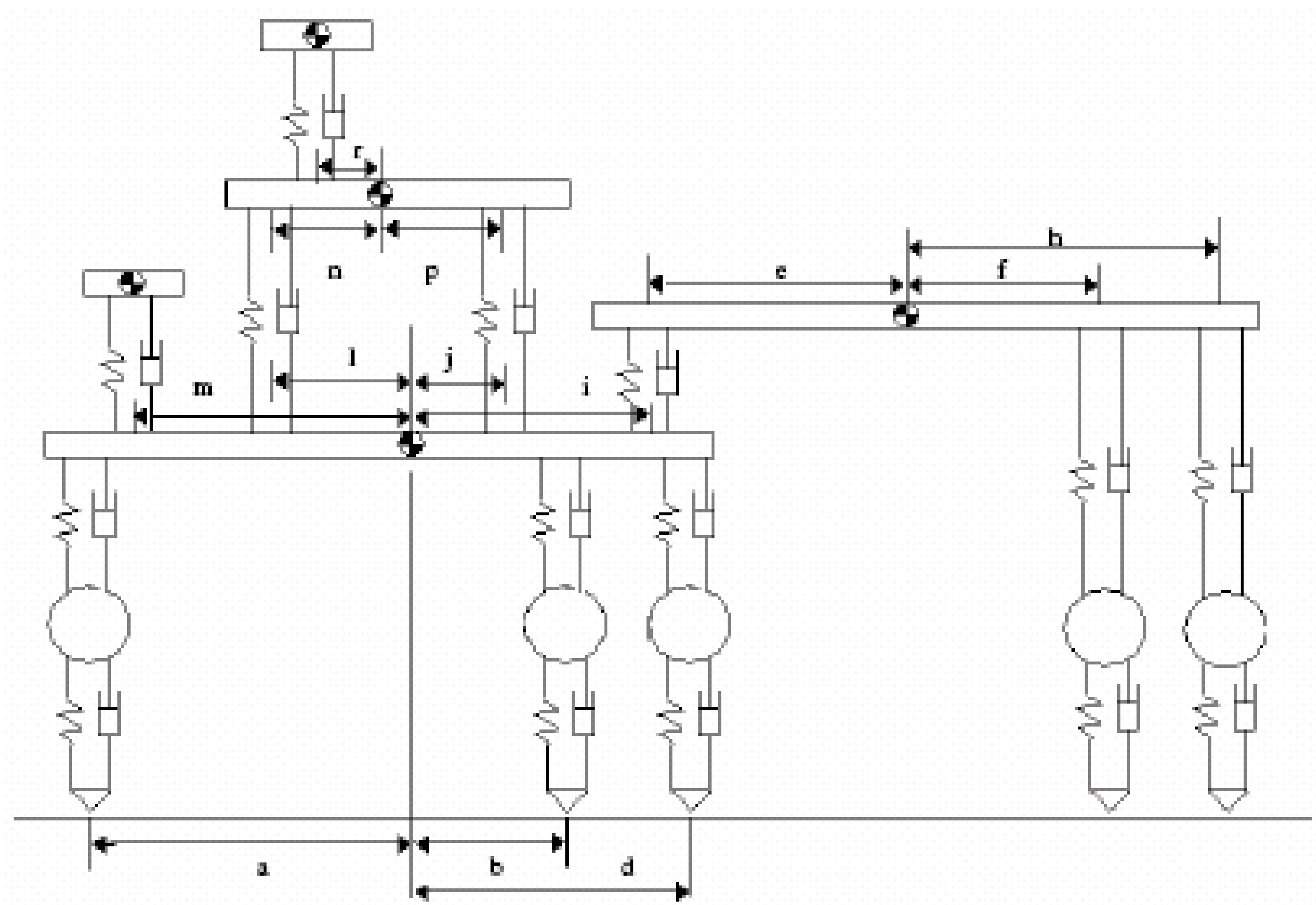


Figure 2.2: Dimensions of the Tractor Semi-Trailer Model

Model Description

To study the dynamic response of the tractor semi-trailer, a mathematical model was developed containing fifteen degrees of freedom. The DOFs for the tractor are listed below.

1. vertical displacements of

    a. Driver's Seat z_S,
    b. Cab CG, z_C,
    c. Engine, z_E,
    d. Tractor CG, z_T,
    e. Tracor Frame Beaming, η_T,
    f. Steer Axle (Axle #1), z_1,
    g. 1st Drive Axle (Axle #2), z_2,
    h. 2nd Drive Axle (Axle #3), z_3,

2. pitch angles of

    a. Tractor Frame, θ_T,
    b. Cab Body, θ_C,

The DOFs for the trailer are

1. vertical displacements of

    a. Trailer Frame CG, z_{TLR},
    b. Trailer Frame Beaming, η_{TLR},
    c. 1st Trailer Axle (Axle #4), z_4,
    d. 2nd Trailer Axle (Axle #5), z_5,

2. pitch angles of

    a. Trailer Frame, θ_{TLR},

All of the displacements are absolute quantities with the exception of the tractor and trailer frame beaming displacements, η_T and η_{TLR} , which are relative to the rigid frames. A description of the tractor semi-trailer model suspension parameters, geometric parameters, and inertial properties can be found in Appendix C along with a visual representation in Figure 2.2. These values were obtained from Law et al [17] and from physical measurements on a Michelin test vehicle, a Freightliner Century Class tractor.

Modeling of Suspended Masses

The tractor semi-trailer model consists of suspended masses which are coupled by parallel linear springs and viscous dampers, as seen in Figure 2.1. The inputs are transmitted from the road to the vehicle via the tires, which are represented as equivalent linear spring and viscous damping "suspensions" which approximate tire stiffness and damping characteristics. The tires are connected to the frame by another equivalent linear spring and damper which approximate the vehicle axle suspension elements.

The tractor frame rides atop three axles, the steer axle at the front of the vehicle and two drive axles at the rear of the vehicle. Likewise, the semi-trailer frame rides atop two axles, both located at the rear of the frame. The semi-trailer is connected to the tractor frame via a fifth wheel connection, modeled by a equivalent linear spring and damper. The fifth wheel can be treated as a pin connection by setting the stiffness value very high, which allows for shear and vertical forces, but no bending moment, to be transferred across the connection.

The engine is modeled as a lumped mass connected to the tractor frame via another linear spring and viscous damper which approximate engine mounts.

The cab sits atop two sets of linear springs and viscous dampers, which allows it to be modeled in any of four configurations: (a) front and rear cab suspension, (b) suspension in only the rear of the cab, (c) suspension in only the front of the cab, (d) no suspension. Finally, the drivers seat has the option of being modeled as an equivalent linear spring and viscous damper, or may be simulated as a rigid connection by setting the stiffness value very high.

Modeling of Suspension Elements

All of the suspension elements found in the model are represented as combinations of linear springs and viscous damping elements. These are meant to provide an appropriate approximation to suspension elements on an actual tractor semi-trailer. The purpose of each of the suspension elements is to decrease the magnitude of the transmission of the road inputs to the vehicle and ultimately to the driver.

There are many different types of suspension elements that can be found on modern tractor semi-trailers. A few of these include coil spring suspensions, parabolic leaf spring suspensions, and air bag suspensions. In this model, all of these suspension types are modeled by parallel spring and damping systems by using the best estimate possible for the stiffness and damping values.

The road inputs are assumed to be identical on the left and right sides of the vehicle. Also, the suspension elements may be lumped into a single per-axle suspension element representative of the left and right sides of the axle.

Tire Modeling

The tires for this tractor semi-trailer are modeled as point masses connected to the road by equivalent linear spring and viscous damping elements. The tire spring constant represents the equivalent tire stiffness and the damping constant simulates the energy dissipation that results from tire deformation [19]. Though the tire damping constant does not vary in this model, it may vary in actual driving conditions depending on temperature and other environmental conditions. The value was held constant since accurate information regarding these effects was not available and it is intended to represent a "nominal" condition. The tire and wheel mass is lumped together with the axle mass and treated as a single mass at the center of the axle.

Tractor and Trailer Frame Bending

The tractor and semi-trailer frames are constructed using simple ladder designs with two longitudinal frame rails on the outside and parallel frame rails between them. This design allows the frames to become excited and flex in bending in response to the road inputs. The fundamental frequencies of the tractor and semi-trailer frame are typically in the range of 20 to 25 Hz, which is within the range of typical excitations caused by the road surface. The bending of the frames affect both the longitudinal and vertical accelerations of the driver's seat as well as other elements of the tractor semi-trailer.

The flexible tractor and semi-trailer frames can be represented in the model in either one of two ways depending on the fifth wheel. When the fifth wheel connection is modeled as a pin connection, the tractor and trailer frame are modeled as free-pinned and pinned-free beams, respectively. However, when a fifth wheel suspension is present, each frame is modeled as a free-free beam. Figures 2.4 and 2.5 depict each of the two mode shapes used to model the frames. The beaming characteristics of each frame are approximated by the first mode shape for that particular configuration. However, provisions for adding higher modes to the model could be done relatively easily.

The equation for bending vibration of a uniform Euler-Bernoulli beam is

$$EI\frac{\partial^4 \eta}{\partial x^4}(x,t)+\rho A\frac{\partial^2 \eta}{\partial t^2}(x,t)=f(x,t) \tag{2.1}$$

where E is the modulus of elasticity, I is the moment of inertia, η(x,t) is the vertical displacement of the beam at some point x along the beam and at some time t, ρ is the density of the beam material, and A is the cross sectional area of the beam. For a beam that is un-damped and in free vibration, f(x,t)=0. Using the separation of variables method,

$$\eta(x,t)=X(x)T(t). \tag{2.2}$$

Applying the separation of variables to Equation 2.1 and rearranging yields,

$$c^2\frac{X''''(x)}{X(x)}=-\frac{\ddot{T}(t)}{T(t)}=\omega^2, \tag{2.3}$$

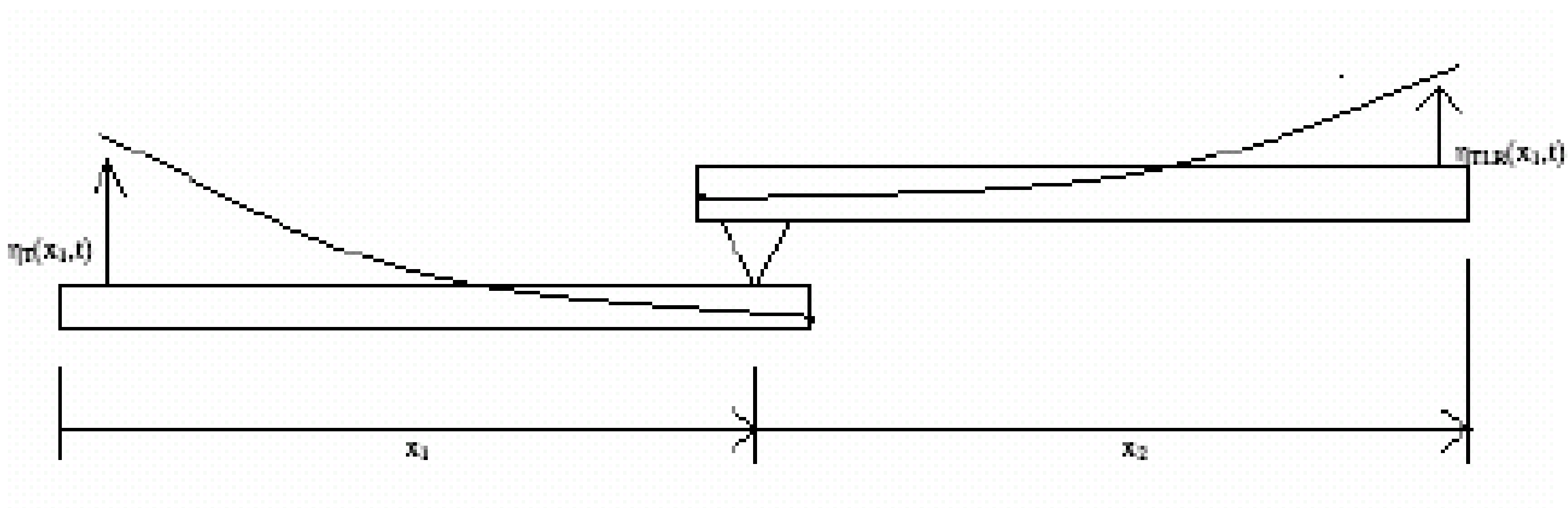


Figure 2.3: Free-Pinned and Pinned-Free Beaming Modes

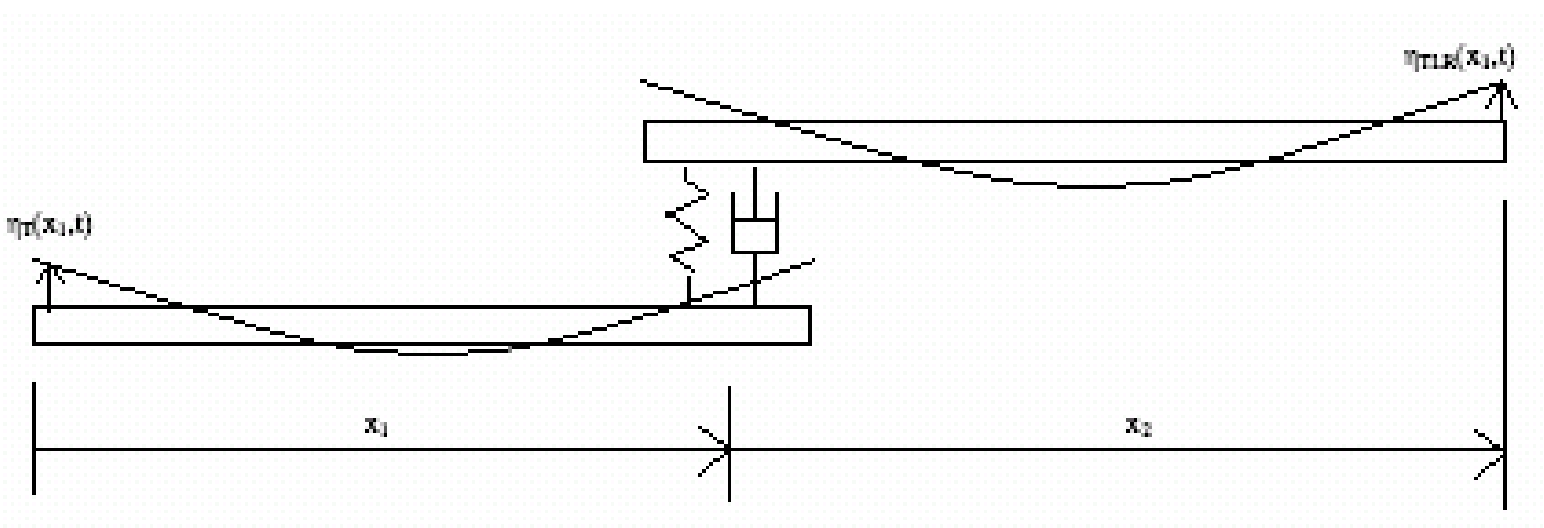


Figure 2.4: Free-Free Beaming Modes

where

$$c^2 = \sqrt{\frac{EI}{\rho A}}. \tag{2.4}$$

The system natural frequency is denoted by the symbol ω. The constant ω^2 is chosen as the separation constant based on the right hand side of Equation 2.3 which forms the temporal equation,

$$\ddot{T}(t) + \omega^2 T(t) = 0, \tag{2.5}$$

which has the solution,

$$T(t) = A\sin\omega t + B\cos\omega t. \tag{2.6}$$

Solving for the left hand side of Equation 2.3 gives the spatial equation,

$$X''''(x) - \frac{\omega^2}{c^2}X(x) = 0. \tag{2.7}$$

By defining

$$\beta^4 = \frac{\omega^2}{c^2} = \frac{\rho A \omega^2}{EI}, \tag{2.8}$$

a general form for the solution to the spatial equation can be calculated to be

$$X(x) = C_1\cos\beta x + C_2\sin\beta x + C_3\cosh\beta x + C_4\sinh\beta x. \tag{2.9}$$

The constants C_1, C_2, C_3, and C_4 are solved for using information provided about the boundary conditions of the beams. For a beam with a "pinned" end, the boundary conditions state that the deflection or displacement and the bending moment at that end are both zero,

$$Deflection = \eta = 0, \tag{2.10}$$

$$BendingMoment = EI\frac{\partial^2\eta}{\partial x^2} = 0. \tag{2.11}$$

For a beam with a free end, the boundary conditions state that the bending moment and the shear force at that end are both zero,

$$BendingMoment = EI\frac{\partial^2 \eta}{\partial x^2} = 0, \tag{2.12}$$

$$ShearForce = \frac{\partial}{\partial x}\left(EI\frac{\partial^2 \eta}{\partial x^2}\right) = 0. \tag{2.13}$$

The derivation of the tractor and trailer beaming equations can be found in Appendix X.

To model the beaming of the tractor and trailer frames using the Lagrangian approach, the assumed modes method is used. The assumed modes method works by separating the distributed parameter system [15]. The displacement due to beaming, η(x.t), can then be approximated by the finite series,

$$\eta(x,t) = \sum_{i=1}^{n} f_i(x)q_i(t), \tag{2.14}$$

where $f_i(x)$ is the i^{th} mode shape beaming function and $q_i(t)$ is the i^{th} generalized coordinate.

The frame models allow the user to input the desired frequency, of the beaming mode, on which is taken into account by the calculation of the flexural rigidity, EI, which is calculated as,

$$EI = (2\pi f)^2\left(\frac{l}{\beta l}\right)^4 \rho A. \tag{2.15}$$

where f is the natural frequency of the bare frame in Hertz (Hz), l is the length of the frame, β is a constant associated with the beam type and mode shape, and ρA is the mass per unit length of the frame [15]. In the simulation, the user defines the desired values for f, ρA, l, and β. The EI calculated in Equation 2.15 is then calculated based on these inputs.

Equations of Motion

The equations of motion for the 15 DOF tractor semi-trailer vehicle model were derived using the Lagrangian approach [15]. The full derivation can be found in Appendix A.

Road Profiles

The road which provides the vehicle model inputs is a random road profile. For the purpose of this analysis, the road profiles are given in terms of their power spectral density functions,

$$S_{Z_R}(\Omega) = C_{sp}\Omega^{-N}, \tag{2.16}$$

where Ω is the spatial frequency measured in cycles per unit length, C_{sp} and N are constants found in Table 2.1, and S_{Z_R} is the power spectral density (PSD) function of the elevation of the elevation of the road surface profile [19]. The PSD of the road profile can be converted to a function of temporal frequency, f measured in Hz, by using the velocity of the vehicle in units of length per second,

$$S_{Z_R}(f) = \frac{S_{Z_R}(\Omega)}{V}, \tag{2.17}$$

through the relationship,

$$f\left(\frac{cyc}{\sec}\right) = \Omega\left(\frac{cyc}{\mathrm{m}}\right) \cdot V\left(\frac{m}{\sec}\right). \tag{2.18}$$

This road profile PSD can then be used to find the PSDs and RMS values for various elements of the model. A full description of this process can be found in Chapter 3.

Table 2.1: Values of C_{sp} and N for PSDs of Various Surfaces [19]

No.	Description	N	C_{sp} (SI)	C_{sp} (English)
1	Smooth Runway	3.8	4.3×10^{-11}	1.6×10^{-11}
2	Rough Runway	2.1	8.1×10^{-6}	2.3×10^{-5}
3	Smooth Highway	2.1	4.8×10^{-7}	1.2×10^{-6}
4	Highway with Gravel	2.1	4.4×10^{-6}	1.1×10^{-5}
5	Pasture	1.6	3.0×10^{-4}	1.6×10^{-3}
6	Plowed Field	1.6	6.5×10^{-4}	3.4×10^{-3}

Note: C_{sp} (SI) is used for computing $S_{Z_R}(\Omega)$ in m^2/(cycle/m) and C_{sp} (English) is used for computing $S_{Z_R}(\Omega)$ in ft^2/(cycle/ft)

CHAPTER 3

SIMULATION

Introduction

The tractor semi-trailer ride simulation uses the vehicle model described in the preceding chapter and Appendices. A MATLAB simulation, titled dof15_freq2.m, was created to investigate the effects various parameters have on the driver ride comfort, vehicle ride heights, and pavement loading. The program allows the user to select desired configurations for the trailer, fifth wheel, cab and seat suspension, and tires. Also developed were simulations that vary certain parameters and create surface plots displaying the corresponding trends in driver ride comfort and trailer CG acceleration.

MATLAB Simulation

The vehicle ride simulation was programmed using MATLAB (Mathworks). The vehicle is described by the fifteen second-order differential equations presented in Appendix A. The equations of motion are arranged in matrix form,

$$M\ddot{X} + C\dot{X} + KX = A\dot{U} + BU \tag{3.1}$$

where M is the mass matrix, C is the damping matrix, K is the stiffness matrix, A is the road input damping matrix, and B is the road input stiffness matrix [15]. The matrix X is the vector of the system unknowns,

$$X^T = \begin{bmatrix} z_s & z_c & \theta_c & z_e & z_t & \theta_t & q_t & z_{tlr} & \theta_{tlr} & q_{tlr} & z_1 & z_2 & z_3 & z_4 & z_5 \end{bmatrix}, \tag{3.2}$$

where, as can be seen in Figure 2.1, z_s is the vertical displacement of the driver's seat, z_c and θ_c are the vertical displacement and pitch angle of the tractor cab, respectively, z_e is the vertical displacement of the engine, z_t, θ_t, and q_t are the tractor vertical displacement, tractor pitch angle, and generalized time dependent coordinate for the beaming of the tractor frame respectively. The trailer vertical displacement, trailer pitch angle, and generalized time dependent coordinate for the beaming of the trailer frame are z_{tlr}, θ_{tlr}, and q_{tlr}, respectively, and z_1, z_2, z_3, z_4, and z_5 are the vertical displacements of the five axles. The matrix U is the vector of the road profile vertical displacement,

$$U^T = \begin{bmatrix} z_{R1} & z_{R2} & z_{R3} & z_{R4} & z_{R5} \end{bmatrix}. \tag{3.3}$$

For each element of the model, the vertical displacements have the positive direction defined as downward movement, and positive pitch rotations are defined as the front of the particular body moving up and the rear moving down. The displacements due to frame beaming are relative to the frame with the positive direction being in the upward direction.

To calculate the frequency responses, PSDs, RMS values, and eigenvalues and eigenvectors, the Laplace transform of the system must be taken,

$$\left\{Ms^2 + Cs + K\right\} X(s) = \left\{As + B\right\} U(s). \tag{3.4}$$

where the M matrix is composed of the mass terms, C is composed of the damping terms, and K is composed of the stiffness terms of each component of the model. The values for the road input in the U vector depend on the user-defined road profile. As discussed in Chapter 2, the road profile is an approximation to the vertical irregularities found on different types of roadways. Each axle is assumed to see the same road profile, but with time delay between the axles. All time delays are calculated relative to the first (steer) axle of the tractor. The magnitude of the time delay, T_i, depends on the velocity at which the vehicle is traveling, v, and the distance, d_i, that particular axle is from the first axle,

$$T_i = \frac{d_i}{v}. \tag{3.5}$$

Applying the time delays to the road input vector, U, the new road input vector in Laplace form becomes,

$$U(s) = \begin{bmatrix} 1 & e^{-sT_2} & e^{-sT_3} & e^{-sT_4} & e^{-sT_5} \end{bmatrix} z_1(s) = b(s) z_1(s). \tag{3.6}$$

Inserting Equation 3.6 into Equation 3.4 results in a much simplified system with only one input due to road irregularities,

$$\left\{Ms^2 + Cs + K\right\} X(s) = \left\{As + B\right\} b(s) z_1(s) \tag{3.7}$$

Model Parameters

The parameters for the vehicle used in this simulation were obtained by combining information found in several different sources. Most of the tractor parameters came from physical measurements conducted by personnel at Michelin on a test tractor, a Freightliner Century Class tractor. A detailed description of the parameters used in this simulation can be found in Appendix C.

Calculation of the Frequency Response

The road inputs into the system affect the dynamic response of each of the individual degrees of freedom. In order to fully analyze how the system reacts to various inputs, it is analyzed over an entire spectrum of frequencies ranging from 0.1 Hz to 50 Hz. Solving for the vector of the system's unknowns, $X(s)$, from Equation 3.7 yields,

$$X(s) = \left(Ms^2 + Cs + K\right)^{-1}\left[\{As + B\}b(s)z_1(s)\right]. \tag{3.8}$$

The vector of the transfer functions in response to the input on the first tractor axle, z_1, is,

$$\frac{X(s)}{z_1(s)} = \left(Ms^2 + Cs + K\right)^{-1}\{As + B\}b(s). \tag{3.9}$$

To obtain the transfer function of a particular coordinate in response to the road, $X(s)$ is pre-multiplied by the appropriate row vector. For example, the transfer function for the vertical displacement of the driver's seat is given by,

$$\frac{z_s(s)}{z_1(s)} = P\left(Ms^2 + Cs + K\right)^{-1}\{As + B\}b(s) \tag{3.10}$$

where,

$$P = \begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}. \tag{3.11}$$

To calculate the velocity or acceleration of any of the degrees in the Laplace domain, the individual motion must be multiplied by s or s^2 respectively.

Similar to the above example, to calculate the transfer function for the vertical acceleration of the driver's seat, Equation 3.10 must be multiplied by,

$$P = \begin{bmatrix} s^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{3.12}$$

In addition to the individual degrees of freedom of the system, other information can be calculated using a combination of the motions. For example, the stroke across the fifth wheel can be calculated by finding the difference of vertical displacements of the points on the tractor frame and trailer frame where the fifth wheel is connected. Equation 3.13 shows the calculations performed to find the stroke across the fifth wheel.

$$z_{stroke} = \left[z_t + i\theta_t - q_t\left(a+i\right) \right] - \left[z_{tlr} - e\theta_{tlr} - q_{tlr}\left(0\right) \right] \tag{3.13}$$

where i is the distance from the CG of the tractor to the fifth wheel connection, a is the distance from the front of the tractor to the CG of the tractor, and e is the distance from the CG of the trailer to the fifth wheel connection. The stroke across the fifth wheel is then calculated through the location vector P,

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & i & -f_t\left(a+i\right) & -1 & e & f_{tlr}\left(0\right) & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{3.14}$$

Transfer functions can also be used to calculate other responses of the system such as the wheel forces. The wheel forces, or pavement loadings, are calculated using the transfer function for each axle. For a given axle, the equation of the per-axle wheel force is,

$$F = c_t\left(\dot{z} - \dot{z}_R\right) + k_t\left(z - z_R\right) \tag{3.15}$$

where c_t is the tire damping coefficient, k_t is the radial tire stiffness coefficient, z is the vertical displacement of the axle, and z_R is the vertical elevation of the road being traversed. Performing a Laplace transform on Equation 3.15 results in,

$$F(s) = (c_t s + k_t) \cdot \left[z(s) - z_R(s) \right]. \tag{3.16}$$

Dividing through Equation 3.16 by the displacement of the road results in the force transfer function for a given axle,

$$\frac{F(s)}{z_R(s)} = (c_t s + k_t) \cdot \left[\frac{z(s)}{z_R(s)} - 1 \right]. \tag{3.17}$$

For the vehicle simulation system, Equation 3.17 provides the force transfer function for each axle relative to the road displacement under that particular axle. In order to present the force transfer function for a particular axle in terms of the roadway, the time delay for that axle must be calculated and applied to Equation 3.17. For example,

$$z_2(s) = z_1(s) e^{-sT_2}, \tag{3.18}$$

where $z_2(s)$ is the displacement of the second axle and T_2 is the time delay between the first and second tractor axles. Likewise, the wheel force transfer function for the second drive axle becomes,

$$\frac{F_2(s)}{z_2(s)} = (c_{t2} s + k_{t2}) \left[\frac{z_{t2}(s)}{z_2(s)} - 1 \right] = (c_{t2} s + k_{t2}) \left[\frac{z_{t2}(s)}{z_1(s)} e^{sT_2} - 1 \right]. \tag{3.19}$$

Calculation of Power Spectral Densities and Root Mean Squares

As discussed in Chapter 2, the vertical profile PSD of the road, given in units of (m^2/cycles/m), is

$$S_{z_1}(\Omega) = C_{sp}\Omega^{-N} \tag{3.20}$$

where C_{sp} and N are constants specific to the individual roadway profiles (Table 2.1), and Ω is the spatial frequency, in units of (cycles/sec). To convert the road PSD into a form that can be used to calculate the PSDs for the responses of the other degrees of freedom of the system, it must be manipulated to be in terms of the temportal frequency, ω, in units of (rad/sec),

$$S_{z_1}(\omega) = \frac{1}{2\pi V} S_{a_1}(\Omega) = \frac{(2\pi V)^{N-1}}{\omega^N} C_{sp} \tag{3.21}$$

where V is the velocity of the vehicle. Using the input PSD from the roadway, the PSDs for the other individual degrees of freedom of the system can be calculated using the Equation 3.22,

$$S(\omega) = \left|H_{z_1}(j\omega)\right|^2 S_{z_1}(\omega), \tag{3.22}$$

where $\left|H_{z_1}(j\omega)\right|$ is the magnitude of the individual transfer function of interest (relative to the road displacement under the first (steer) tractor axle).

As specified in ISO 2631 [5,7], the RMS vertical and longitudinal accelerations are calculated over a series of one-third octave bands with specified center frequencies. The lower and upper frequencies of each band, f_1 and f_2, are related to the center frequency, f_c, by the equations

$$f_1 = 0.89 f_c \tag{3.23}$$

and

$$f_2 = 1.26 f_2 = 1.12 f_c. \tag{3.24}$$

The mean square value of a particular acceleration is equal to the area under the PSD curve for that particular acceleration. In each one-third octave band, this area is approximated by

$$\Delta E\left(\ddot{z}^2\right) = \frac{S(\omega_2) + S(\omega_c)}{2}(\omega_2 - \omega_c) + \frac{S(\omega_c) + S(\omega_1)}{2}(\omega_c - \omega_1). \tag{3.25}$$

To calculate the total mean square over the entire frequency range of interest, which includes all the center frequencies, all of the mean squares in the one-third octave band are summed. The RMS over the entire frequency range is then the square root of this value,

$$RMS = \sqrt{\sum \Delta E\left(\ddot{z}^2\right)}. \tag{3.26}$$

The standard set forth in ISO 2631 specifies that the RMS values of acceleration in each band must be plotted and compared with the ISO-specified comfort curves (Figure 1.1).

The standards set forth in ISO 2631 also define the calculation and use of a single weighted RMS acceleration number for the measurement of ride comfort. The overall weighted RMS acceleration, a_0, is the root mean square,

$$a_0 = \left[\sum (w_i a_i)^2\right]^{1/2}, \tag{3.27}$$

where w_i is the ISO-specified weighting factor at the center frequency for the i^{th} one-third octave band, and a_i is the RMS acceleration in the same one-third octave band. Table 3.1 lists the ISO 2631 weighting factors for driver RMS vertical and longitudinal accelerations.

Table 3.1: ISO 2631 Weighting Factors for Driver RMS Accelerations [7]
1997 ISO 2631 Standards, Section 6.2, Table 3, pg. 7

Hz	**Vertical**	**Longitudinal**
0.100	0.0312	0.0624
0.125	0.0486	0.0973
0.16	0.0790	0.1580
0.20	0.1210	0.2430
0.25	0.1820	0.3650
0.315	0.2630	0.5300
0.40	0.3520	0.7130
0.50	0.4180	0.8530
0.63	0.4590	0.9440
0.80	0.4770	0.9920
1.00	0.4820	1.0110
1.25	0.4840	1.0080
1.6	0.4940	0.9680
2.0	0.5310	0.8900
2.5	0.6310	0.7760
3.15	0.8040	0.6420
4.0	0.9670	0.5120
5.0	1.0390	0.4090
6.3	1.0540	0.3230
8.0	1.0360	0.2530
10.0	0.9880	0.2120
12.5	0.9020	0.1610
16.0	0.7680	0.1250
20.0	0.6360	0.1000
25.0	0.5130	0.0800
31.5	0.4050	0.0632
40.0	0.3140	0.0494
50.0	0.2460	0.0388

The purpose of the ISO weighting factors is to assign greater importance to the frequencies which cause the driver to experience larger amounts of discomfort. These values in turn have a greater effect on the overall weighted RMS acceleration value, a_0. This value is calculated by the equation [1997 ISO Standards, Section 6.4.2 Paragraph 3, pg. 12]

$$a_0 = \sqrt{\left(k_x a_{0\_L}\right)^2 + \left(k_z a_{0\_V}\right)^2} \tag{3.28}$$

where a_{0_L} is the longitudinal weighted RMS acceleration, a_{0_V} is the vertical weighted RMS acceleration, k_x is the longitudinal acceleration frequency weighting, and k_z is the vertical acceleration frequency weighting. When evaluating vehicle ride comfort, k_x and k_z are both equal to one. The overall weighted RMS acceleration value, a_0, can then be compared to the comfort ranges in Table 3.2.

Table 3.2: Weighted RMS Acceleration Comfort Levels [7]
1997 ISO Standards, Section C.2.3 Paragraph 2, pg. 25

Overall Weighted Acc. (a_0)	**ISO 2631 Comfort Level**
Less than 0.315 m/s^2	Not Uncomfortable
0.315 to 0.63 m/s^2	A Little Uncomfortable
0.5 to 1.0 m/s^2	Fairly Uncomfortable
0.8 to 1.6 m/s^2	Uncomfortable
1.25 to 2.5 m/s^2	Very Uncomfortable
Greater than 2.0 m/s^2	Extremely Uncomfortable

CHAPTER 4

RESULTS

Introduction

The tractor semi-trailer simulations allow for any number of parameter configurations and model characteristics to be changed in whatever order desired. In the time and frequency domain programs, properties can be altered and the effects these properties have on the system response can be closely studied. Also, the parameter variation programs allow for different configurations in order to study the effect of certain parameters on the variation of specific components. The responses which are most important and therefore most intensely studied are the ride comfort levels experienced at the driver's seat and the vertical acceleration at the center of gravity of the trailer. Also important are the dynamic stroke at the fifth wheel connection, the vehicle ride height, and the static pavement loading at the tire/road interface. These axle loads must not exceed the load limits regulated by the federal government. The simulation outputs also offer the option of examining wheel force transfer functions, and while these are not discussed in this thesis, this could be an area of interest for future research. The vehicle model outlined in Chapter 2 is utilized in the MATLAB simulation which is outlined in Chapter 3.

The specific cases studies examined and discussed in the following pages are listed below.

- Tractor Axle Suspension Parameter Variation
- Tractor Tire Parameter Variation
- Trailer Suspension and Beaming Parameter Variation
- Tractor and Trailer Beaming Parameter Variation
- Fifth Wheel Suspension Parameter Variation
- Vehicle with Full Set of Adjusted Parameters
- Rollover Analysis

Baseline Simulation

A "standard" or "nominal" vehicle was developed with the nominal parameters defined in Appendix C. Some of the values representing the nominal vehicle were originally provided to Vaduri and Law [17] by Michelin. Other values were obtained either through physical measurements or literature by Ribartis et al [20] and represent a common cab-over style tractor semi-trailer.

The other set of parameters specific to the "standard" or nominal vehicle include the road conditions, velocity, beaming frequencies, and suspension configurations. The nominal vehicle is assumed to be traveling at 60 mph over a smooth highway. The "Smooth Highway" road profile used is defined by Wong [19] and also appears in Table 2.1. There is no fifth wheel suspension system on the nominal vehicle, and therefore the tractor and trailer frames are modeled as free-pinned and pinned-free respectively. Table 4.1 provides a complete list of the options selected for the nominal vehicle in the order they appear in the MATLAB simulation.

Table 4.1: User Inputs for Nominal Vehicle

Parameter	**Input**
Vehicle Selection	Ideal Tractor Semi-Trailer
Seat Suspension	Yes
Cab Suspension	Rear Cab Suspension
Trailer Configuration	Loaded Trailer
Fifth Wheel Configuration	Without Fifth Wheel Suspension
Tractor Beaming Frequency [Hz]	20
Trailer Beaming Frequency [Hz]	20
Steer Axle Tire	XZA2 275/80R22.5
Steer Axle Tire Pressure [psi]	80
Drive Axle Tire	XONE XDA 445/50R22.5
Drive Axle Tire Pressure [psi]	104
Trailer Axle Tire	XONE XTA 445/50R22.5
Trailer Axle Tire Pressure [psi]	104
Vehicle Velocity [mph]	60
Road Profile	Smooth Highway

The eigenvalues representing the nominal vehicle simulation are shown in Table 4.2. A brief description of the corresponding mode shapes or eigenvectors for the nominal vehicle is given in Table 4.3. In the simulation, positive displacements are defined as down and positive rotations are defined as nose up. Details for each can be found in Appendix D which lists the normalized eigenvectors.

Table 4.2: Eigenvalues for the Nominal Vehicle

No.	Eigenvalue Pairs	Frequency (Hz)	Damping Ratio
1	-20.647 ± 33462i	532.6	0.0006
2	-17.876 ± 5008.6i	797.2	0.0036
3	-5.0655 ± 2814.3i	447.9	0.0018
4	-8.6229 ± 135.34i	21.58	0.0635
5	-0.8085 ± 80.984i	12.89	0.0100
6	-66.914 ± 21.211i	11.17	0.9533
7	-54.921 ± 47.320i	11.54	0.7576
8	-16.477 ± 69.086i	11.30	0.2320
9	-21.954 ± 64.320i	10.82	0.3230
10	-23.393 ± 62.945i	10.69	0.3484
11	-4.7004 ± 14.485i	2.424	0.3087
12	-5.6963 ± 1.6980i	0.946	0.9583
13	-5.6689 ± 7.5895i	1.508	0.5984
14	-0.89408 ± 9.8286i	1.571	0.0906
15	-1.7590 ± 9.0625i	1.469	0.1905

Table 4.3: Summary of Modal Characteristics for the Nominal Vehicle

No.	Freq. (Hz)	Damp Ratio	Dominant Modes	Mag	Phase (°)	Details
1	532.6	0.0006	z_T	1.000	0.00	The high frequency is due to the rigid fifth wheel configuration. The connection is modeled as an extremely stiff spring to emulate a rigid connection.
			η_T	0.569	-179.99	
2	797.2	0.004	z_C	1.000	0.00	The high frequency is due to the rear-only cab suspension configuration. The front cab suspension is modeled as an extremely stiff spring to emulate a rigid connection.
			θ_C	0.825	179.90	
			η_T	0.439	-179.95	
			z_E	0.186	-10.85	
3	447.9	0.002	z_E	1.000	0.00	The engine mounts to the frame are modeled as very stiff springs.
			z_C	0.510	-179.98	
			θ_C	0.421	0.21	
			η_T	0.396	-179.89	
4	21.58	0.06	η_{TLR}	1.000	0.00	The user-defined trailer frame beaming frequency is 20 Hz. Due to coupling with other suspension elements, the resonant frequency is shifted slightly higher.
			z_5	0.905	132.06	
			z_T	0.523	177.92	
			z_4	0.444	134.09	

Table 4.3: Summary of Modal Characteristics for the Nominal Vehicle (Continued)

5	12.89	η_T	1.000	0.00	The user-defined tractor frame beaming frequency is 20 Hz. Due to coupling with other suspension elements, the resonant frequency is shifted lower.
		z_T	0.898	179.67	
	0.01	z_E	0.560	1.20	
		z_3	0.412	137.06	
6	11.17	z_5	1.000	0.00	Wheel hop frequency; trailer axles
	0.95	z_4	0.729	2.72	
7	11.54	z_4	1.000	0.00	Wheel hop frequency; trailer axles
	0.76	z_5	0.737	-172.69	
8	11.30	z_1	1.000	0.00	Wheel hop frequency; steer axle
	0.23				
9	10.82	z_2	1.000	0.00	Wheel hop frequency; drive axles
	0.32	z_3	0.813	-164.23	
10	10.69	z_3	1.000	0.00	Wheel hop frequency; drive axles
	0.35	z_2	0.803	14.39	
11	2.424	z_5	1.000	0.00	The two trailer axles and tractor heave are the largest components in this mode.
		z_T	0.962	144.06	
	0.31	z_4	0.794	0.33	
		z_{TLR}	0.698	-37.43	
12	0.946	z_S	1.000	0.00	The driver's seat is dominant in this mode which has a frequency approximately equal to that of the driver and seat mass on the seat spring.
	0.96				
13	1.508	z_S	1.000	0.00	The driver's seat and cab heave and pitch are the largest components of this mode.
	0.60	z_C	0.552	76.62	
14	1.571	z_E	1.000	0.00	Engine and tractor heave are large and approximately in phase. Cab and seat heave are also large.
		z_C	0.993	-21.53	
	0.09	z_S	0.900	-78.02	
		z_T	0.670	-1.72	
15	1.469	z_S	1.000	0.00	Heave of the driver's seat, cab, trailer, and tractor are all large in this mode.
		z_C	0.867	56.15	
	0.19	z_{TLR}	0.827	78.67	
		z_T	0.781	88.30	

Tractor Axle Suspension Parameter Variation

Axle Suspension Stiffness

Axle suspension stiffness and damping characteristics were varied using opt_axleK_freq.m and opt_axleC_freq.m which are described in the Appendices G and H. The stiffness and damping values were varied individually, and the individual results analyzed to obtain the set of parameters that resulted in the best performance. First, the stiffness of the tractor drive and steer axle suspensions were varied.

Figure 4.1 shows the weighted RMS combined accelerations (Equation 3.28) of the driver and trailer CG varied against the steer axle properties and single drive axle properties. The stiffness values for each of the drive axles are assumed to be the same, so they are varied together in the program. The weighted RMS acceleration of the driver shows the greatest sensitivity to the steer axle stiffness. Over the entire range of steer axle stiffness input into the program, there is a 28% change in the total weighted RMS acceleration. Reducing drive axle stiffnesses caused approximately a 3% total reduction. The trends show that as the stiffnesses of the tractor steer and drive axles decrease, the total weighted RMS acceleration can be lowered from 0.45 m/s^2 to 0.34 m/s^2, which is a 24.4% reduction in total weighted acceleration of the driver from its nominal value.

It is important to analyze the effect of the tractor suspension parameter variation on the vertical accelerations experienced at the trailer CG in order to ensure that functionality is not compromised. Figure 4.1 shows that for the best combination of tractor axle stiffnesses, the weighted acceleration of the trailer CG is raised by only 0.006 m/s^2, which is approximately a 3% increase.

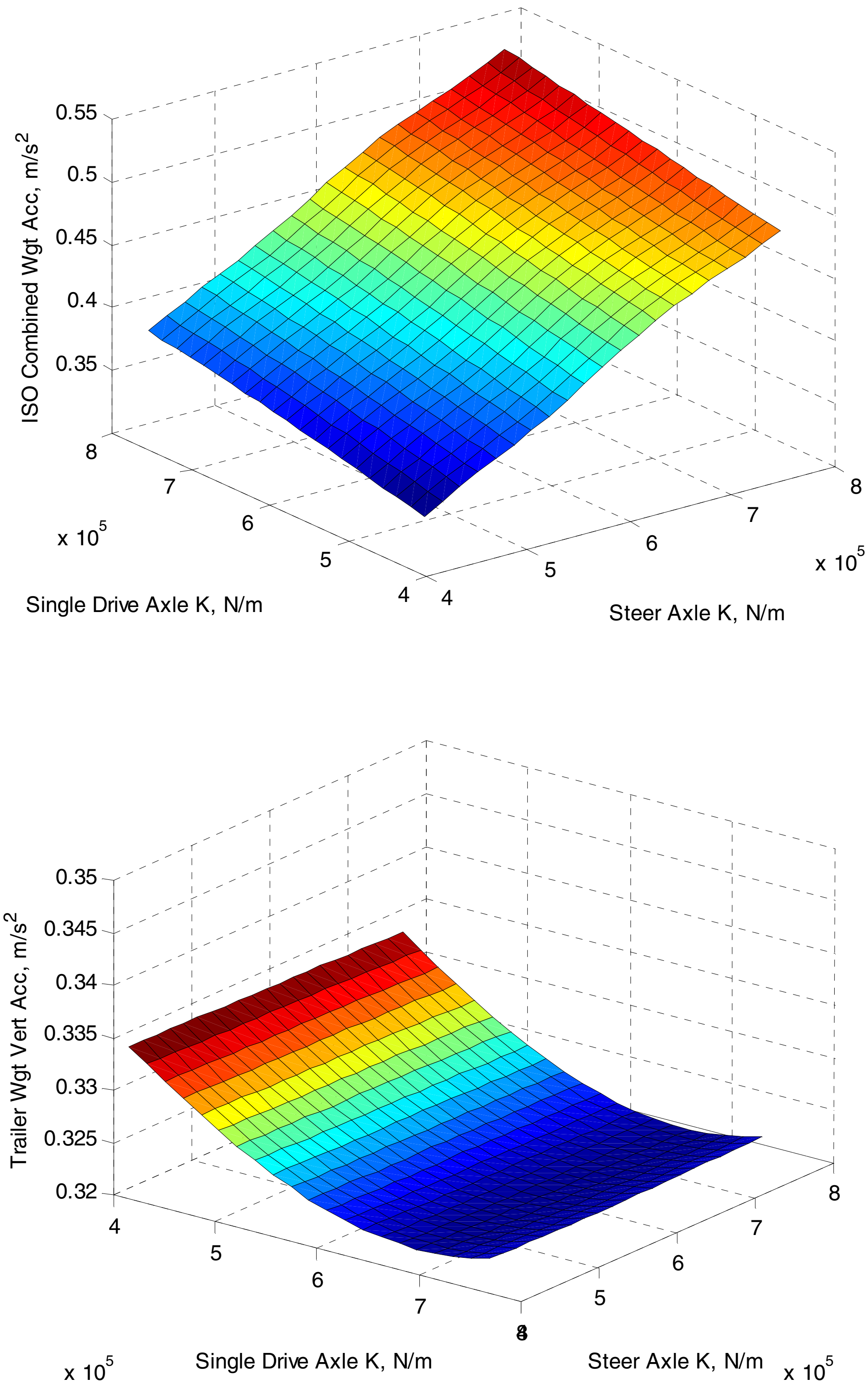


Figure 4.1: Axle Suspension Stiffness Parameter Variation

When reducing stiffness values for the tractor axles, it is important that the vehicle ride height not be affected to an extent that it may become detrimental to its performance. Table 4.4 shows the ride height reduction experienced by the tractor semi-trailer in a loaded condition.

Table 4.4: Ride Height Reduction with Adjusted Suspension Parameters

Axle	**Stiffness Value (N/m)**	**Ride Height Reduction (in)**
Steer Axle	406910	1.29
#1 Drive Axle	410830	1.86
#2 Drive Axle	410830	1.99
#1 Trailer Axle	1000000	0.17
#2 Trailer Axle	1000000	-0.15

The maximum reduction in ride height was found to be approximately two inches on the second drive axle for the loaded vehicle.

Adjusting the suspension stiffness affects the static axle loads, so it is important to analyze these values to ensure that the vehicle stays within the acceptable limits. A summarization of the federal government regulated axle load limits was obtained through and e-mail correspondence between Mrs. Sue Nelson, Manager of Truck Tire Innovation at Michelin Americas R&D Corporation, and Dr. E. Harry Law [21]. The correspondence is shown below.

> "Current standard limits for Class 8 6x4 tractors (1 steer axle, tandem drive axle) with a tandem axle trailer are:
>
> Steer axle: 12000 lb
> Drive tandem: 17000 lb/axle (34000 total all drive)
> Trailer tandem: 17000 lb/axle (34000 total all trailer)
>
> Loads are the same for dual or single tire configurations."

The nominal vehicle used by Vaduri [3] Trangsrud [1] and in this thesis exceeds the limits set by the federal government. However, South Carolina regulations [22] state that a five axle vehicle may not exceed a gross weight of 90,000 lbs, and two-axle tandems may not carry a load greater than 40,000 lbs with the issue of a permit, so these regulations with be treated as the legal limits. Table 4.5 displays the loads seen by each of the axles in the nominal vehicle as well as the vehicle using the reduced tractor suspension stiffnesses. Both vehicles represented have the same gross vehicle weight (GVW) of 76788 lbs.

Table 4.5: Static Axle Loads with Adjusted Suspension Parameters

Vehicle Configuration	**Steer Axle Load (lbs)**	**#1 Drive Axle Load (lbs)**	**#2 Drive Axle Load (lbs)**	**#1 Trailer Axle Load (lbs)**	**#1 Trailer Axle Load (lbs)**
Nominal Vehicle	9964	14704	15768	18619	17733
		30472		36352	
Adjusted Tractor Axle Suspension Parameters	9963	14667	15722	19312	17125
		30389		36437	
SC Legal Load Limits with Permit	20000	40000		40000	
Federal Legal Load Limits	12000	34000		34000	

Adjusting the suspension values had very little effect on the loads experienced by the steer and drive axles, but did have some significant effect on the trailer axle loads. However, these loads are still within the acceptable range allowed by South Carolina regulations, so the changes in axle loads were determined not to be a factor in the suspension parameter variation process.

The parameter variation program allows the user to formulate a cost function penalty. This penalty weighs the combined RMS acceleration of the driver and the vertical RMS acceleration of the trailer CG using weights assigned by the user. Equation 4.1 shows the function used to calculate the penalty function.

$$JPenalty = K1 * (aV / aV_0) + K2 * (a0\_V\_tlr / a0\_V\_tlr_0) \tag{4.29}$$

Where K1 = Driver Comfort Weight $\left(0 \le K1 \le 1\right)$

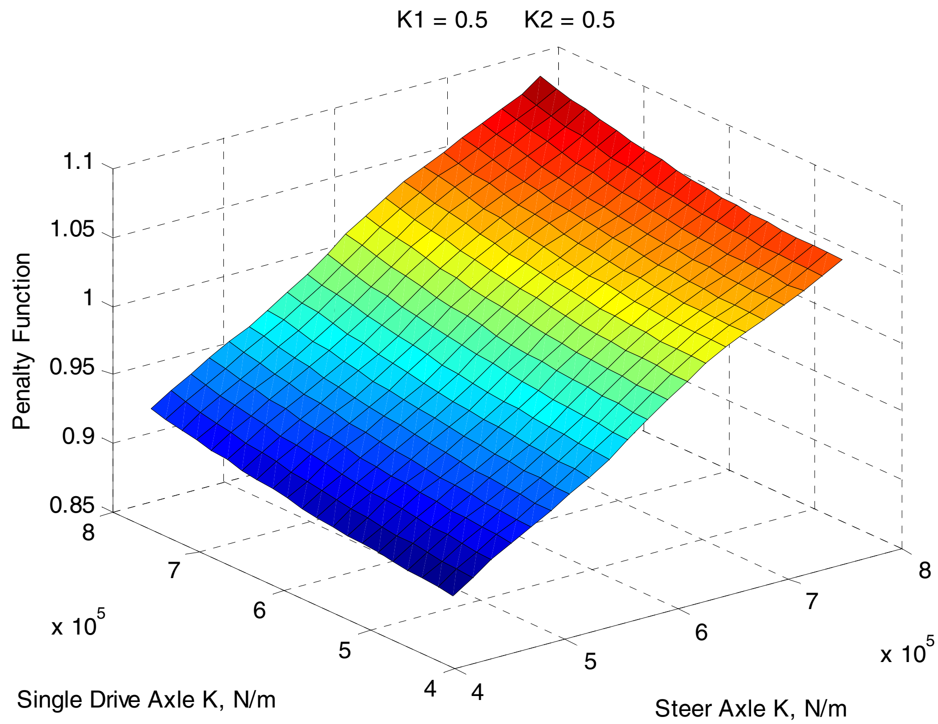
K2 = Trailer Acceleration Weight $\left(0 \le K2 \le 1\right)$

aV = Driver Combined ISO RMS Acceleration (m/s^2)

aV_0 = Driver Combined ISO RMS Acceleration Nominal Value (m/s^2)

a0_V_tlr = Weighted Trailer Vertical RMS Acceleration (m/s^2)

$a0\_V\_tlr_0$ = Weighted Trailer Vertical RMS Acceleration Nominal Value (m/s^2)



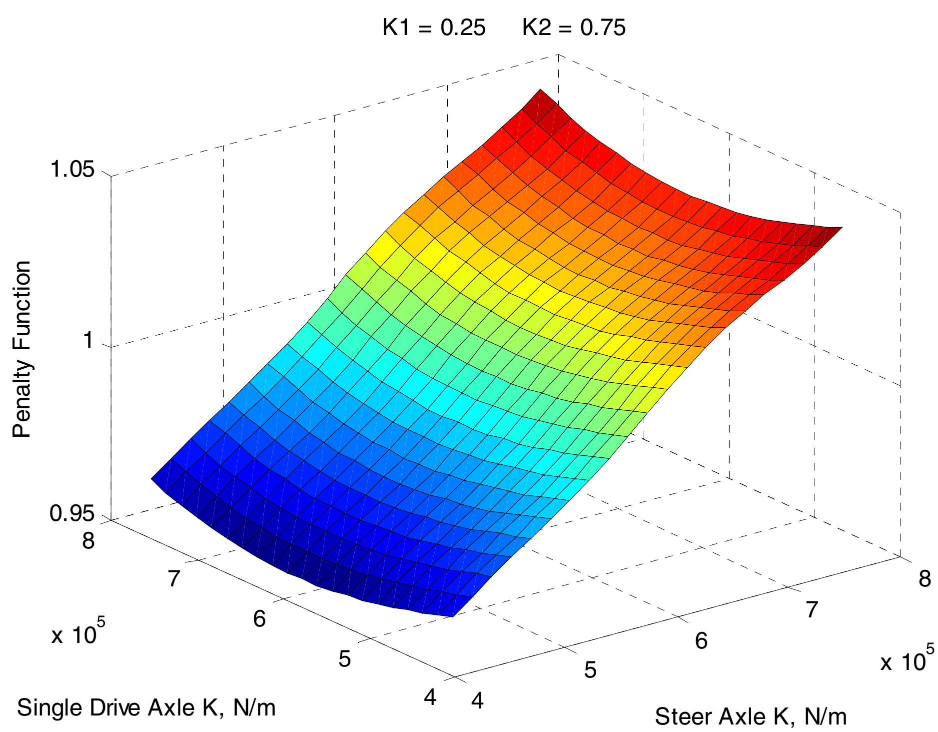
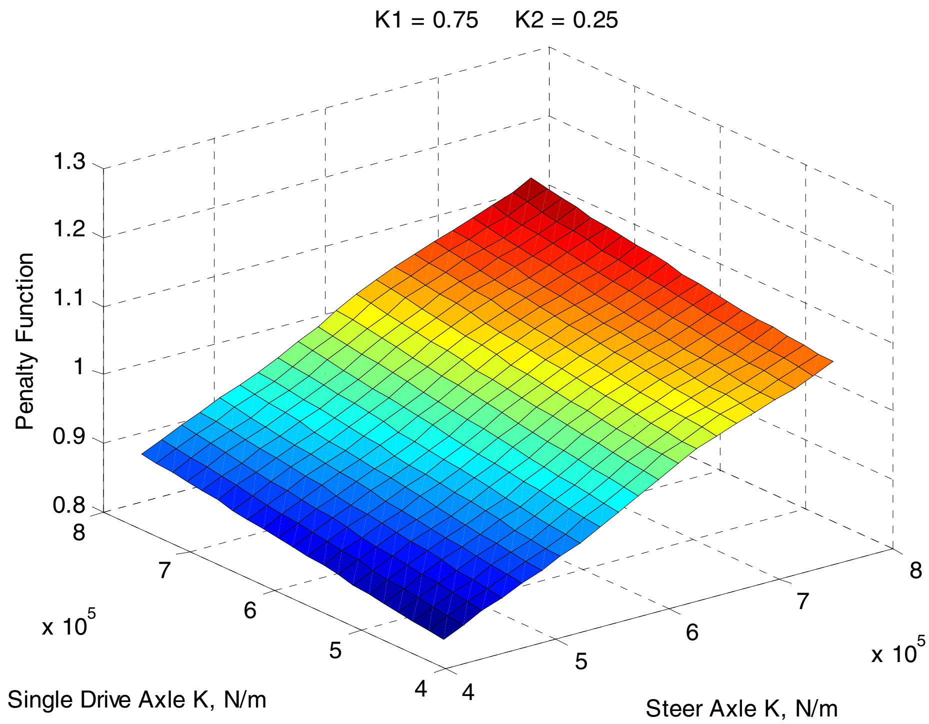


Figure 4.2: J Penalty Formulation for Axle Suspension Stiffness



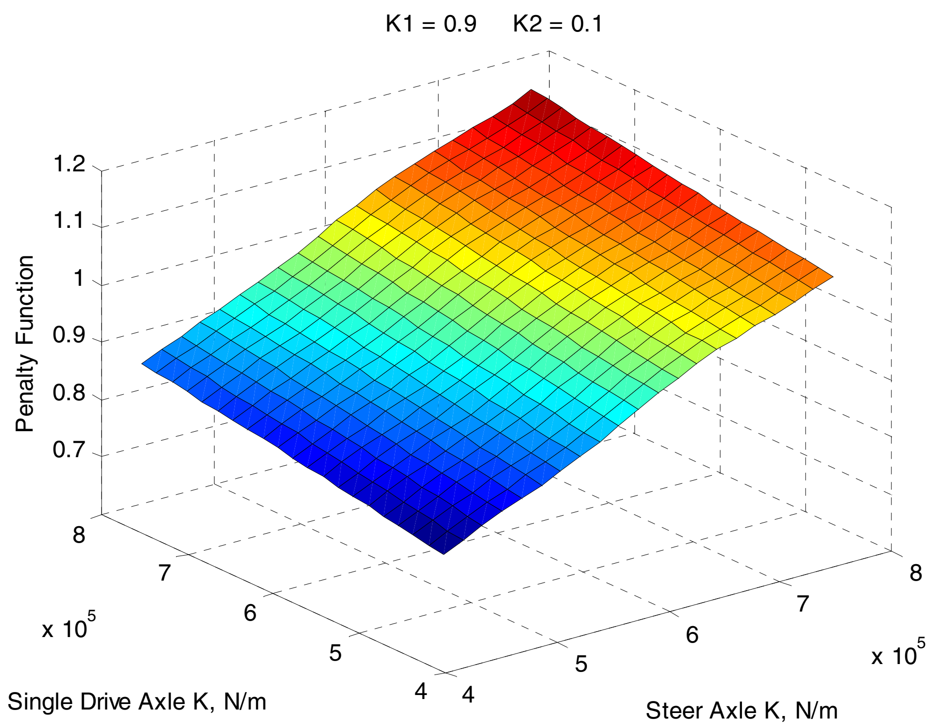


Figure 4.3: J Penalty Formulation for Axle Suspension Stiffness

The importance of the ride comfort of the driver is denoted by the value K1 and the trailer CG by K2. Both values should combine to a value of one. For example, if the driver ride comfort is much more important than the vertical acceleration of the trailer, then the user may assign a value of 0.8 to K1 and 0.2 to K2. The values could be reversed if the opposite were true. Figures 4.2 and 4.3 show the J penalty function results with varying values for K1 and K2.

The plots in Figure 4.2 suggest that when driver ride comfort and trailer CG vertical acceleration are of equal importance, then the trend of decreasing the axle stiffnesses to improve ride comfort performance remains steady throughout all four cases. However, when great importance is placed on the vertical acceleration of the trailer CG (i.e., K1=0.25, K2=0.75), then there is a minimum point along the stiffness range for the drive axles at which the best performance can be achieved. In this case, the trend of J with decreasing stiffness of the steer axle remains the same as trends seen when great importance is placed on driver ride comfort.

It is also interesting to note that when great importance is placed on driver ride comfort, as in Figure 4.3 (K1=0.9, K2=0.1), it is possible to obtain a J penalty function as low as 0.77. Table 4.6 shows the results of the J penalty formulation with varying K1 and K2 values. Also in the table are the corresponding axle stiffness values and the percent improvement in the ISO weighted acceleration values as compared to the nominal values.

Table 4.6: J Penalty Formulation Results with Adjusted Axle Suspension Stiffness

	Weighting Factor Values			
	K1=0.5 K2=0.5	**K1=0.25 K2=0.75**	**K1=0.75 K2=0.25**	**K1=0.9 K2=0.1**
Minimum J Penalty Value	0.900	0.957	0.833	0.793
Steer Axle Stiffness (N/m)	406910	406910	406910	406910
Drive Axle Stiffness (N/m)	410830	622114	410830	410830
ISO Combined Driver Weighted Acc. (m/s^2)	0.34	0.37	0.34	0.34
% Improvement Relative to Nominal Value	+ 24.4	+ 17.8	+ 24.4	+ 24.4
ISO Vertical Trailer Weighted Acc. (m/s^2)	0.33	0.32	0.33	0.33
% Improvement Relative to Nominal Value	- 3.1	0.0	- 3.1	- 3.1

The data in Tble 4.6 shows that as long as the ride comfort of the driver is weighted as least as heavily as the vertical acceleration of the trailer CG (i.e. $K1 \geq 0.5$), then the lowest value of the RMS driver acceleration will be the same for all K1 and K2 scenarios. The corresponding axle stiffnesses will be the minimum allowable for the axles. However, when the majority of the emphasis is placed on the vertical acceleration of the trailer CG rather than the ride comfort of the driver, the results indicate a significantly higher value for the drive axle stiffness is required. However, by looking at the improvements in the weighted acceleration values, it is evident that only a very minor decrease in the trailer weighted vertical acceleration is possible, even when it is weighed most heavily. The driver ride comfort, which can improve by as much as 24.4%, is definitely the area of greater focus with this parameter variation program.

Axle Suspension Damping

As stated earlier, the suspension damping was varied independently of the suspension stiffness. Figure 4.4 shows the results obtained by the suspension damping parameter variation program. Since the damping values have no effect on the vehicle ride height or axle loads, only the effect of the damping values on the weighted accelerations were studied.

Figure 4.4 suggests that larger damping values for the steer and drive axles result in a lower combined weighted acceleration for the driver. As with the stiffness parameter variation, the plot shows that the acceleration has a much greater sensitivity to the damping constant of the steer axle than the drive axles. There is a 12.5% reduction in the RMS weighted acceleration of the driver at the highest damping values. Sensitivity to the damping constants of the drive axles is much lower, and the plot shows only a 3% reduction in combined weighted acceleration over the range of values.

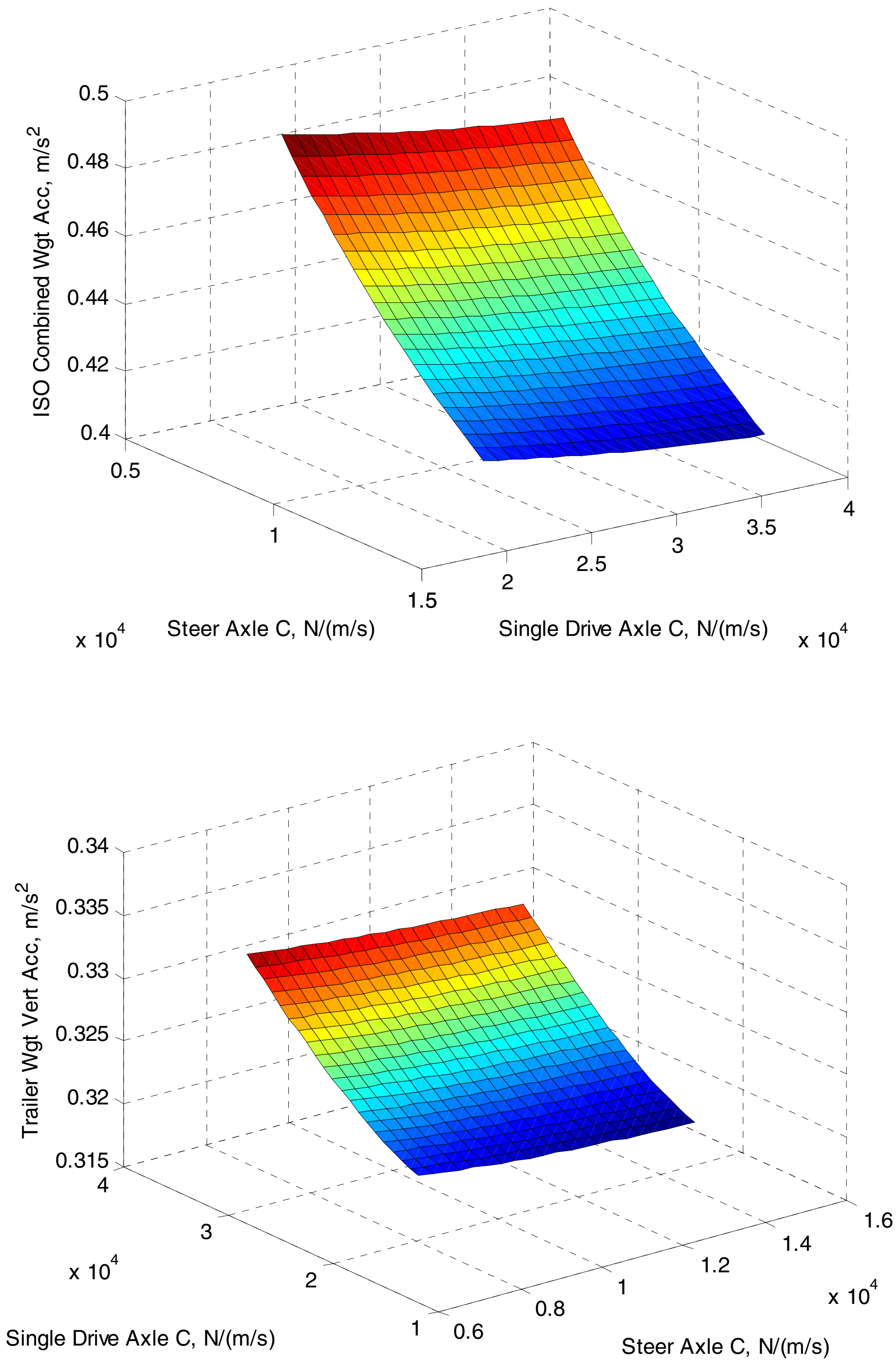


Figure 4.4: Axle Suspension Damping Parameter Variation

Like the stiffness parameter variation, adjusting the damping constants does not have a significant effect on the weighted vertical acceleration of the trailer. Figure 4.4 shows only a 3% change in the weighted RMS vertical acceleration over the entire range of damping constants, and is therefore determined to be insignificant and not a factor in the damping constant variation.

Figures 4.5 and 4.6 display the J penalty formulation plots for varying K1 and K2 values. When the driver ride comfort and trailer vertical acceleration are weighed equally (K1=K2=0.5), there is a minimum J value in the middle of the range of drive axle damping values. However, overall, the results are not very sensitive to drive axle damping. Second, when greater importance is placed on the trailer CG vertical acceleration (K1=0.25, K2=0.75), the lowest J value shifts to maximum steer axle damping and minimum drive axle damping. Finally, when the weighting factors shift the other way toward driver ride comfort (K1=0.75, K2=0.15 & K1=0.9, K2=0.1), the lowest J value shifts to maximum steer axle damping and maximum drive axle damping. Table 4.7 shows the results with minimum J penalty values, damping values, weighted accelerations and their percent improvements relative to nominal values.

Table 4.7: J Penalty Formulation Results with Adjusted Axle Suspension Damping

	Weighting Factor Values			
	K1=0.5 K2=0.5	**K1=0.25 K2=0.75**	**K1=0.75 K2=0.25**	**K1=0.9 K2=0.1**
Minimum J Penalty Value	0.971	0.979	0.952	0.937
Steer Axle Damping (N/(m/s))	14651	14651	14651	14651
Drive Axle Damping (N/(m/s))	26675	20075	35750	35750
ISO Combined Driver Weighted Acc. (m/s^2)	0.42	0.43	0.42	0.42
% Improvement Relative to Nominal Value	+ 6.7	+ 4.4	+ 6.7	+ 6.7
ISO Vertical Trailer Weighted Acc. (m/s^2)	0.32	0.32	0.33	0.33
% Improvement Relative to Nominal Value	0.0	0.0	-3.1	-3.1

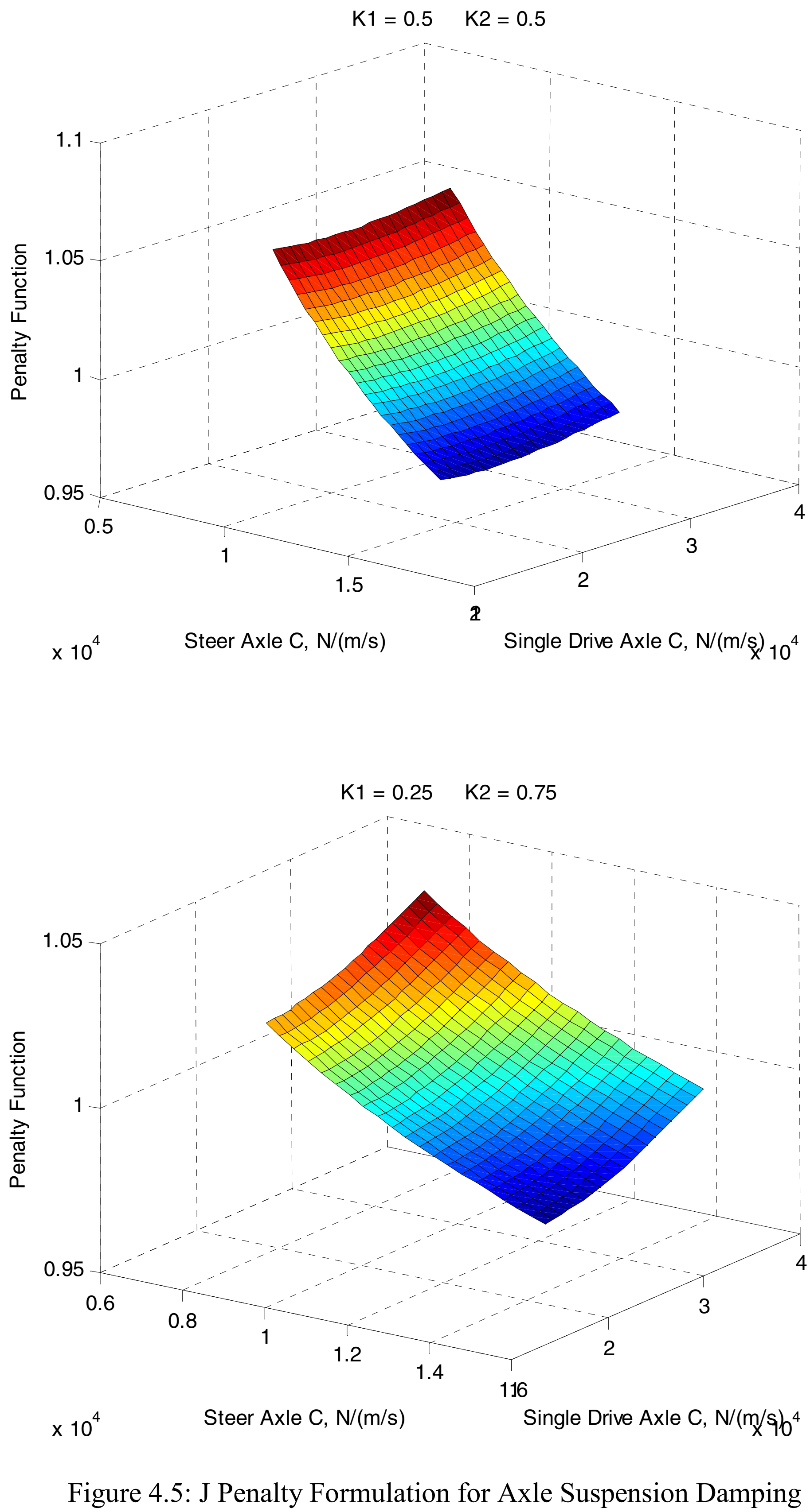


Figure 4.5: J Penalty Formulation for Axle Suspension Damping

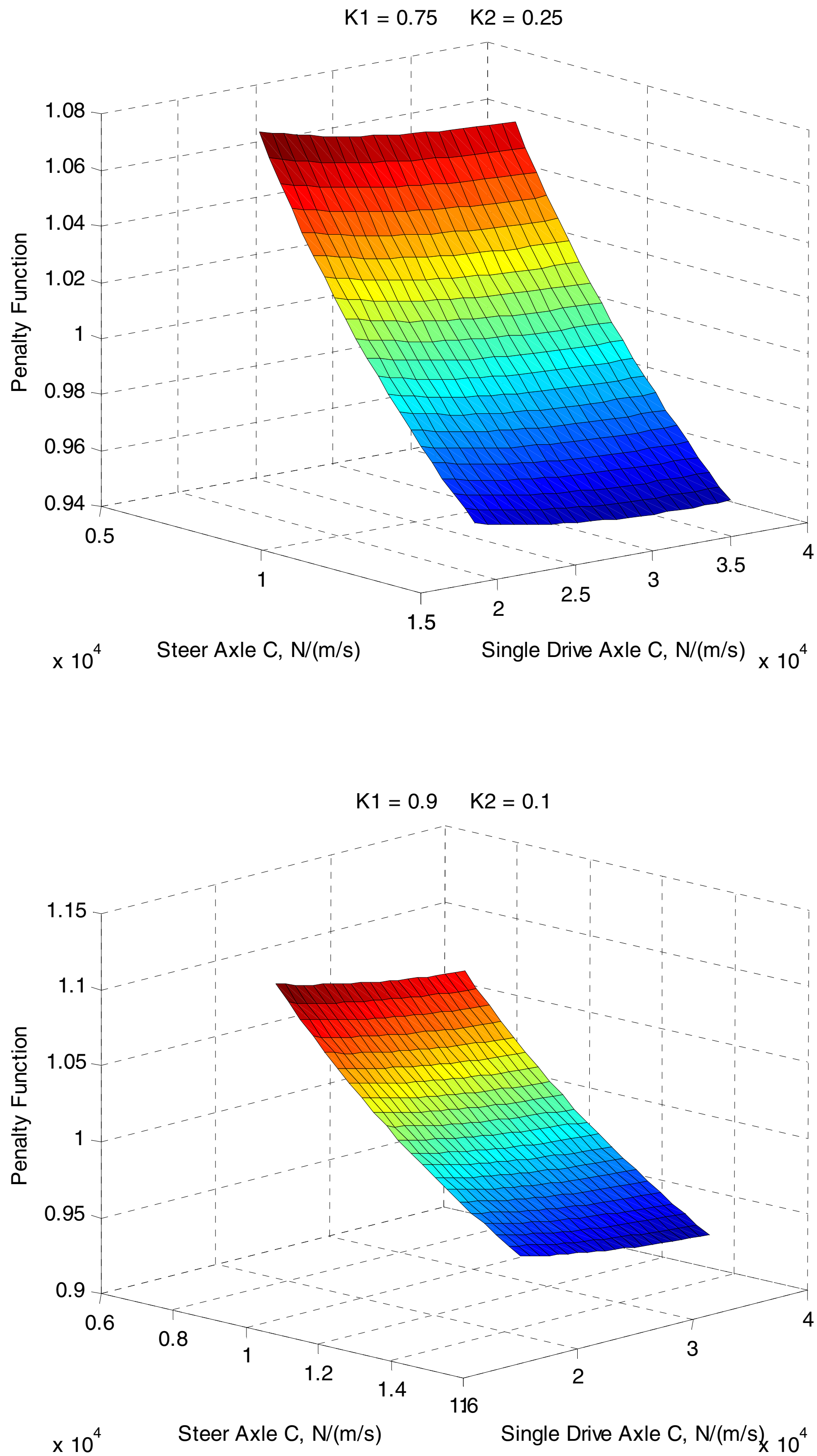


Figure 4.6: J Penalty Formulation for Axle Suspension Damping

The results in Table 4.7 show that when even greater importance is placed on the trailer vertical CG acceleration (K1=0.25, K2=0.75), improvement is seen in the driver ride comfort relative to the nominal value. When there is equal importance placed on both (K1=0.5, K2=0.5), a 4.4% improvement is seen in the driver ride comfort relative to the nominal value and there is no visible change in trailer vertical CG acceleration. However, when greater importance is placed on the driver ride comfort (K1=0.75, K2=0.2 and K1=0.9, K2=0.1), a maximum improvement of 6.7% can be obtained in the driver ride comfort but the trailer vertical CG acceleration is increased by 3.1% relative to their nominal values.

Table 4.8 shows the stiffness and damping values chosen for best ride performance. These values were chosen factoring in their effect on the combined RMS weighted acceleration of the driver and the trailer CG. Also considered are the effects of the adjusted stiffness values on the static ride heights (Table 4.4) of the tractor semi-trailer and the static axle loads (Table 4.5) on each of the axles.

Table 4.8: Nominal and Adjusted Suspension Stiffness and Damping Constants
60 mph, Smooth Highway

Axle	Nominal Stiffness Constant (N/m)	Adjusted Stiffness Constant (N/m)	Nominal Damping Constant (N/(m/s))	Adjusted Damping Constant (N/(m/s))
Steer Axle	581300	406910	11270	14651
#1 Drive Axle	586900	410830	27500	35750
#2 Drive Axle	586900	410830	27500	35750
#1 Trailer Axle	1000000	1000000	70000	70000
#2 Trailer Axle	1000000	1000000	70000	70000

Axle Suspension with Adjusted Stiffness and Damping Values

Table 4.9 shows the vertical, longitudinal, and combined weighted RMS accelerations of the driver with the nominal and adjusted values and the percent improvement relative to the nominal values.

Table 4.9: Combined Weighted Acceleration with Adjusted Suspension Stiffness and Damping
60 mph, Smooth Highway

Vehicle Suspension Configuration	**Vertical Weighted Acceleration (m/s^2)**	**Longitudinal Weighted Acceleration (m/s^2)**	**Combined Weighted Acceleration (m/s^2)**	**ISO Comfort Level**
Nominal Parameters	0.28	0.35	0.45	A Little Uncomfortable
Adjusted Parameters	0.22	0.24	0.32	A Little Uncomfortable
% Improvement Relative to Nominal Value	+ 21.4	+ 31.4	+ 28.9	

The results indicate that the greatest area of improvement lies in the longitudinal acceleration of the driver. The longitudinal weighted acceleration of the driver showed a 31.4% improvement compared to a 21.4% improvement in the vertical weighted acceleration. The combined value showed a 28.9% improvement.

It is important to analyze not only the total improvement in weighted acceleration, but also where these improvements occur in the frequency range. Figure 4.7 shows the vertical and longitudinal weighted RMS accelerations along with the International Standards Organization's (ISO) specified 2.5 and 8 hour comfort boundaries [5:1974]. These boundaries represent the maximum level of acceleration that the vehicle operator can tolerate for the specified amount of time. On the plots are the curves using nominal parameters and the results when using the adjusted parameters.

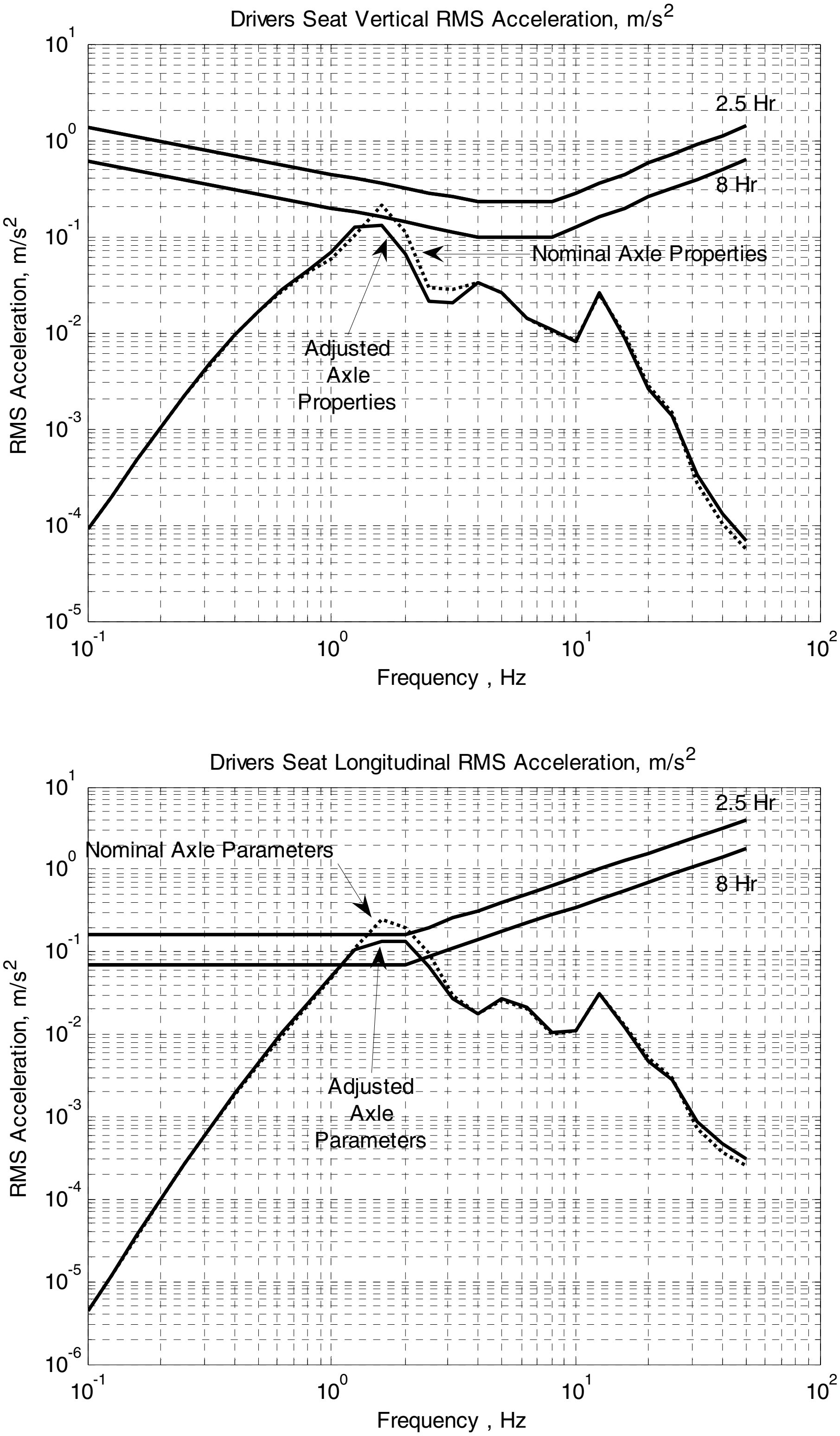


Figure 4.7: Effect of Adjusted Suspension Stiffness and Damping on Driver Ride Comfort

Both plots in Figure 4.7 show that the greatest improvements occur low in the frequency range. The maximum improvements for both the vertical and longitudinal weighted RMS accelerations occur between 1.5 and 4 Hertz, which correspond to the range for vehicle body modes.

Table 4.10 shows the weighted RMS accelerations at the frequencies corresponding to body modes of the nominal tractor semi-trailer (Table 4.3) and their corresponding percent improvements. On the table are the values when nominal parameters and adjusted parameters are input into the program. As expected, the greatest improvements occur at 1.6 and 2 Hz.

Table 4.10: Weighted RMS Accelerations at Body Mode Frequencies
for Axle Suspension Parameter Variation
60 mph, Smooth Highway

Vertical Weighted RMS Acceleration (m/s^2) Improvement

Frequency (Hz)	**Nominal Parameters**	**Adjusted Parameters**	**% Improvement**
1.60	0.2070	0.1308	+ 36.8
2.00	0.1089	0.0661	+ 39.3
2.50	0.0289	0.0213	+ 26.2
3.15	0.0279	0.0202	+ 27.5

Longitudinal Weighted RMS Acceleration (m/s^2) Improvement

Frequency (Hz)	**Nominal Parameters**	**Adjusted Parameters**	**% Improvement**
1.60	0.2431	0.1323	+ 45.6
2.00	0.1953	0.1310	+ 32.9
2.50	0.0952	0.0674	+ 29.2
3.15	0.0300	0.0275	+ 8.2

Tire Parameter Variation

Tire Stiffness

Tire stiffness and damping characteristics were varied using opt_tireK_freq.m and opt_tireC_freq.m which are described in Appendices I and J. The tractor is equipped with wide-base singles, which is reflected in the axle mass. The stiffness and damping values were varied individually, and the individual results analyzed to obtain the best set of parameters. First, the tire stiffness was varied. Figure 4.8 shows the weighted accelerations of the driver and trailer CG varied against the steer tire properties and single drive tire properties. The stiffness values for each of the drive tires are assumed to be the same, so they are varied together in the program.

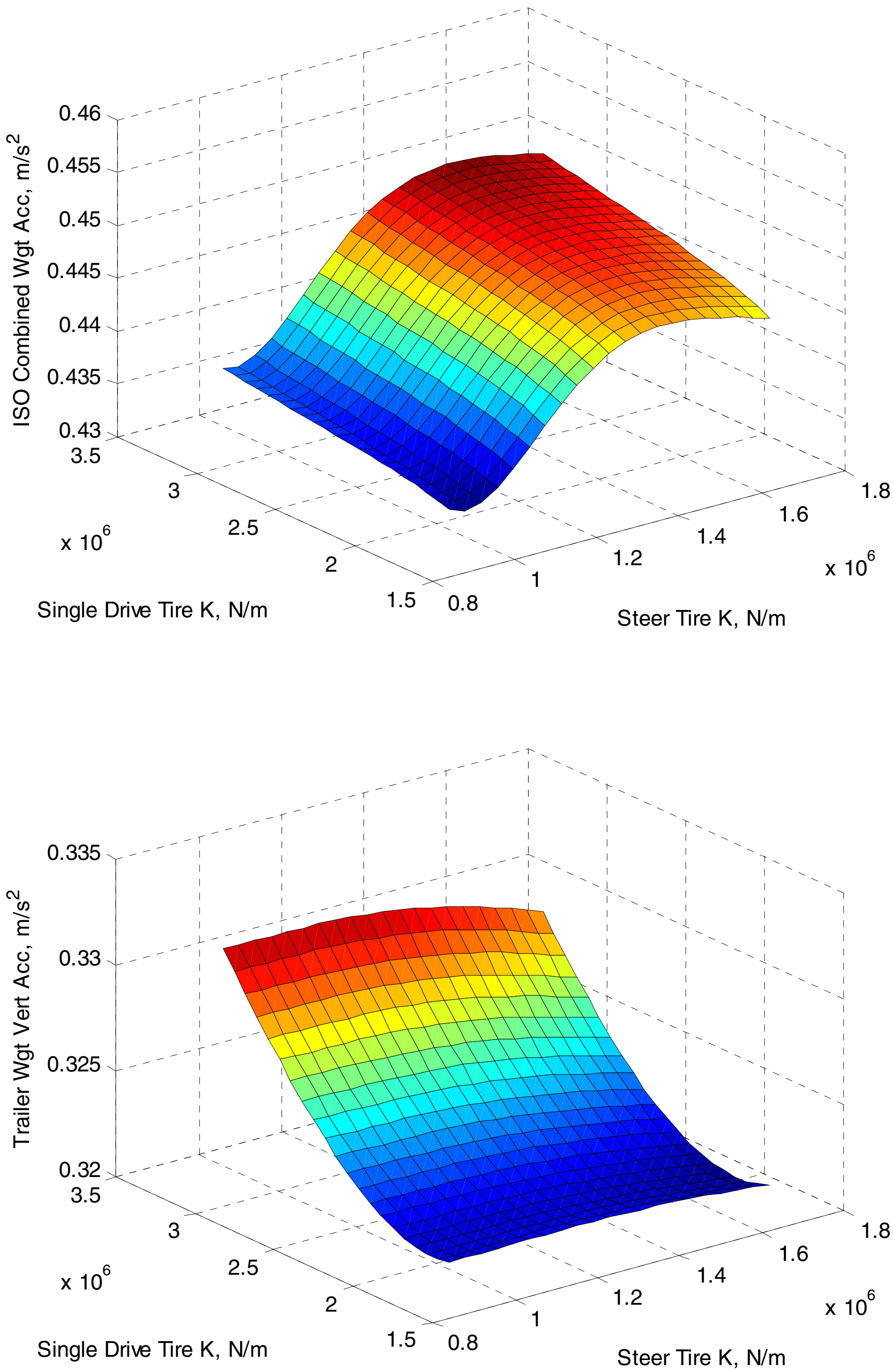


Figure 4.8: Tire Stiffness Parameter Variation

The weighted acceleration of the driver shows the greatest sensitivity to the steer tire stiffness. Over the range of steer tire stiffnesses input into the program, there is a 3% reduction in the total weighted acceleration.

Figure 4.8 suggests that decreasing the stiffness of the drive tires resulted in a lower total weighted acceleration. The reduced drive tire stiffnesses caused approximately a 0.3% reduction in the combined weighted RMS acceleration of the driver. Also, the vertical weighted RMS acceleration of the trailer is relatively insensitive to variations in the steer tire stiffness. However, the adjustment of the drive axle tire stiffness lowered the vertical weighted RMS acceleration at the trailer CG by 2.1%.

As with the suspension parameter variation, it is crucial that the vehicle ride height not be affected to an extent that changes are detrimental to the vehicle performance when altering the stiffness values of the tractor tires. Improved ride performance corresponds to reduced stiffness values for the steer as well as for the drive tires, so the changes in ride height were calculated at these values. Table 4.11 shows the ride height changes experienced by the tractor semi-trailer in a loaded condition.

Table 4.11: Ride Height Reduction with Adjusted Tire Parameters

Axle	**Tire Stiffness Value Per-Tire (kN/m)**	**Ride Height Reduction (in)**
Steer Axle	472.68	0.50
#1 Drive Axle	835.87	0.46
#2 Drive Axle	835.87	0.50
#1 Trailer Axle	1,194.1	0.00
#2 Trailer Axle	1,194.1	0.00

The maximum reduction in ride height was found to be approximately half of an inch on the steer axle and drive axles when the vehicle is loaded. This value represents a small percentage of the total ride height.

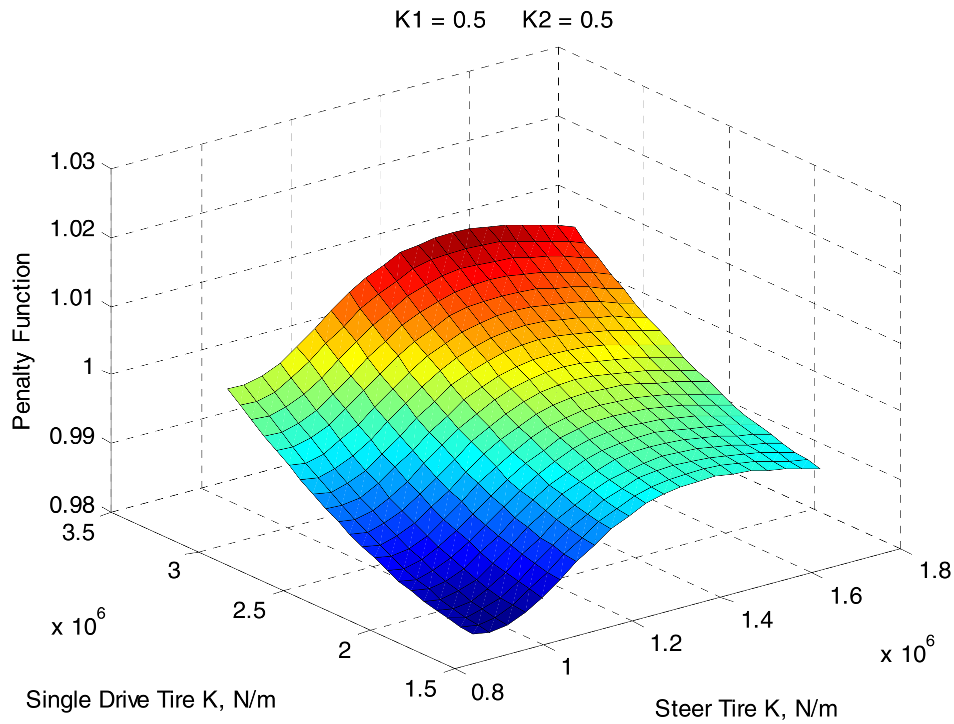
Table 4.12 displays the static axle loads in the nominal vehicle as well as the vehicle using the adjusted tractor tire stiffnesses. Both vehicles represented have a fully laden trailer.

Table 4.12: Static Axle Loads with Adjusted Tire Stiffness Values

Vehicle Configuration	**Steer Axle Load (lbs)**	**#1 Drive Axle Load (lbs)**	**#2 Drive Axle Load (lbs)**	**#1 Trailer Axle Load (lbs)**	**#1 Trailer Axle Load (lbs)**
Nominal Vehicle	9964	14704	15768	18619	17733
		30472		36352	
Adjusted Tractor Tire Parameters	9964	14704	15768	18619	17733
		30472		36352	
SC Legal Load Limits with Permit	20000	40000		40000	
Federal Legal Load Limits	12000	34000		34000	

Adjusting the tire stiffness values proved to have no discernable effect on the static axle loads. Since the load limits were not exceeded by the nominal vehicle, it was determined that the variation of the tire stiffnesses does not generate any risk for exceeding these limits.

J penalty plots were formed to study the parameter variation trends of the tire parameters when different weights were placed on drive ride comfort and trailer vertical CG acceleration. Figure 4.9 shows the J penalty results with varying K1 and K2 values.



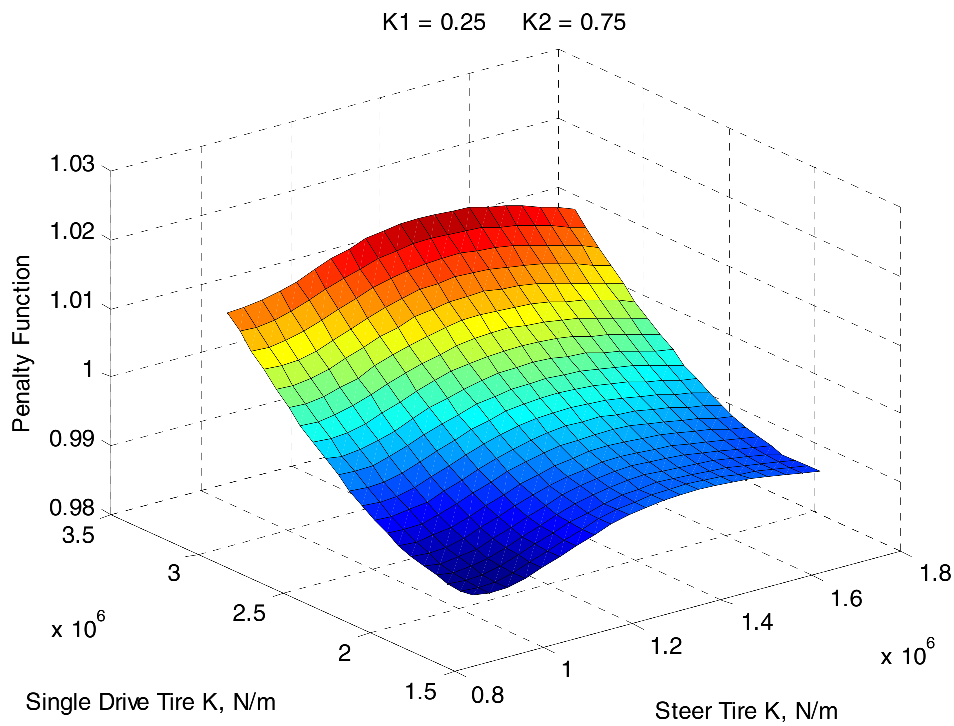


Figure 4.9: J Penalty Formulation for Tire Stiffness

The plots in Figure 4.9 suggest that when K1=0.25 and K2=0.75, there is a greater sensitivity to the drive tire stiffness than for the steer axle. However, as the driver ride comfort becomes more important (i.e., as K1 increases), the J penalty becomes less sensitive to the drive axle tire stiffness and remains highly sensitive to the steer tire stiffness. Table 4.13 shows the results of the J penalty formulation with minimum J penalty values, stiffness values, weighted accelerations and the percent improvements relative to nominal values.

Table 4.13: J Penalty Formulation Results with Adjusted Tire Stiffness Values

	Weighting Factor Values			
	K1=0.5 K2=0.5	**K1=0.25 K2=0.75**	**K1=0.75 K2=0.25**	**K1=0.9 K2=0.1**
Minimum J Penalty Value	0.981	0.987	0.975	0.971
Steer Tire Stiffness (N/m)	945350	945350	945350	945350
Drive Tire Stiffness (N/m)	1671741	1743387	1671741	1671741
ISO Combined Driver Weighted RMS Acc. (m/s^2)	0.43	0.43	0.43	0.43
% Improvement Relative to Nominal Value	+ 4.4	+ 4.4	+ 4.4	+ 4.4
ISO Vertical Trailer Weighted RMS Acc. (m/s^2)	0.32	0.32	0.32	0.32
% Improvement Relative to Nominal Vehicle	0.0	0.0	0.0	0.0

Table 4.13 suggests that whether driver ride comfort or trailer vertical CG acceleration is weighed heavier, the best steer tire stiffness remains at 945.35 kN/m and results in a 4.4% improvement in driver ride comfort.

Tire Damping

Results indicate that tire damping variations show almost no measurable effect on the total weighted acceleration of the driver, or the weighted vertical acceleration at the trailer CG. The combined ISO weighted acceleration of the driver showed only a 0.0006 m/s^2 difference and the vertical weighted acceleration of the trailer CG showed no difference over the entire range (± 30% about the nominal value) of tire damping values.

Table 4.14 shows the stiffness and damping values for each of the tractor and trailer tires that result in the best ride performance. These values were chosen by factoring in their effect on the combined driver weighted RMS acceleration and the vertical weighted RMS acceleration of the trailer CG. The corresponding static ride heights and the static axle loads are given in Tables 4.11 and 4.12.

Table 4.14: Nominal and Adjusted Tire Stiffness and Damping Constants

Axle	**Nominal Stiffness Constant (N/m)**	**Adjusted Stiffness Constant (N/m)**	**Nominal Damping Constant (N/(m/s))**	**Adjusted Damping Constant (N/(m/s))**
Steer Axle	1295000	945350	517	517
#1 Drive Axle	2388200	1671741	648.3	648.3
#2 Drive Axle	2388200	1671741	648.3	648.3
#1 Trailer Axle	2388200	2388200	648.3	648.3
#2 Trailer Axle	2388200	2388200	648.3	648.3

Ride Performance with Adjusted Tire Stiffness and Damping Values

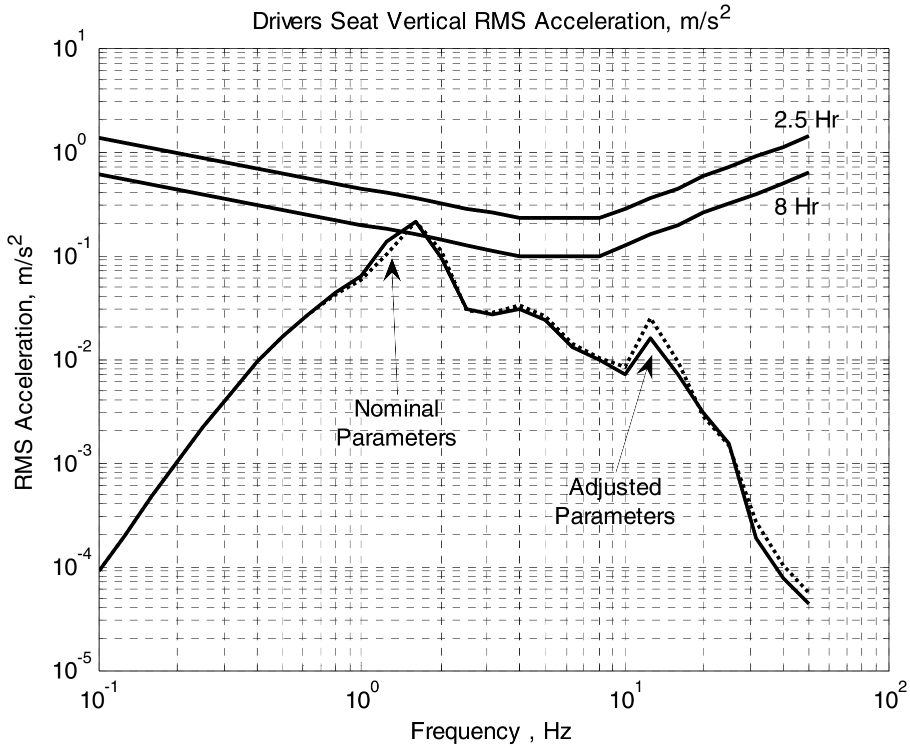
Combining the optimized tire stiffness and damping constants results in a notable improvement of the combined weighted acceleration of the driver. Table 4.15 shows the vertical, longitudinal, and combined weighted accelerations of the driver with the nominal and adjusted values and the percent improvement relative to the nominal values.

Table 4.15: Combined Weighted RMS Driver Acceleration for Tire Parameter Variation

Vehicle Tire Configuration	**Vertical Weighted Acceleration (m/s^2)**	**Longitudinal Weighted Acceleration (m/s^2)**	**Combined Weighted Acceleration (m/s^2)**	**ISO Comfort Level**
Nominal Parameters	0.28	0.35	0.45	A Little Uncomfortable
Optimized Parameters	0.29	0.33	0.43	A Little Uncomfortable
% Improvement	- 3.6	+ 5.7	+ 4.4	

The values in Table 4.15 suggest that there is an increase in the vertical weighted RMS acceleration, but improvements in the longitudinal weighted RMS acceleration result in a 4.4% decrease in the combined weighted RMS acceleration.

It is important to analyze not only the total improvement in weighted RMS acceleration, but also where these improvements occur in the frequency range. Figure 4.10 shows the vertical and longitudinal weighted accelerations along with the International Standards Organization's (ISO) specified 2.5 and 8 hour comfort boundaries [5:1974]. On the plots are the curves using nominal parameters (dotted line) and those with adjusted parameters (solid line).



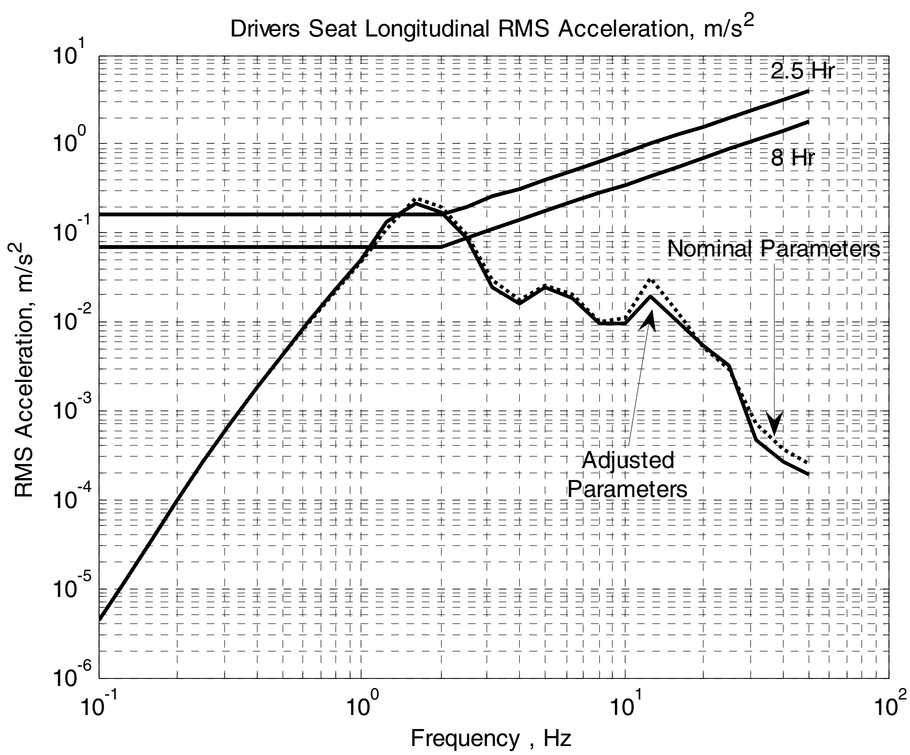


Figure 4.10: Effect of Adjusted Tire Parameters on Driver Ride Comfort

The vertical weighted ISO curve shows that the weighted RMS acceleration is slightly higher at 1.25 Hz. Also, Figure 4.11 shows lower vertical and longitudinal RMS accelerations between 10 and 16 Hz and lower accelerations at frequencies higher than 30 Hz. Wheel hop frequencies occur in the area around 12.5 Hz, so the improvements in this range can be attributed to the lower tire stiffness values.

Table 4.16 shows the weighted RMS accelerations at the frequencies that showed the largest differences and their corresponding percent improvements. On the table are the values when nominal parameters and adjusted parameters are input into the program.

Table 4.16: Weighted RMS Accelerations at Specific Frequencies
for Tire Parameter Variation
60 mph, Smooth Highway

Vertical Weighted RMS Acceleration (m/s^2) Improvement

Frequency (Hz)	**Nominal Parameters**	**Adjusted Parameters**	**% Improvement**
1.25	0.1038	0.1376	- 32.6
10.0	0.0084	0.0072	+ 14.3
12.5	0.0247	0.0159	+ 35.6

Longitudinal Weighted RMS Acceleration (m/s^2) Improvement

Frequency (Hz)	**Nominal Parameters**	**Adjusted Parameters**	**% Improvement**
12.5	0.0304	0.0197	+35.2

Ride Performance with Adjusted Suspension and Tire Parameters

Table 4.17 shows the vertical, longitudinal, and combined weighted RMS accelerations of the driver with the nominal and adjusted tire and suspension values and the percent improvement relative to the nominal values.

Table 4.17: Acceleration with Adjusted Suspension and Tire Parameters
60 mph, Smooth Highway

Vehicle Suspension Configuration	**Vertical Weighted RMS Acceleration (m/s^2)**	**Longitudinal Weighted RMS Acceleration (m/s^2)**	**Combined Weighted RMS Acceleration (m/s^2)**	**ISO Comfort Level**
Nominal Parameters	0.28	0.35	0.45	A Little Uncomfortable
Adjusted Parameters	0.23	0.23	0.32	A Little Uncomfortable
% Improvement	+ 17.9	+ 34.3	+ 28.9	

The results in Table 4.17 suggest that an overall 28.9% decrease in combined weighted RMS acceleration is possible when the vehicle is equipped with the adjusted suspension and tire parameters. The vertical weighted RMS acceleration is improved by 17.9%, which is down from 21.4% when using only the adjusted suspension parameters. This is due to the adjusted tire parameters, which were shown to cause a 3.6% increase in the vertical weighted RMS acceleration of the driver. However, the adjusted tire parameters provided for a

5.7% decrease in longitudinal weighted RMS acceleration, and when combined with the adjusted suspension parameters, resulted in an overall 34.3% reduction in the longitudinal weighted RMS acceleration.

Figure 4.11 shows the vertical and longitudinal weighted accelerations along with the International Standards Organization's (ISO) specified 2.5 and 8 hour comfort boundaries [5:1974]. The plots suggest that the greatest improvements in the vertical weighted ISO acceleration comes between 1.6 and 4 Hz. Also, there are much lower accelerations between 8 and 16 Hz. The area between 1.6 and 4 Hz represent improvements in the body mode frequencies caused by the adjusted suspension elements. The improvements between 8 and 10 Hz are representative of wheel hop frequencies, and can be attributed to both the adjusted suspension and tire elements. The only area in which there is no improvement occurs at 1.25 Hz. By observing Figure 4.10, it becomes evident that this can be attributed to the adjustment of the tire parameters.

Also seen in Figure 4.11 is the large improvement in the longitudinal weighted RMS acceleration. The greatest improvement occurs in the area between 1.25 and 4 Hz. Improvements in this area suggest that the adjusted suspension and tire parameters are reducing the amount of pitching experienced by the tractor. Also there are much lower acceleration values between 8 and 16 Hz, which can be attributed to the adjusted tire stiffness values.

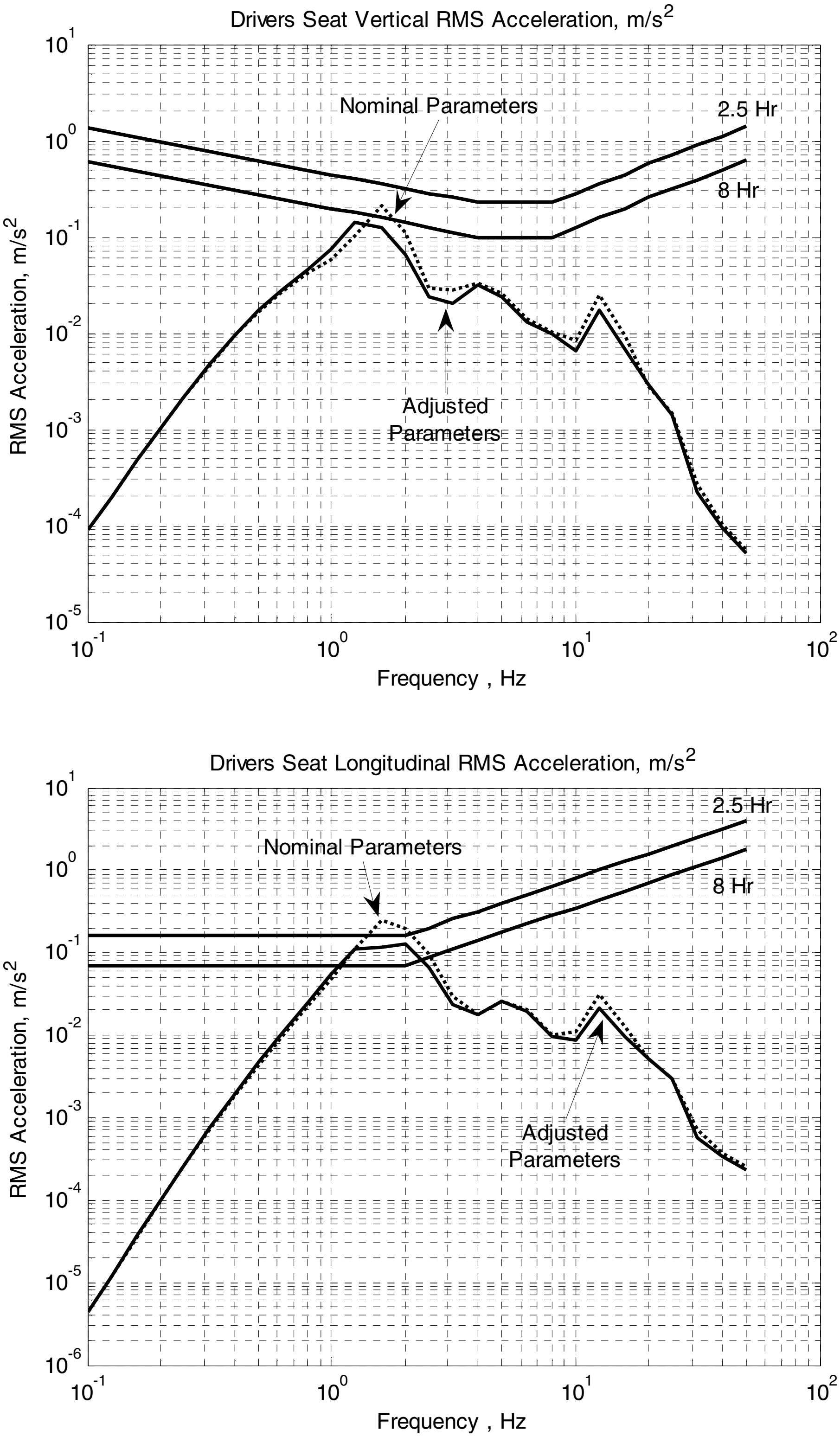


Figure 4.11: Effect of Adjusted Suspension and Tire Parameters on Driver Ride Comfort

Table 4.18 shows the weighted RMS accelerations at the frequencies that showed the largest differences and their corresponding percent improvements. On the table are the values when nominal parameters and adjusted parameters are input into the program.

Table 4.18: Weighted RMS Accelerations at Specific Frequencies
for Axle Suspension and Tire Parameter Variation
60 mph, Smooth Highway

Vertical Weighted RMS Acceleration (m/s^2) Improvement

Frequency (Hz)	**Nominal Parameters**	**Adjusted Parameters**	**% Improvement**
1.00	0.0585	0.0743	- 27.0
1.25	0.1038	0.1420	- 36.8
1.60	0.2070	0.1248	+ 39.7
2.00	0.1089	0.0642	+ 41.0
2.50	0.0289	0.0240	+ 17.0
3.15	0.0279	0.0204	+ 26.9
8.00	0.0102	0.0096	+ 5.9
10.0	0.0084	0.0065	+ 22.6
12.5	0.0247	0.0175	+ 29.1

Longitudinal Weighted RMS Acceleration (m/s^2) Improvement

Frequency (Hz)	**Nominal Parameters**	**Adjusted Parameters**	**% Improvement**
1.25	0.1098	0.1101	- 0.3
1.60	0.2431	0.1191	+ 51.3
2.00	0.1953	0.1253	+ 35.8
2.50	0.0952	0.0653	+ 31.4
3.15	0.0279	0.0237	+ 0.4
8.00	0.0102	0.0094	+ 7.8
10.0	0.0112	0.0086	+ 23.2
12.5	0.0304	0.0217	+ 28.6

Trailer Suspension and Beaming Parameter Variation

The same techniques used in the previous sections were used to study the effects of the trailer parameters on the ride characteristics of the tractor semi-trailer. The parameter variation was performed by the program opt_tlr_axlebeam.m which is described in Appendix K. Trailer axle stiffness was varied ±30% about the nominal value and the beaming frequency of the trailer frame was varied from 10 Hz to 30 Hz. Figure 4.12 shows the surface plots generated by the parameter variation program.

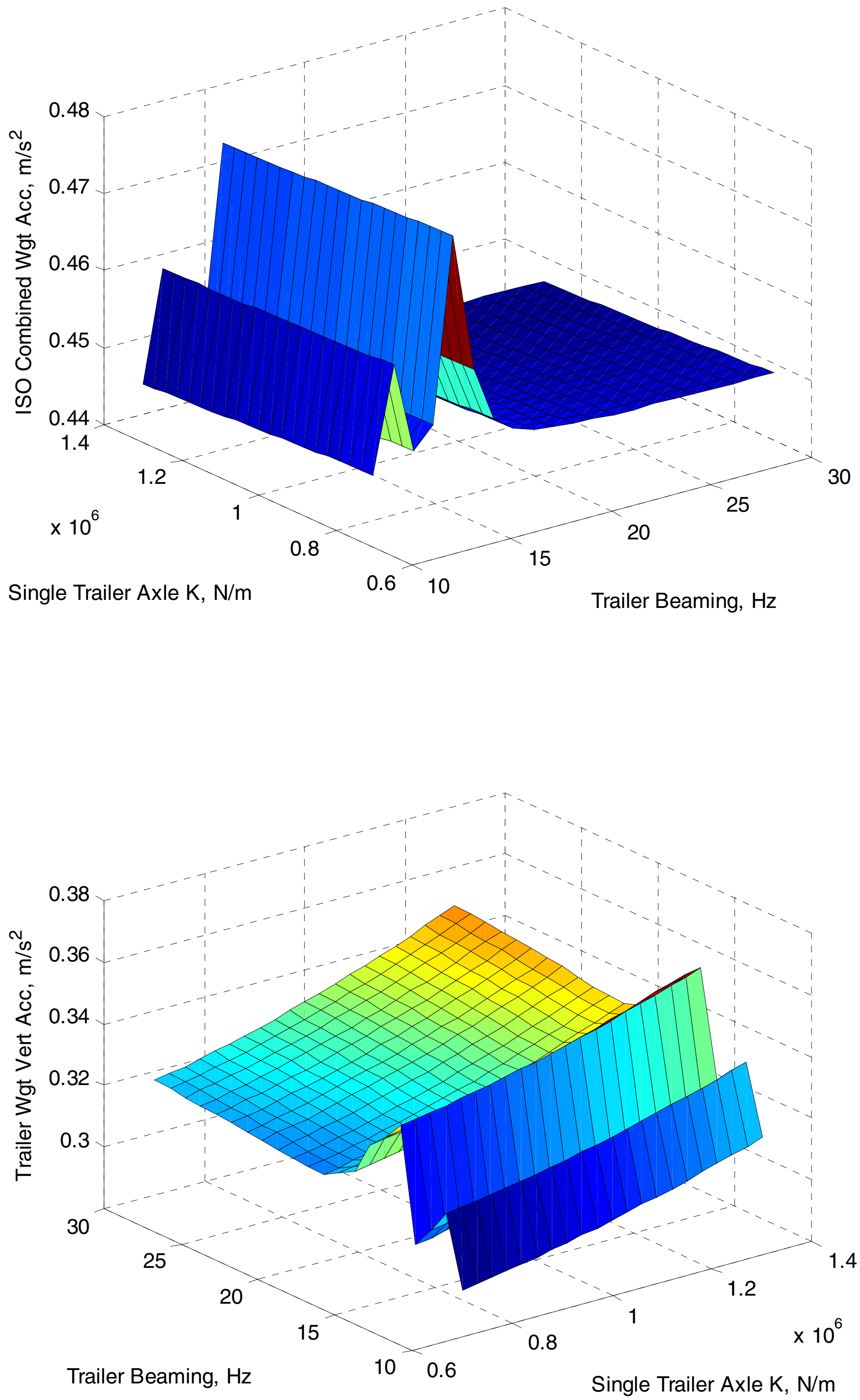


Figure 4.12: Trailer Suspension and Beaming Parameter Variation

Variations in the trailer frame beaming frequencies suggested that values at and above 18 Hz had very little effect on the acceleration of the trailer CG or the driver ride comfort. The same is true for the 12 and 13 Hz frequencies. The most notable features of Figure 4.12 were the relatively large acceleration values at frequencies of 11, 14, 15, 16, and 17 Hz. The spike at 11 Hz can be attributed to wheel hop modes. The largest spike, which occurred at 14 Hz, can most likely be attributed to the coupling with the tractor frame beaming frequency, which was held constant at 14 Hz (Table 4.3).

The lowest RMS combined acceleration of the driver occurs very close to a trailer beaming frequency of 10 Hz. However, this is very close to the acceleration spikes which occur from 11 to 17 Hz. The most practical course is to set the trailer beaming frequency at or above 20 Hz to avoid any acceleration spikes.

Figure 4.12 suggests that the suspension stiffness of the trailer axles has very little effect on the combined weighted RMS acceleration of the driver and only a small effect on the vertical weighted RMS acceleration of the trailer CG. When the beaming frequency of the trailer frame is held constant, there is only a 0.7% change in the combined weighted RMS acceleration of the driver and an 8.7% change in the vertical weighted RMS acceleration of the trailer CG over the range of trailer axle suspension stiffnesses.

The trends in Figure 4.12 show that the combined weighted RMS acceleration of the driver decreases slightly as the trailer axle suspension stiffness is increased. However, as the suspension stiffness decreases, there is a considerable reduction in the vertical weighted RMS acceleration at the trailer CG. Also, there is no change in the sensitivity of the combined weighted RMS acceleration of the driver or the vertical weighted RMS acceleration of the trailer CG at different beaming frequencies.

Table 4.19 shows the ride height reduction experienced by the tractor semi-trailer in a loaded condition when using the reduced trailer axle stiffnesses.

Table 4.19: Ride Height Reduction with Adjusted Trailer Suspension Parameters

Axle	Axle Stiffness Value (N/m)	Ride Height Reduction (in)
Steer Axle	581300	0.00
#1 Drive Axle	586900	0.01
#2 Drive Axle	586900	0.01
#1 Trailer Axle	700000	1.24
#2 Trailer Axle	700000	1.47

The maximum reduction in ride height was found to be just below one and a half inches on the second trailer axle when the vehicle is loaded.

Table 4.20 displays the loads seen by each of the axles in the nominal vehicle as well as the vehicle using the minimum tractor suspension stiffnesses. Both vehicles represented have a fully laden trailer.

Table 4.20: Static Axle Loads with Adjusted Trailer Suspension Stiffness

<table>
<tr><th>Vehicle Configuration</th><th>Steer Axle Load (lbs)</th><th>#1 Drive Axle Load (lbs)</th><th>#2 Drive Axle Load (lbs)</th><th>#1 Trailer Axle Load (lbs)</th><th>#1 Trailer Axle Load (lbs)</th></tr>
<tr><td rowspan="2">Nominal Vehicle</td><td rowspan="2">9964</td><td>14704</td><td>15768</td><td>18619</td><td>17733</td></tr>
<tr><td colspan="2">30472</td><td colspan="2">36352</td></tr>
<tr><td rowspan="2">Adjusted Trailer Axle Parameters</td><td rowspan="2">9965</td><td>14730</td><td>15801</td><td>18132</td><td>18160</td></tr>
<tr><td colspan="2">30531</td><td colspan="2">36292</td></tr>
<tr><td>SC Legal Load Limits with Permit</td><td>20000</td><td colspan="2">40000</td><td colspan="2">40000</td></tr>
<tr><td>Federal Legal Load Limits</td><td>12000</td><td colspan="2">34000</td><td colspan="2">34000</td></tr>
</table>

Adjusting the trailer suspension values had very little effect on the loads experienced by the steer and drive axles, but did have a greater effect on the trailer axle loads. However, these loads are still within the acceptable range allowed by South Carolina regulations.

The J penalty function was calculated to study the trends in the driver ride comfort and the vertical weighted acceleration of the trailer CG when varying levels of importance were placed on each of them. Figures 4.13 and 4.14 show the surface plots for the J penalty functions with varying values for K1 and K2.

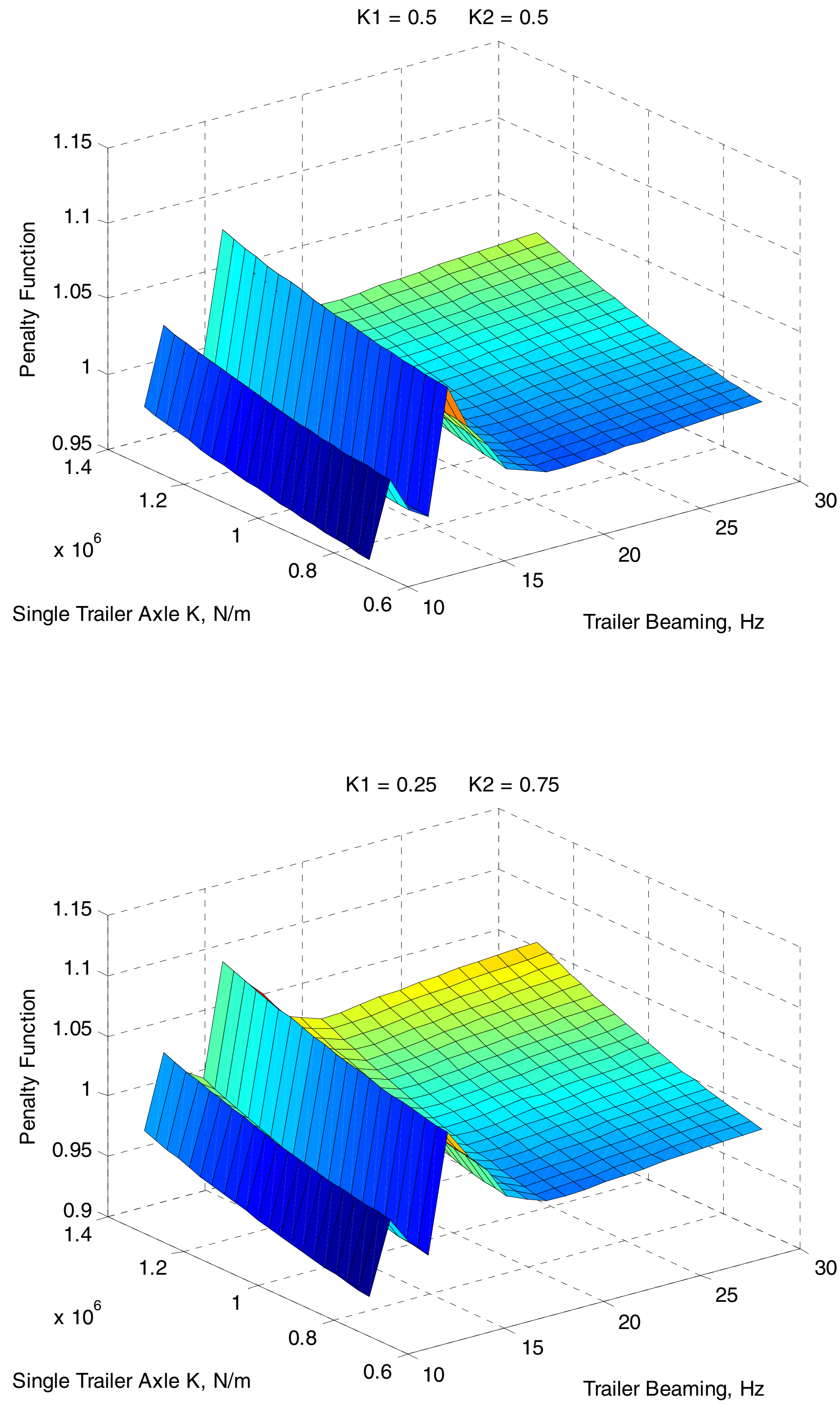


Figure 4.13: J Penalty Formulation for Trailer Parameters

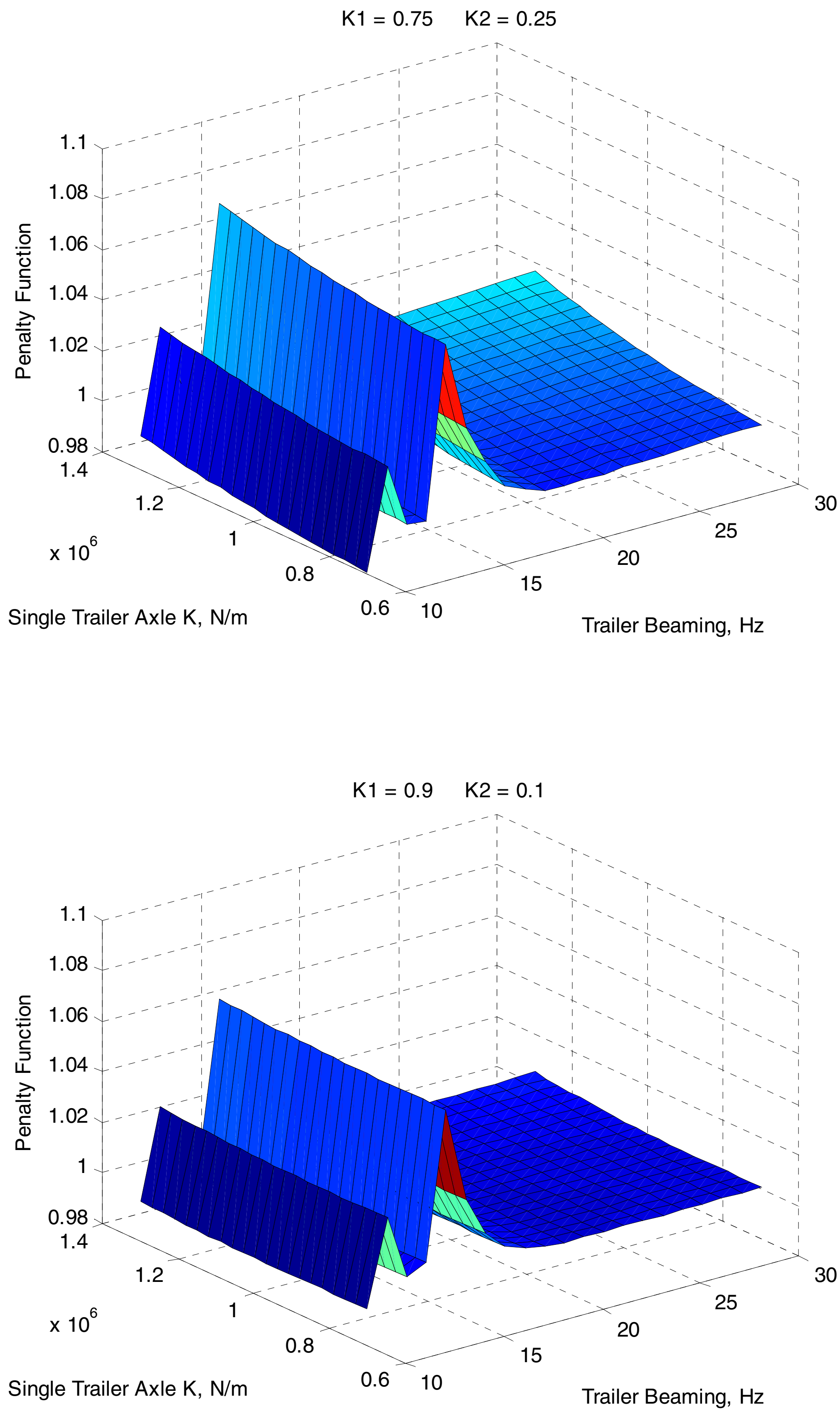


Figure 4.14: J Penalty Formulation for Trailer Parameters

Figures 4.13 and 4.14 suggest that reducing trailer axle suspension stiffness results in increased performance. Also, the J penalty surface plots confirm that the beaming frequency should remain at or above 20 Hz to avoid any acceleration spikes.

Table 4.21 shows the stiffness values for the trailer axles and the beaming frequency of the trailer frame chosen to achieve maximum ride comfort for both the driver and the trailer CG. These values were chosen factoring in their effect on the combined driver weighted RMS acceleration and the vertical weighted RMS acceleration of the trailer CG. Also considered are the effects of the adjusted stiffness values on the static ride heights of the tractor semi-trailer and the static loads on each of the axles.

Table 4.21: Nominal and Adjusted Trailer Suspension Stiffness and Beaming Frequency
60 mph, Smooth Highway

Component	Nominal Value	Adjusted Value
#1 Trailer Axle (N/m)	1000000	700000
#2 Trailer Axle (N/m)	1000000	700000
Trailer Frame Beaming Frequency (Hz)	20	20

Trailer Parameters with Adjusted Stiffness and Beaming Frequency Values

Using the adjusted suspension stiffness and keeping the nominal trailer frame beaming frequency has a positive effect on the vertical and a negative effect on the longitudinal weighted RMS accelerations of the driver. However, these balance out to zero effect on the combined weighted RMS acceleration. There is also and improvement in the vertical weighted acceleration of the trailer CG. Table 4.22 shows the vertical, longitudinal, and combined weighted RMS accelerations of the driver as well as the vertical weighted RMS acceleration of the trailer CG with the nominal and adjusted values and the percent improvement relative to the nominal values.

Table 4.22: Weighted Accelerations with Adjusted Trailer Suspension and Beaming Parameters
60 mph, Smooth Highway

Vehicle Parameter Configuration	**Driver Vertical Weighted Acceleration (m/s^2)**	**Driver Longitudinal Weighted Acceleration (m/s^2)**	**Driver Combined Weighted Acceleration (m/s^2)**	**Trailer Vertical Weighted Acceleration (m/s^2)**
Nominal Parameters	0.28	0.35	0.45	0.32
Adjusted Parameters	0.27	0.36	0.45	0.31
% Improvement	3.6	-2.9	0.0	+ 3.1

The vehicle ride height (Table 4.19) is affected only a small amount, the axle load limits (Table 4.20) are not violated, and there is improvement in the trailer vertical weighted RMS acceleration. Consequently, the values listed in Table 4.21 are confirmed as the preferred values.

Tractor and Trailer Beaming Variations

The same parameter variation techniques used in the previous sections were used to study the individual effects of the tractor and trailer beaming frequencies on the vertical and longitudinal ISO weighted accelerations of the driver individually. The parameter variations were performed by the program opt_beam_freq.m which is described in Appendix L. The beaming frequencies of the tractor and trailer were ranged from 10 Hz to 30 Hz Figure 4.15 shows the surface plots generated by the parameter variation program.

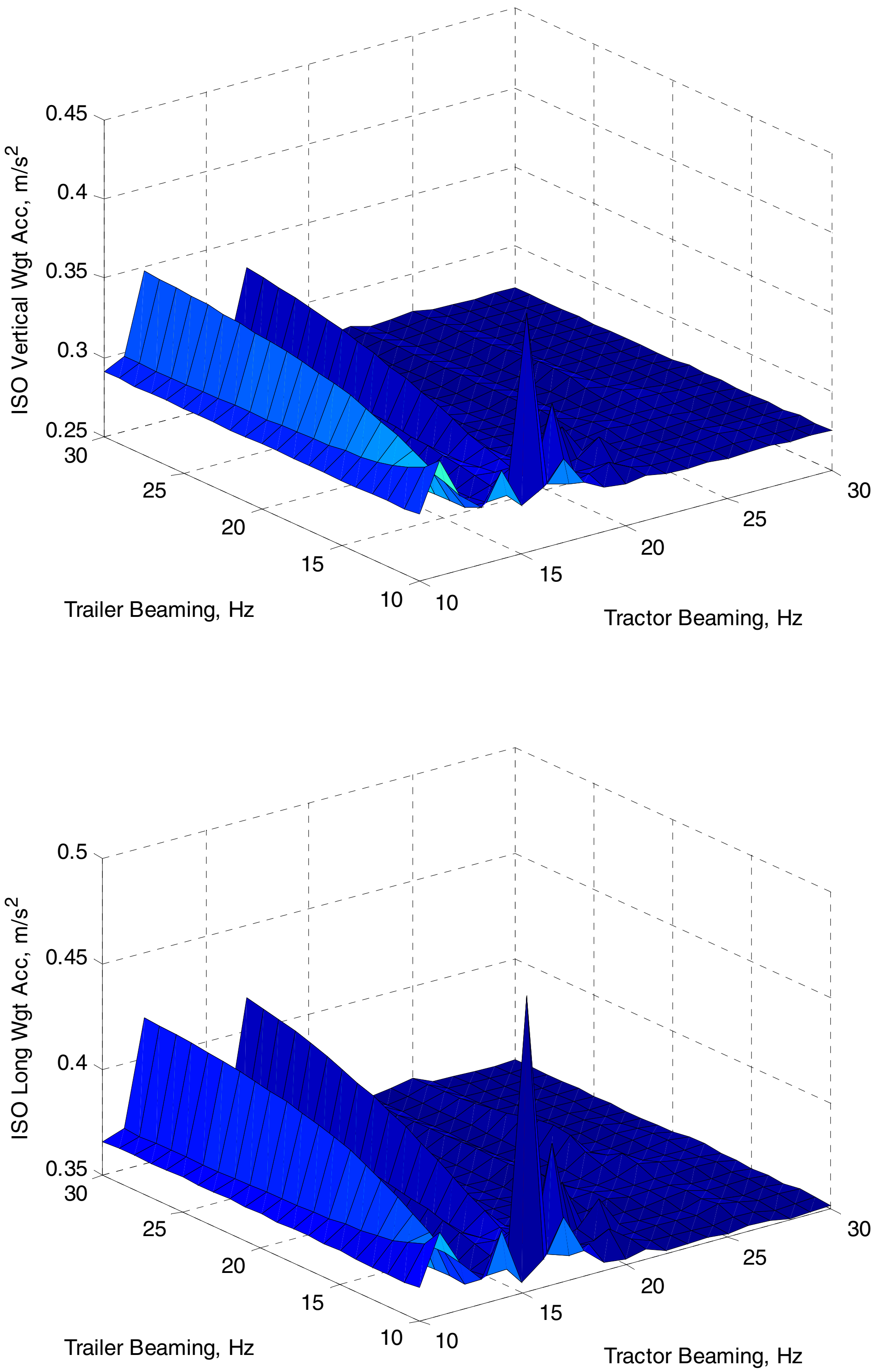


Figure 4.15: Effect of Tractor and Trailer Beaming Frequency on Driver Ride Comfort

Both plots in Figure 4.15 show spikes in the weighted RMS accelerations when the tractor beaming frequency equals 12 Hz and 17 Hz. The magnitude of the spikes increase as the trailer beaming frequency increases. However, there is a very large spike in the weighted RMS acceleration when the tractor beaming frequency equals 17 Hz and the trailer beaming frequency is at its lowest value, which is 10 Hz.

At its lowest value, the longitudinal weighted RMS acceleration is approximately 0.35 m/s^2, and the vertical weighted RMS acceleration is approximately 0.27 m/s^2. The longitudinal weighted RMS acceleration ranges from 0.35 m/s^2 to approximately 0.45 m/s^2, while the vertical weighted RMS acceleration ranges only from 0.27 m/s^2 to approximately 0.35 m/s^2.

Both plots in Figure 4.15 suggest that in order to avoid detrimental acceleration spikes, the tractor beaming frequency should be set at or above 20 Hz. When the tractor beaming is set at or above this value, the trailer beaming frequency has only a very limited effect on the vertical and longitudinal weighted RMS accelerations.

Fifth Wheel Suspension Parameter Variation

The vertical stiffness and damping constants across the fifth wheel were varied using the MATLAB program opt_5wKC_freq.m which is described in Appendix M. The stiffness constant was varied from 50,000 N/m to 1,000,000 N/m and the damping constant was varied from 2,000 N/(m/s) to a maximum of 40,000 N/(m/s). The minimum value of stiffness constant represents a fairly soft suspension system, and the maximum value represents a rigid connection. Figure 4.16 shows the surface plots generated by the parameter variation programs.

With the implementation of a fifth wheel vertical suspension system, the stroke across this connection becomes an important factor. Figure 4.17 shows the RMS stroke across the fifth wheel vertical suspension system as the parameters vary.

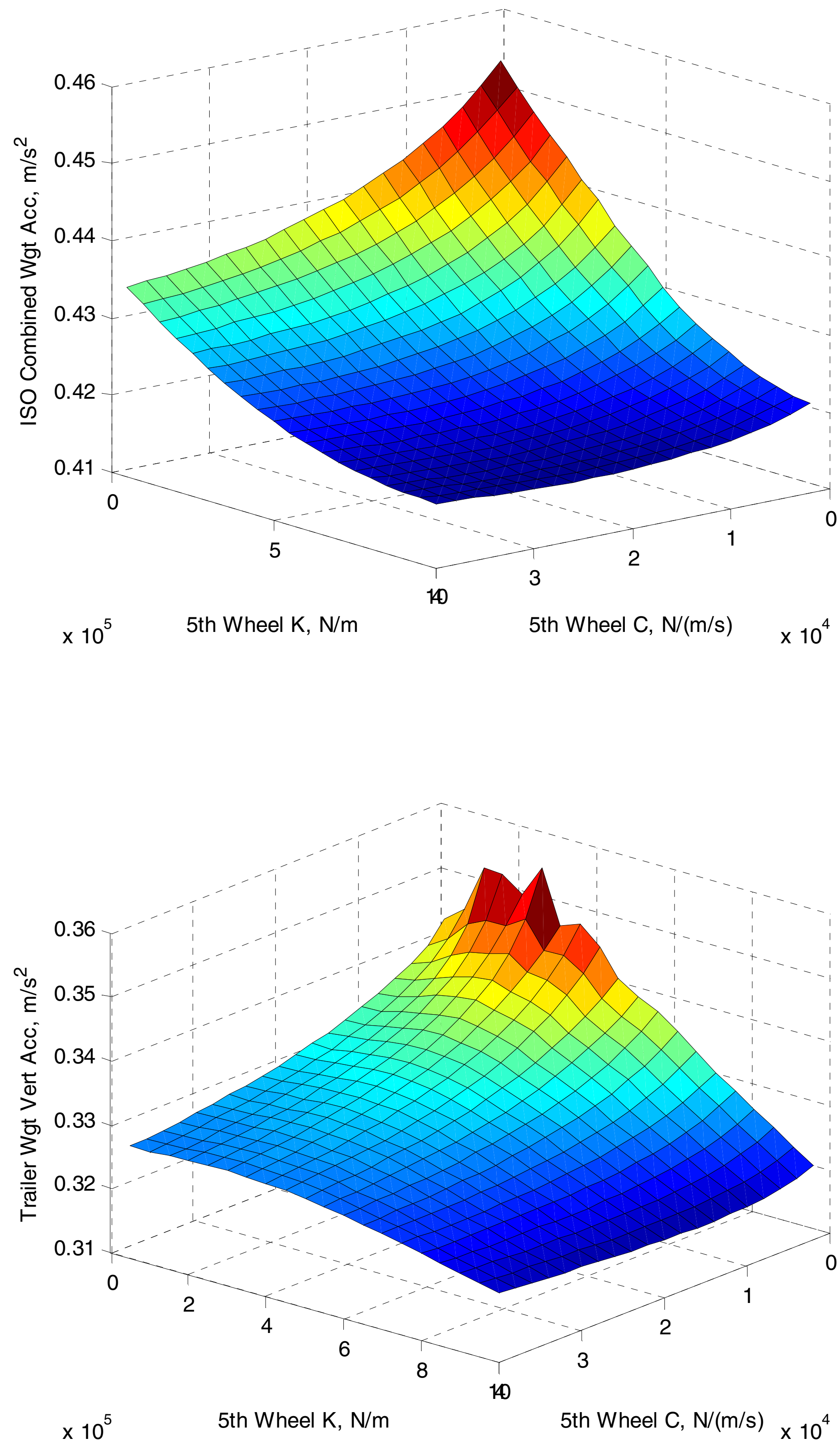


Figure 4.16: Parameter Variation for the Fifth Wheel Suspension System

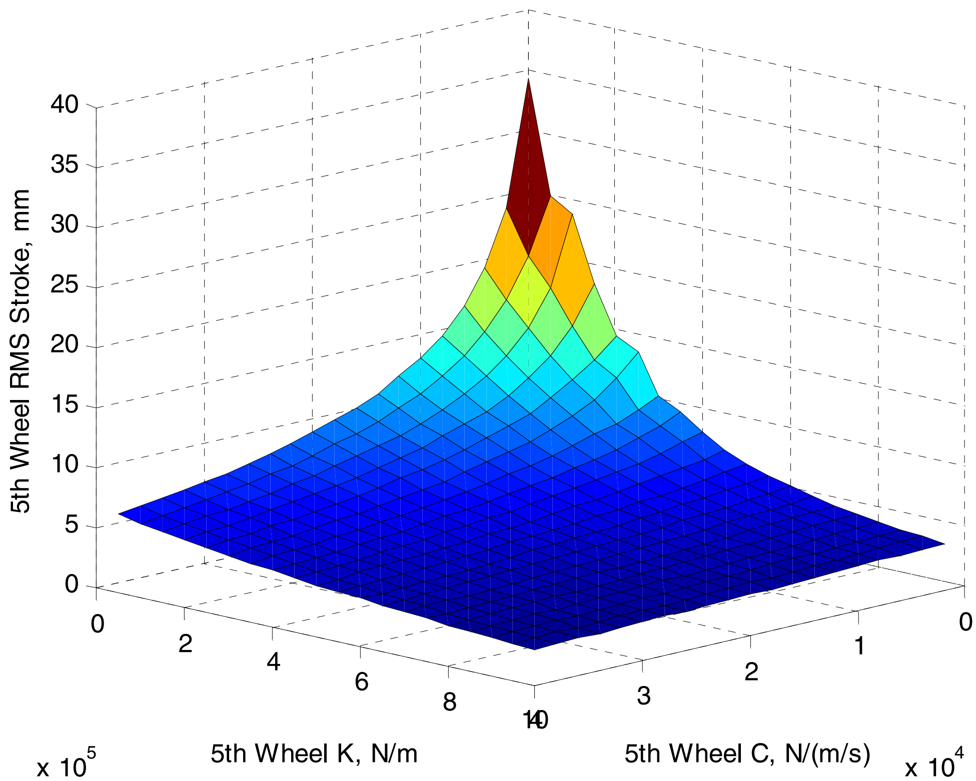
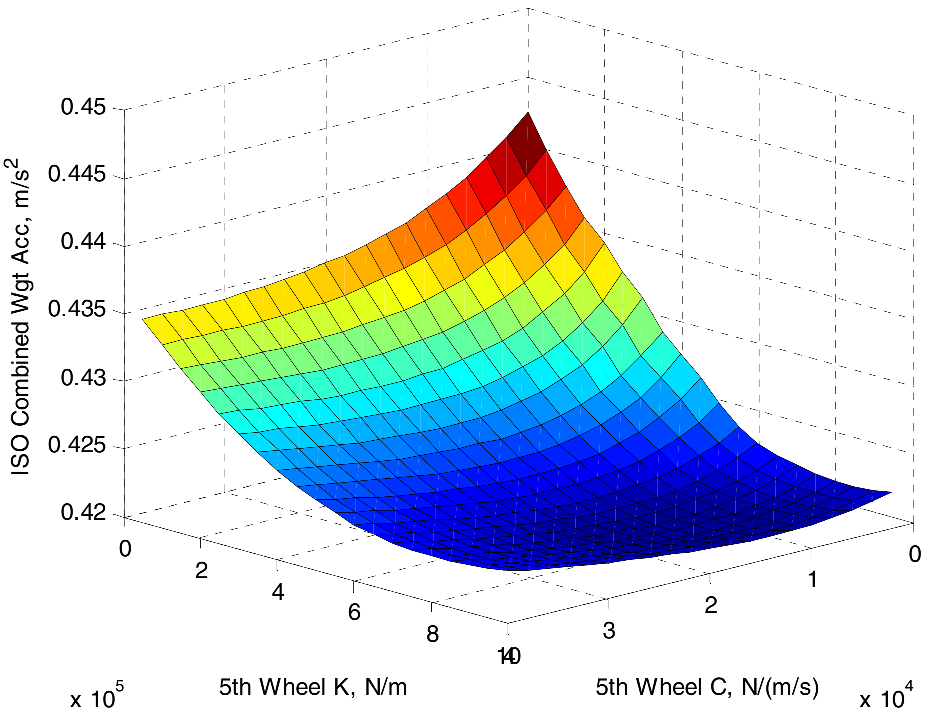


Figure 4.17: RMS Stroke Across the Fifth Wheel Vertical Suspension System

Figure 4.16 suggests that the best performance is achieved when no fifth wheel suspension is present. When a “rigid” connection is present, the combined weighted RMS acceleration is lower than the nominal value, which is 0.45 m/s^2. This is caused by the fact that the beaming modes are being modeled as “free-free” Euler-Bernoulli beams when a fifth wheel suspension system is utilized.

Figure 4.17 shows that the RMS stroke across the fifth wheel vertical suspension system is not an area of concern for any stiffness or damping value.

It should be noted that further research has shown that changing other parameters in the tractor semi-trailer can have strong effects on the trends seen in the surface plots created by the fifth wheel suspension parameter variation program. Figure 4.18 shows the surface plots generated when a full cab suspension system is chosen. The parameters for the full cab suspension are given in Table C.4.



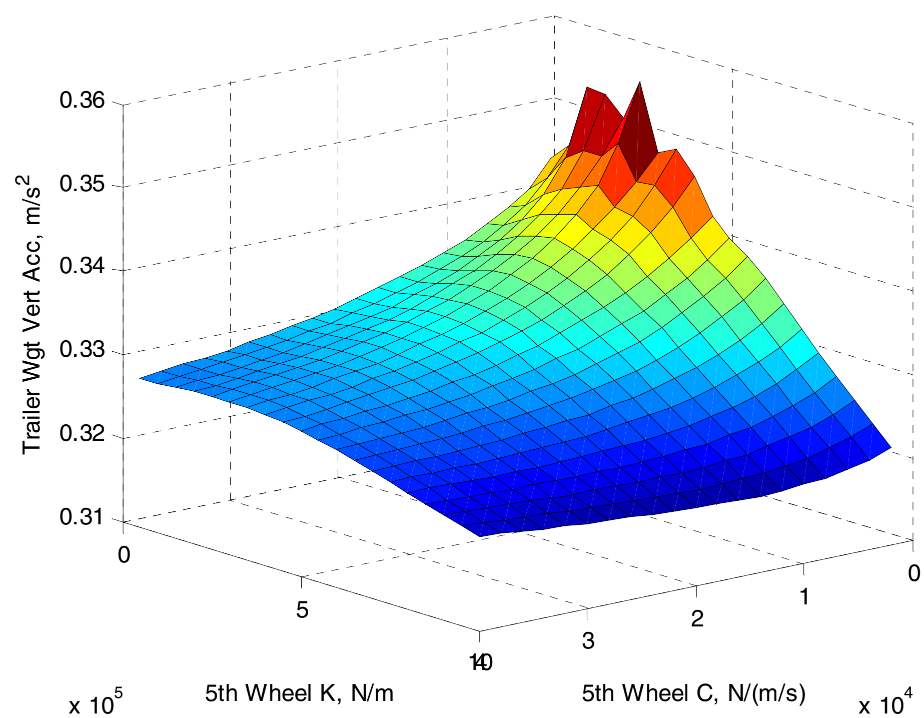


Figure 4.18: Parameter Variation for the Fifth Wheel Suspension with Full Cab Suspension

When a full cab suspension system is in place, the recommended fifth wheel suspension stiffness value becomes 800,000 N/m, as opposed to a “rigid” fifth wheel connection when using a rear-only cab suspension system. The new recommended value for the fifth wheel damping constant is 14,000 N/(m/s).

Trends in the trailer vertical weighted RMS acceleration plot remain largely the same. The surface plots show that the best performance for trailer ride is obtained by a “rigid” fifth wheel connection. However, there are only minor detrimental effects on the trailer acceleration when a suspension system is implemented. Significant improvements in the driver ride comfort with minor trade-offs in the trailer CG vertical RMs acceleration suggest that a fifth wheel suspension and full cab suspension may be beneficial.

Vehicle with Full Set of Adjusted Parameters

After the variation of the suspension parameters, tire parameters, trailer parameters, beaming frequencies, and fifth wheel parameters individually, it is possible to analyze the effects that all of these factors have together on the weighted RMS accelerations of the driver and trailer CG. Table 4.23 shows the nominal values for each of the suspension elements, tires, and the fifth wheel suspension system along with their adjusted values. The input beaming frequencies for the tractor and trailer frames remained at the nominal values of 20 Hz. The fifth wheel vertical suspension system parameter variation resulted in improvements in the driver ride comfort and the trailer CG vertical RMS acceleration when coupled with a full cab suspension system, so these values are also listed in Table 4.23.

Table 4.23: Nominal and Adjusted Parameters for Vehicle
60 mph, Smooth Highway

Parameter	Nominal Stiffness Constant (N/m)	Adjusted Stiffness Constant (N/m)	Nominal Damping Constant (N/(m/s))	Adjusted Damping Constant (N/(m/s))
Steer Axle Suspension	581300	406910	11270	14651
#1 Drive Axle Suspension	586900	410830	27500	35750
#2 Drive Axle Suspension	586900	410830	27500	35750
#1 Trailer Axle Suspension	1000000	700000	70000	70000
#2 Trailer Axle Suspension	1000000	700000	70000	70000
Steer Axle Tire	1295000	945350	517	517
#1 Drive Axle Tire	2388200	1671741	648.3	648.3
#2 Drive Axle Tire	2388200	1671741	648.3	648.3
#1 Trailer Axle Tire	2388200	2388200	648.3	648.3
#2 Trailer Axle Tire	2388200	2388200	648.3	648.3
Fifth Wheel Suspension (with full cab suspension)	10000000	800000	N/A	14000

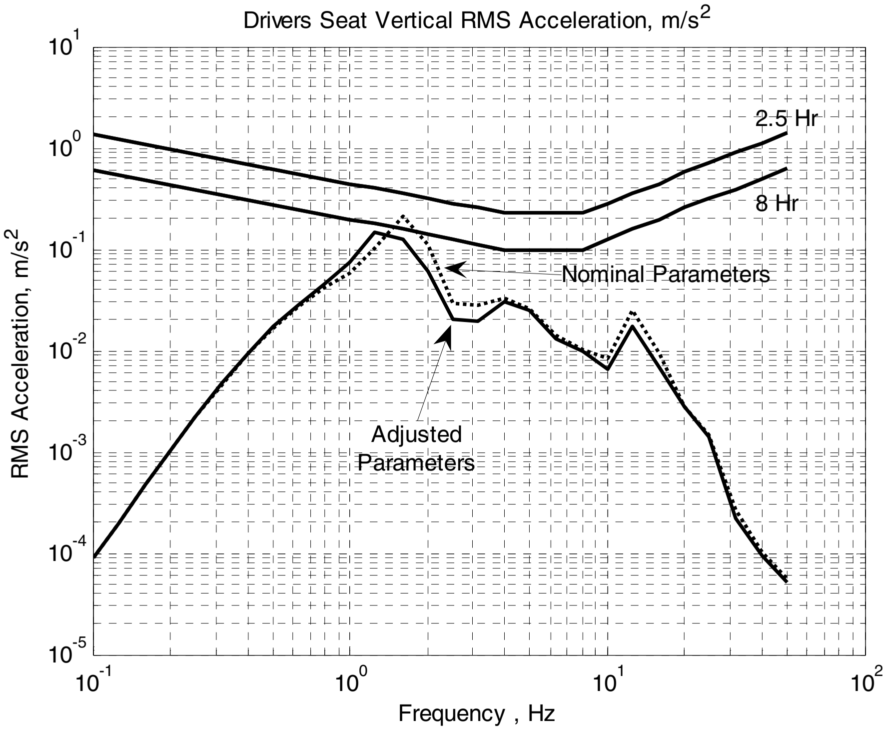
Table 4.24 shows the vertical, longitudinal, and combined weighted RMS accelerations of the driver and trailer CG with the nominal and adjusted values and the percent improvement relative to the nominal values.

Table 4.24: Acceleration with the Full Set of Adjusted Parameters
60 mph, Smooth Highway

Vehicle Suspension Configuration	**Vertical Weighted Acceleration (m/s^2)**	**Longitudinal Weighted Acceleration (m/s^2)**	**Combined Weighted Acceleration (m/s^2)**	**Trailer Vertical Weighted Acceleration (m/s^2)**	**ISO Comfort Level**
Nominal Parameters	0.28	0.35	0.45	0.32	A Little Uncomfortable
Adjusted Parameters (no 5th Wheel Susp. System)	0.23	0.23	0.32	0.32	A Little Uncomfortable
% Improvement Relative to Nominal Values	+ 17.9	+ 34.3	+ 28.9	0.0	
Adjusted Parameters (5th Wheel Susp. System and Full Cab Susp.)	0.26	0.19	0.32	0.31	A Little Uncomfortable
% Improvement Relative to Nominal Values	+ 7.1	+ 45.7	+ 28.9	+ 3.1	

The values in Table 4.24 show considerable improvement in the vertical weighted RMS acceleration at the driver's seat, but the area of greatest improvement lies in the longitudinal weighted RMS acceleration. When no fifth wheel vertical suspension system and a rear-only cab suspension system are present, the vertical weighted RMS acceleration of the driver showed a 17.9% improvement, but the longitudinal weighted RMS acceleration of the driver displayed a significantly larger 34.3% improvement. This resulted in a 28.9% increase in the combined weighted RMS acceleration of the driver. The trailer CG vertical weighted acceleration remained constant. However, when a fifth wheel vertical suspension system is implemented and the cab is fully suspended, there is a 7.1% improvement in the vertical weighted RMS acceleration of the driver and a 45.7% improvement in the longitudinal weighted RMS acceleration of the driver. This resulted in a 28.9% increase in the combined weighted RMS acceleration of the driver. Also, there was a 3.1% improvement in the trailer CG vertical weighted RMS acceleration.

It is important to analyze not only the total improvement in weighted RMS acceleration, but also where these improvements occur in the frequency range. Figure 4.19 shows the vertical and longitudinal weighted RMS accelerations of the driver when no fifth wheel suspension system and a rear-only cab suspension system are present, along with the ISO specified 2.5 and 8 hour comfort boundaries. On the plots are the curves using nominal parameters and the results when inserting adjusted parameters.



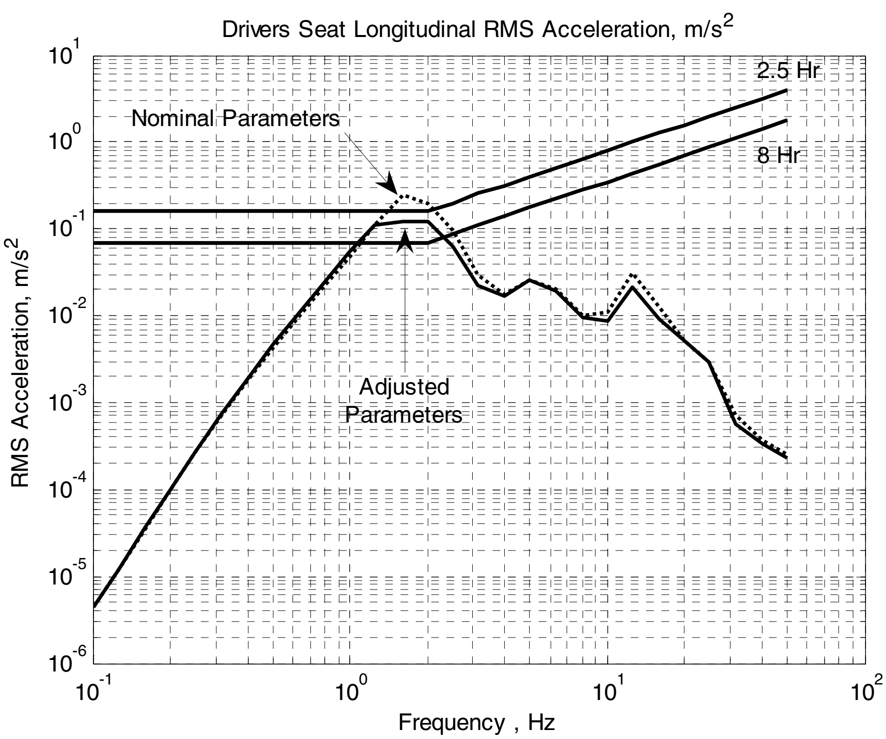


Figure 4.19: Effects of Adjusted Parameters with No 5th Wheel Vertical Suspension System on Driver Ride Comfort

The plots in Figure 4.19 suggest that the greatest improvements are in the body mode frequency region. There is also considerable improvement in the wheel hop frequency region, which occurs between 10 and 16 Hz.

In the vertical RMS acceleration plot, the improvements reduce the frequencies that were violating the 8 hour comfort curve to acceptable values. The longitudinal RMS acceleration curve violates both the 8 and 2.5 hour comfort curves when nominal parameters are utilized, but with the adjusted set of parameters only the 8 hour curve is violated.

Figure 4.20 shows the vertical and longitudinal weighted RMS accelerations of the driver when a fifth wheel suspension system and a full cab suspension system are present, along with the ISO specified 2.5 and 8 hour comfort boundaries. On the plots are the curves using nominal parameters and the results when inserting adjusted parameters.

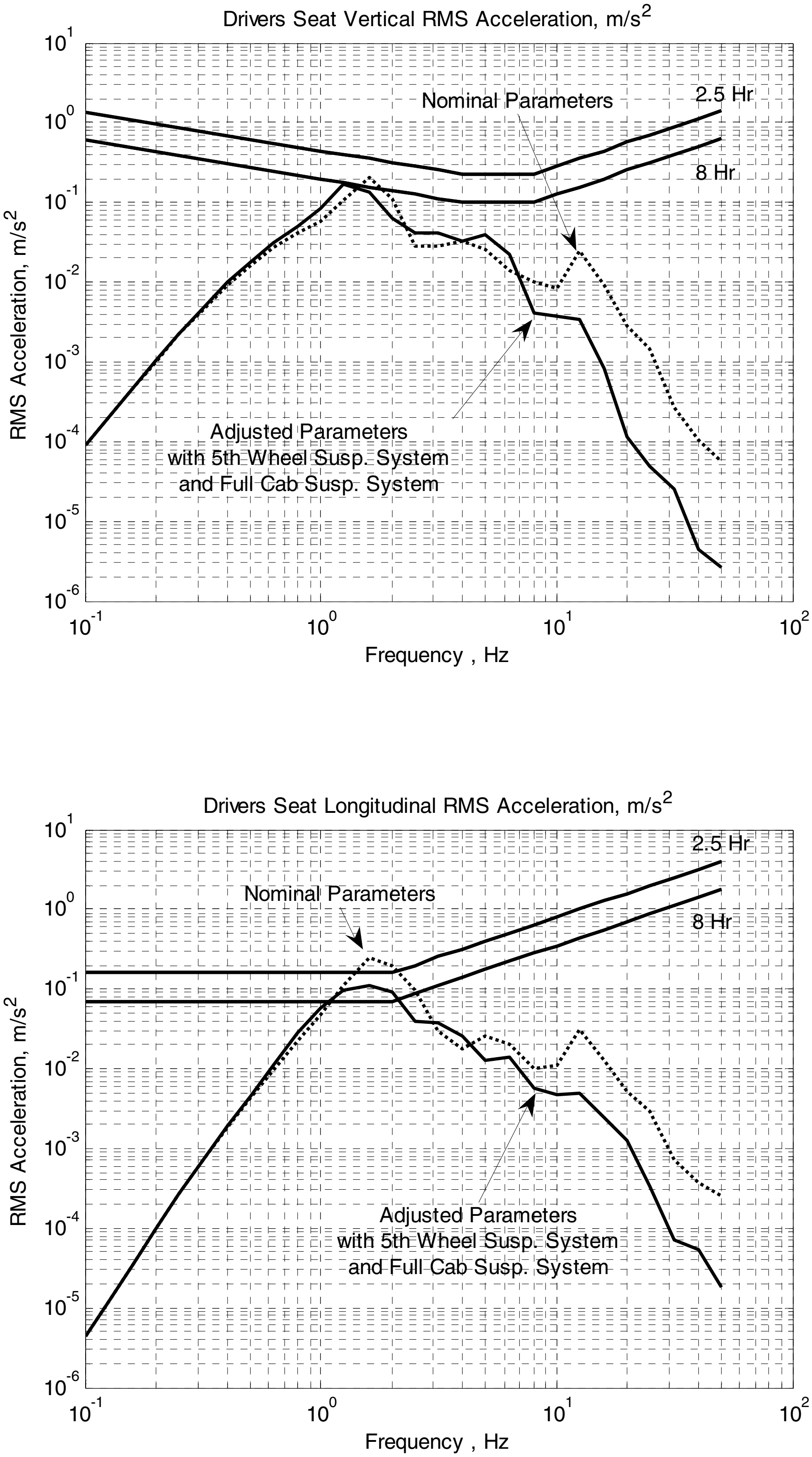


Figure 4.20: Effects of Adjusted Parameters with 5th Wheel Vertical Suspension System on Driver Ride Comfort

The plots in Figure 4.20 suggest that again the greatest improvements are in the body mode frequency region. In the vertical RMS acceleration plot, the presence of the fifth wheel vertical suspension system and full cab suspension has detrimental effects at 1.6, 3.15, 5 and 6.3 Hz. However, at frequencies higher than 6.3 Hz, there are large improvements. In the longitudinal RMS acceleration plot, there are improvements along the entire frequency spectrum except for 3.15 and 4 Hz.

In the vertical RMS acceleration plot, the improvements reduce the frequencies that were violating the 8 hour comfort curve to acceptable values. The longitudinal RMS acceleration curve violates both the 8 and 2.5 hour comfort curves when nominal parameters are utilized, but with the adjusted set of parameters only the 8 hour curve is violated.

The vertical, longitudinal, and combined RMS weighted acceleration at the driver's seat were also analyzed for the case in which the trailer is unloaded. This was performed to ensure that no detrimental effects were created when inserting the adjusted parameters. Table 4.25 shows the results of this study.

Table 4.25: Acceleration with an Unloaded Trailer
60 mph, Smooth Highway

Vehicle Suspension Configuration	Vertical Weighted Acceleration (m/s^2)	Longitudinal Weighted Acceleration (m/s^2)	Combined Weighted Acceleration (m/s^2)	ISO Comfort Level
Nominal Vehicle with Loaded Trailer	0.28	0.35	0.45	A Little Uncomfortable
Nominal Vehicle with Unloaded Trailer	0.28	0.36	0.46	A Little Uncomfortable
% Improvement Relative to Nominal Vehicle with Loaded Trailer	0	- 2.9	- 2.2	
Adjusted Vehicle with Unloaded Trailer	0.24	0.24	0.33	A Little Uncomfortable
% Improvement Relative to Nominal Vehicle with Loaded Trailer	+ 14.3	+ 31.4	+ 26.7	

Table 4.25 suggests that the unloaded vehicle equipped with nominal parameters will experience accelerations very similar to those experienced by the nominal vehicle. When the unloaded vehicle is equipped with the adjusted parameters, there is a 26.7% improvement in the accelerations experienced at the driver's seat, which is slightly down from 28.9% when the loaded vehicle is equipped with the adjusted parameter values.

Rollover Analysis

Research was conducted to study the effects that the adjusted suspension and tire parameters had on the rollover characteristics. The rollover simulation was performed using ROLL10WB3.m which was developed by Law [16]. A model and analysis were developed and implemented in Matlab to predict the lateral acceleration for impending rollover (or rollover threshold) under steady cornering of tractor semi-trailer trucks. The model includes the effects of vertical and lateral tire compliance, nonlinear axle roll suspensions, vertical suspensions for all axles, and a nonlinear "rocking" model for the fifth wheel connection. Provisions are made in the program for "switching" the appropriate equations and continuing the calculations after inside wheel lift-off is predicted for a given axle. Similar provisions are made to represent the tipping or rocking of the trailer on the fifth wheel. Outputs from the simulation include the inside wheel load (N), the tractor and trailer roll angles (deg), the axle roll angles (deg), and the trailer roll angle minus the tractor roll angle (deg) plotted against the lateral acceleration of the tractor semi-trailer (Gs). Like the parameter variation programs, the rollover simulation was performed using Michelin's new wide-base tire. Figure 4.21 shows the output plots from ROLL10WB3.m for the nominal vehicle.

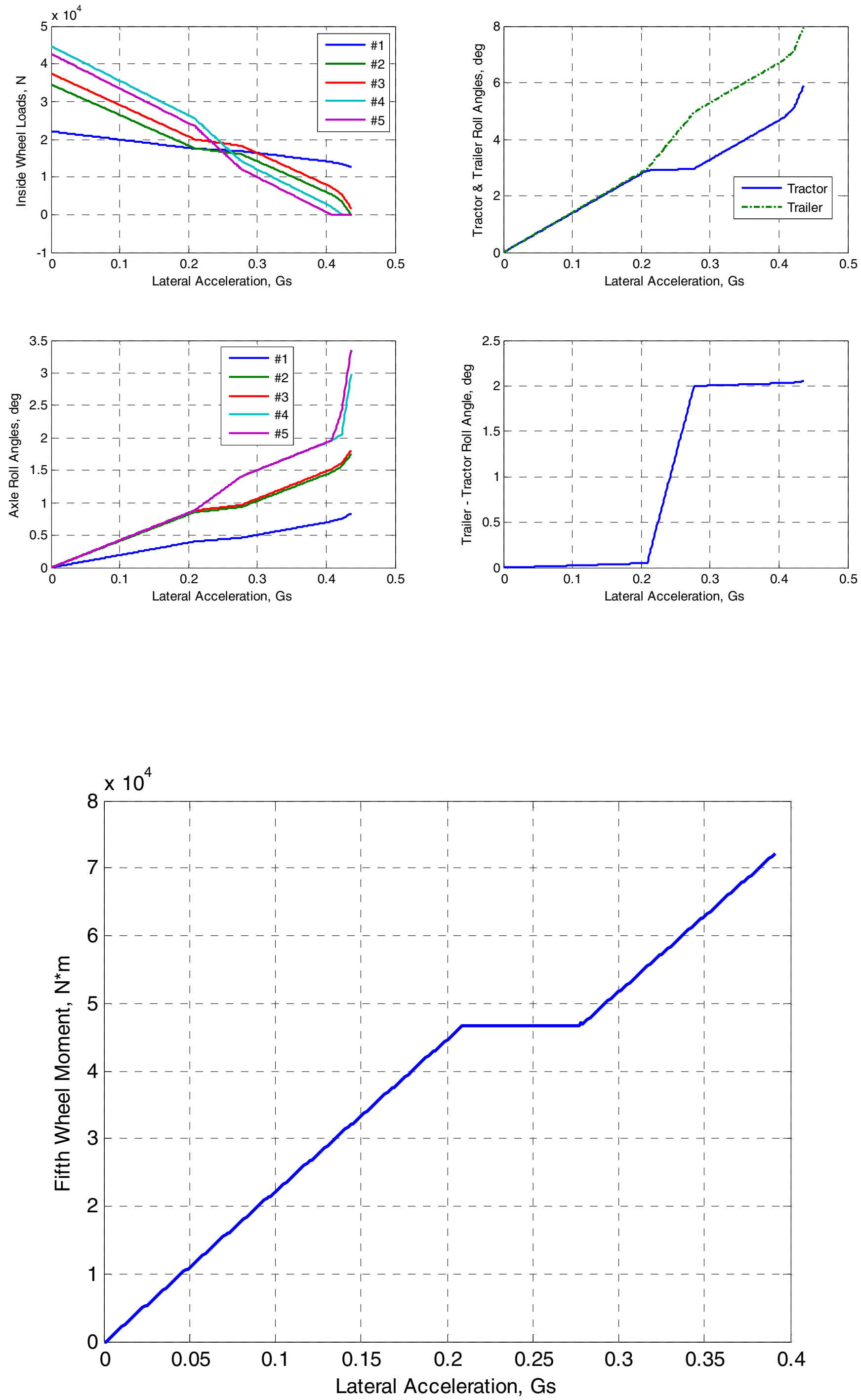


Figure 4.21: Rollover Results for the Nominal Vehicle

Figure 4.21 indicates that the nominal vehicle will experience inside wheel lift-off for the fourth axle at approximately 0.42 Gs of lateral acceleration. At this acceleration, the fifth (trailer) axle has already lifted off. This is indicated by the inside wheel loads on the fourth and fifth axles reaching zero. Also, at 0.42 Gs the axle and vehicle roll angles begin to rapidly increase.

This simulation was also performed for the vehicle using the adjusted values obtained in the previous sections. The adjusted values included the spring constants for each of the five axles, the vertical stiffnesses for each of the tires on the tractor semi-trailer, and the initial and secondary roll stiffnesses for each of the five axles. The secondary roll stiffnesses of the axles are used to more accurately describe the behavior of the suspension springs during large deflections. This could be representative of bump-stops on the axles or simply an increase in vertical stiffness as the displacements becomes greater. The secondary roll stiffness values are estimated by increasing the initial roll stiffness values by a factor of ten.

There was no option to include a fifth wheel suspension system in the rollover program, as a program with this feature has not yet been developed. Tables 4.26 and 4.27 display the nominal and adjusted values used in the rollover simulation, and Figure 4.22 shows the results from the rollover simulation using the adjusted values.

Table 4.26: Nominal and Adjusted Tire Parameters for the Rollover Simulation

Parameter	**Nominal Value Per-Side (kN/m)**	**Adjusted Value Per-Side (kN/m)**
Steer Axle Tire Vertical Stiffness	647.5	472.68
#1 Drive Axle Tire Vertical Stiffness	1194.1	835.87
#2 Drive Axle Tire Vertical Stiffness	1194.1	835.87
#1 Trailer Axle Tire Vertical Stiffness	1194.1	1194.1
#2 Trailer Axle Tire Vertical Stiffness	1194.1	1194.1

Table 4.27: Nominal and Adjusted Suspension Parameters for the Rollover Simulation

Parameter (Per Axle)	**Nominal Value**	**Adjusted Value**
Steer Axle Suspension Stiffness (N/m)	581300	406910
#1 Drive Axle Suspension Stiffness (N/m)	586900	410830
#2 Drive Axle Suspension Stiffness (N/m)	586900	410830
#1 Trailer Axle Suspension Stiffness (N/m)	1000000	700000
#2 Trailer Axle Suspension Stiffness (N/m)	1000000	700000
Steer Axle Initial Roll Stiffness (N*m/rad)	119697	83787.9
#1 Drive Axle Initial Roll Stiffness (N*m/rad)	614662	430263
#2 Drive Axle Initial Roll Stiffness (N*m/rad)	614662	430263
#1 Trailer Axle Initial Roll Stiffness (N*m/rad)	744064	520845
#2 Trailer Axle Initial Roll Stiffness (N*m/rad)	744064	520845
Steer Axle Secondary Roll Stiffness (N*m/rad)	1196970	837879
#1 Drive Axle Secondary Roll Stiffness (N*m/rad)	6146620	4302630
#2 Drive Axle Secondary Roll Stiffness (N*m/rad)	6146620	4302630
#1 Trailer Axle Secondary Roll Stiffness (N*m/rad)	7440640	5208450
#2 Trailer Axle Secondary Roll Stiffness (N*m/rad)	7440640	5208450

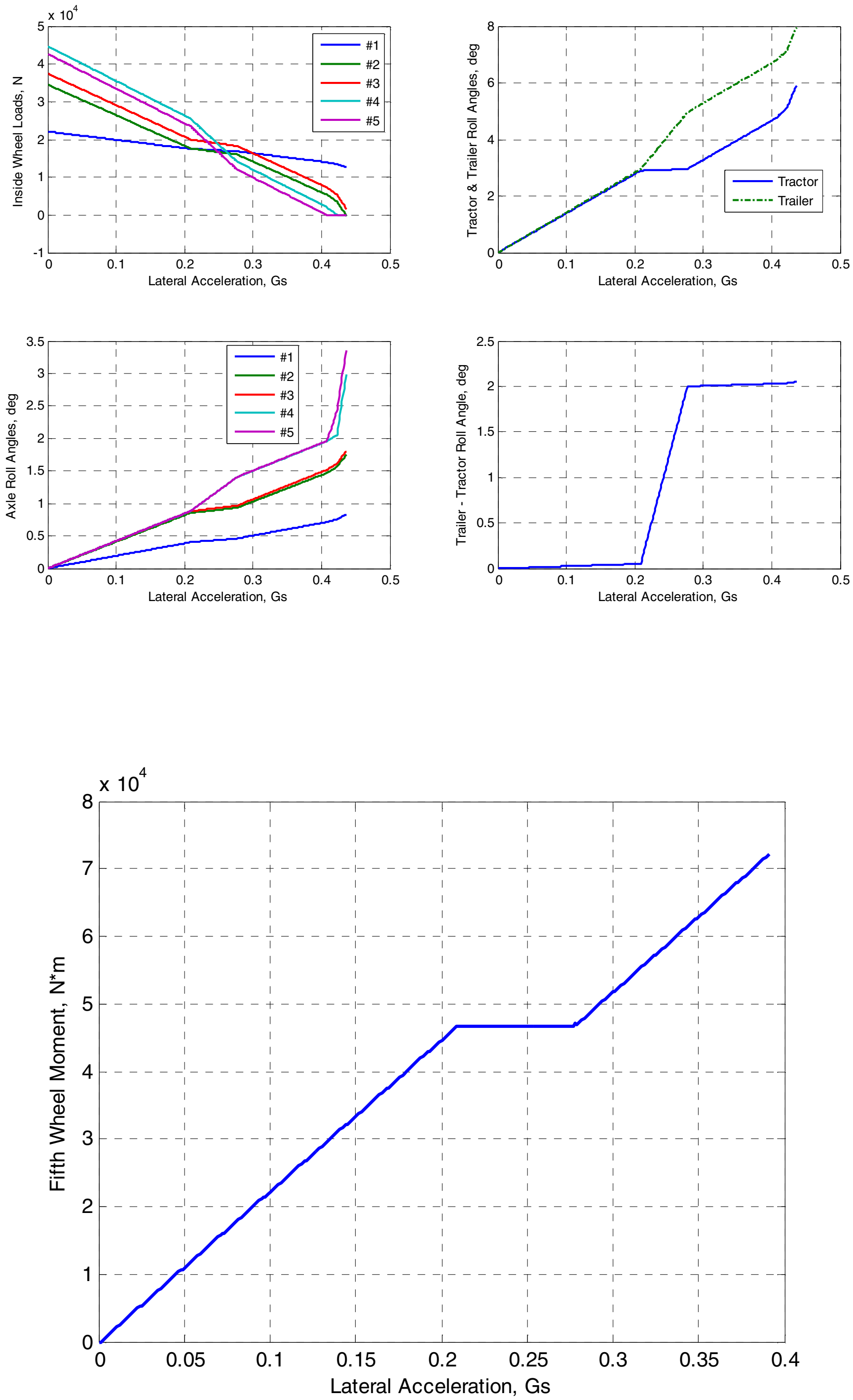


Figure 4.22: Rollover Results for the Vehicle with Adjusted Parameters

Figure 4.22 suggests that the vehicle equipped with adjusted parameters experiences liftoff of the second trailer axle between 0.40 and 0.41 Gs, as opposed to 0.42 Gs in the nominal vehicle. Values for the roll angles of the tractor and trailer are slightly higher, as well as axle roll angle values. Also, the axle and vehicle roll angles begin to rapidly increase around 0.41 Gs. The use of the adjusted suspension parameters resulted in a 31.2% improvement in the driver ride comfort, but only a slight decrease in the lateral acceleration at which the tractor semi-trailer experiences rollover.

CHAPTER 5

SUMMARY AND CONCLUSIONS

Summary

In this thesis, a 15 DOF model that describes the vertical dynamic response of a tractor semi-trailer was developed. A 14 DOF model was previously developed by Trangsrud [1], and this model was used as a basis of comparison for the new model. With the exception of the addition of the fifth wheel suspension system, the physical model is identical to Trangsrud's. However, in this model the equations of motion were developed using Lagrange's equation as well as Newton's Laws for further validation.

The new model was simulated in MATLAB and used to explore the effects of various parameters on ride comfort, vehicle ride heights, and pavement loading. Many different vehicle configurations and operating scenarios may be simulated. The options for the vehicle configurations include: (a) a choice of six different tire types with the option to select the inflation pressure, (b) the presence or absence of seat suspension, (c) a choice of front, rear, full, or no cab suspension, (d) the presence of a fifth wheel suspension with the option to input the values for the suspension parameters, (e) variable tractor and trailer frame bending stiffnesses and, (f) the option of using a loaded or unloaded trailer. In addition to these options, specific values for the vehicle geometry, suspension characteristics, and inertial properties may be chosen by the user. The options for the vehicle operating scenarios include: (a) the user's choice from four different road profile types and, (b) vehicle velocity.

The MATLAB simulation performs all necessary calculations in the frequency domain. In the simulation program, dof15_freq2.m, the user has the option of examining any of the following outputs: (a) eigenvalues and eigenvectors of the system, (b) driver weighted acceleration values, (c) transfer functions of vehicle motions, (d) weighted RMS accelerations of the driver and how they relate to ISO 2631 comfort criteria (Table 3.2), (e) fifth wheel RMS stroke, (f) road profile RMS plots, (g) weighted RMS accelerations of the driver over the frequency range from 0.1 to 50 Hz and how they relate to ISO 2631 comfort boundaries [3:1974], (h) static loads and deflection at each of the axles and, (i) per-axle wheel force transfer functions.

The parameter variation programs allow the user the same input options as dof15_freq2.m, but the output options are somewhat limited by the nature of the programs. Each of the programs produce surface plots of the ISO combined weighted acceleration of the driver and surface plots of the ISO vertical weighted acceleration at the trailer CG. Some of the programs also offer surface plots of the J penalty function and the RMS stroke across the fifth wheel.

The 15 DOF model was compared to a “nominal” vehicle to assess the effect that the parameter variations have on the system response. Different case studies were performed to investigate the effects of each of the different parameters. The results from these case studies are summarized in Tables 5.1 through 5.3 and the numbered items following. The vehicle was assumed to be traveling at 60 mph over a “Smooth Highway”.

Table 5.1: Weighted RMS Accelerations
60 mph, Smooth Highway

Vehicle Suspension Configuration	**Vertical Weighted Acceleration (m/s^2)**	**Longitudinal Weighted Acceleration (m/s^2)**	**Combined Weighted Acceleration (m/s^2)**	**Trailer Vertical Weighted Acceleration (m/s^2)**
Nominal Parameters	0.28	0.35	0.45	0.32
Adjusted Tractor Axle Suspension Parameters / % Improvement Relative to Nominal Value	0.22 / +21.4	0.24 / +31.4	0.32 / +28.9	0.34 / -6.3
Adjusted Tractor Tire Parameters / % Improvement Relative to Nominal Value	0.29 / -3.6	0.33 / +5.7	0.43 / +4.4	0.32 / 0
Adjusted Tractor Axle Suspension and Tire Parameters / % Improvement Relative to Nominal Value	0.23 / +17.9	0.23 / +34.3	0.32 / +28.9	0.33 / -3.1
Adjusted Trailer Beaming and Axle Parameters / % Improvement Relative to Nominal Value	0.27 / +3.6	0.36 / -2.9	0.45 / 0	0.31 / +3.1
Adjusted Tractor and Trailer Beaming Parameters / % Improvement Relative to Nominal Value	0.28 / 0	0.35 / 0	0.45 / 0	0.32 / 0
Full Set of Adjusted Parameters (Table 4.23) with No 5th Wheel Susp. System / % Improvement Relative to Nominal Value	0.23 / +17.9	0.23 / +34.3	0.32 / +28.9	0.32 / 0
Full Set of Adjusted Parameters (Table 4.23) with 5th Wheel Susp. System / % Improvement Relative to Nominal Value	0.26 / +7.1	0.19 / +45.7	0.32 / +28.9	0.31 / +3.1

Table 5.2: Vehicle Ride Height Reductions

Axle	Axle #1	Axle #2	Axle #3	Axle #4	Axle #5
Ride Height Reduction with Adjusted Tractor Axle Suspension Parameters (in)	1.29	1.86	1.99	0.17	-0.15
Ride Height Reduction with Adjusted Tractor Tire Parameters (in)	0.50	0.46	0.50	0	0
Ride Height Reduction with Adjusted Tractor Axle Suspension and Tire Parameters (in)	1.78	2.32	2.49	0.17	-0.15
Ride Height Reduction with Adjusted Trailer Axle Parameters (in)	0	0.01	0.01	1.24	1.47
Ride Height Reduction with Full Set of Adjusted Parameters (Table 4.23) (in)	1.78	2.34	2.51	1.41	1.33

Table 5.3: Static Axle Loads and Legal Load Limits [38]

<table>
<tr><th>Vehicle Configuration</th><th>Steer Axle Load (lbs)</th><th>#1 Drive Axle Load (lbs)</th><th>#2 Drive Axle Load (lbs)</th><th>#1 Trailer Axle Load (lbs)</th><th>#1 Trailer Axle Load (lbs)</th></tr>
<tr><td rowspan="2">Nominal Vehicle</td><td rowspan="2">9964</td><td>14704</td><td>15768</td><td>18619</td><td>17733</td></tr>
<tr><td colspan="2">30472</td><td colspan="2">36352</td></tr>
<tr><td rowspan="2">Adjusted Tractor Axle Suspension Parameters</td><td rowspan="2">9963</td><td>14667</td><td>15722</td><td>19312</td><td>17125</td></tr>
<tr><td colspan="2">30389</td><td colspan="2">36437</td></tr>
<tr><td rowspan="2">Adjusted Tractor Tire Parameters</td><td rowspan="2">9964</td><td>14704</td><td>15768</td><td>18619</td><td>17733</td></tr>
<tr><td colspan="2">30472</td><td colspan="2">36352</td></tr>
<tr><td rowspan="2">Adjusted Tractor Axle Suspension and Tire Parameters</td><td rowspan="2">9963</td><td>14667</td><td>15722</td><td>19312</td><td>17125</td></tr>
<tr><td colspan="2">30389</td><td colspan="2">36437</td></tr>
<tr><td rowspan="2">Adjusted Trailer Axle Parameters</td><td rowspan="2">9965</td><td>14730</td><td>15801</td><td>18132</td><td>18160</td></tr>
<tr><td colspan="2">30531</td><td colspan="2">36292</td></tr>
<tr><td rowspan="2">Full Set of Adjusted Parameters (Table 4.23)</td><td rowspan="2">9964</td><td>14704</td><td>15768</td><td>18619</td><td>17733</td></tr>
<tr><td colspan="2">30472</td><td colspan="2">36352</td></tr>
<tr><td>SC Legal Load Limits with Permit</td><td>20000</td><td colspan="2">40000</td><td colspan="2">40000</td></tr>
<tr><td>Federal Legal Load Limits</td><td>12000</td><td colspan="2">34000</td><td colspan="2">34000</td></tr>
</table>

1. The tractor axle suspension stiffness variation suggested lowering the steer axle suspension stiffness from 581,300 N/m per axle to 406,910 N/m per axle which is a 30% decrease. It also suggested lowering the drive axle stiffnesses from 586,900 N/m per axle to 410,830 N/m per axle which is also a 30% decrease. This resulted in 24.4% improvement in the combined weighted RMS acceleration of the driver with only a 3.1% increase in the vertical weighted RMS acceleration of the trailer CG. These changes resulted in an acceptable reduction in vehicle ride height and did not cause the vehicle to violate any axle load limitation regulations.

2. The tractor axle suspension damping variation suggested raising the steer axle damping value from 11,270 N/(m/s) per axle to 14,651 N/(m/s) per axle which is a 30% increase. It also suggested raising the drive axle damping values from 27,500 N/(m/s) per axle to 35,750 N/(m/s) per axle which is also a 30% increase. This resulted in a 6.7% improvement in the combined weighted RMS acceleration of the driver with only a 3.1% increase in the vertical weighted RMS acceleration of the trailer CG.

3. Inserting the adjusted values for tractor axle suspension stiffness and damping resulted in a 21.4% improvement in the vertical weighted RMS acceleration, a 31.4% improvement in the longitudinal weighted RMS acceleration, and a 28.9% improvement in the combined weighted RMS acceleration of the driver. The areas of greatest improvement were found to occur at frequencies that correspond to body mode frequencies. All of the values for the weighted accelerations were listed as "A Little Uncomfortable" by ISO 2631 standards.

4. The tractor tire stiffness variation suggested lowering the steer tire stiffness from 647.5 kN/m per tire to 472.68 kN/m per tire which is a 27% decrease. It also suggested decreasing the drive tire stiffness from 1,194.1 kN/m per tire to 835.87 kN/m per tire which is a 30% decrease. This resulted in a 4.4% improvement in the combined weighted RMS acceleration of the driver and had insignificant effects on the vertical weighted RMS acceleration of the trailer CG. These changes resulted in an acceptable change in vehicle ride height and did not cause the vehicle to violate any axle load limitation regulations.

5. The tractor tire damping variation did not cause any significant changes in the accelerations experienced by the driver or the trailer CG.

6. Inserting the adjusted values for tractor axle suspension stiffness and damping as well as the adjusted values for the tire stiffness resulted in a 17.9% improvement in the vertical weighted acceleration, a 34.3% improvement in the longitudinal weighted acceleration, and a 28.9% improvement in the combined weighted acceleration of the driver. The areas of greatest improvement were found to occur at frequencies that correspond to body mode frequencies as well as frequencies corresponding to wheel hop frequencies. All of the values for the weighted accelerations were listed as “A Little Uncomfortable” by ISO 2631 standards.

7. The trailer suspension stiffness and beaming frequency variation suggested that lowering the trailer axle stiffness to 700,000 N/m and maintaining a trailer frame beaming frequency higher than 20 Hz resulted in no change in the combined weighted RMS acceleration of the driver and a 3.1% decrease in the vertical weighted RMS acceleration of the trailer CG. These changes resulted in an acceptable reduction in vehicle ride height and did not cause the vehicle to violate any axle load limitation regulations. All of the values for the weighted accelerations were listed as “A Little Uncomfortable” by ISO 2631 standards.

8. The tractor and trailer beaming frequency variation study suggested that maintaining beaming frequencies of the tractor and trailer frame above 20 Hz will cause the system to avoid any acceleration spikes caused by coupling of the beaming frequencies of the frames with wheel hop frequencies. There were no improvements in the weighted RMS accelerations for beaming frequencies higher than 20 Hz. However, all of the values for the weighted accelerations were listed as “A Little Uncomfortable” by ISO 2631 standards.

9. The fifth wheel suspension parameter variation study suggested that implementing a fifth wheel vertical suspension system would be detrimental to the ride comfort of the driver. This means that the best performance would be achieved using a conventional fifth wheel connection with the rear cab suspension. However, with a full cab suspension on the tractor a local minimum is present for the fifth wheel suspension and damping values. The recommended stiffness value becomes 800,000 N/m and the recommended damping value becomes 14,000 N/(m/s).

10. The implementation of the full set of adjusted parameters without a fifth wheel suspension system resulted in 17.9% decrease in the vertical weighted RMS acceleration of the driver, a 34.3% decrease in the longitudinal RMS weighted acceleration of the driver, and a 28.9% decrease in the combined weighted RMS acceleration of the driver. Also, no detrimental effects were witnessed in the trailer vertical weighted RMS acceleration. The implementation of the full set of adjusted parameters with a fifth wheel suspension system and full cab suspension resulted in 7.1% decrease in the vertical weighted RMS acceleration of the driver, a 45.7% decrease in the longitudinal weighted RMS acceleration of the driver, and a 28.9% decrease in the combined weighted RMS acceleration of the driver. Also, there was a 3.1% decrease in the vertical weighted RMS acceleration at the trailer CG. Changes in the vehicle ride height were acceptable, and no axle load limit regulations were violated.

11. The rollover study showed that very large improvements in ride characteristics could be obtained with only a very minor reduction of the lateral acceleration for inside wheel lift-off.

Recommendations

This research built on the foundation laid by Vaduri [3] and Trangsrud [1]. There were multiple changes and additions made to this simulation, as well as the creation of new programs to explore parameter variations. These factors make the simulation more valuable as a predictive tool, and open up some new areas of research for future engineers. Some possible additions to the model and simulation are presented below.

1. Developing a three dimensional model with unequal left and right road irregularities would allow vehicle and cab lateral motion and roll to be included.

2. The inclusion of higher order bending modes in the tractor and trailer frames could possibly give a more accurate picture of the dynamic response of the model.

3. Correlating the simulated data with physical test data would lend additional credibility to the model and encourage future use of the model and simulation.

4. More scenarios and/or additional vehicles could be studied with the parameter variation programs. Different combinations of vehicle parameters can have a significant effect on the outcome of the variation programs, and different vehicles could behave differently.

5. A parameter variation program that included a rollover indicator would help to analyze the tradeoffs experienced when finding the best set of parameters for ride quality. Also, a rollover program that included options for a fifth wheel suspension system would give good information on the effect of the fifth wheel suspension system on the rollover characteristics of the vehicle.

APPENDICES

Appendix A: Equations of Motion

The equations of motion for the fifteen degree-of-freedom tractor semi-trailer are derived in this appendix using the Lagrangian Method. The fifteen degrees of freedom are (in the order in which they are derived): vertical displacement of the seat, vertical displacement of the cab, pitch of the cab, vertical displacement of the engine, vertical displacement of the tractor frame, pitch of the tractor frame, beaming of the tractor frame, vertical displacement of the trailer frame, pitch of the trailer frame, beaming of the trailer frame, and the vertical displacements of all five axles.

The Lagrangian Method uses the kinetic and potential energy of the system, along with the generalized force, which is determined from the work done by the applied force in some virtual displacement. The Lagrangian function is defined,

$$L = T^* - V \tag{A.1}$$

where T^* represents the kinetic coenergy of the system, and V represents the potential energy of the system. The potential energy is a function of ξ_j while T^* is a function of $\dot{\xi}_j$, ξ_j, and time t. Therefore, the Lagrangian can be represented by the variables,

$$L = L\left(\dot{\xi}_1, \dot{\xi}_2, ..., \dot{\xi}_n, \xi_1, \xi_2, ..., \xi_n, t\right) \tag{A.2}$$

and its variation can be defined as,

$$\delta L = \sum_{j=1}^{n}\left(\frac{\delta L}{\delta \dot{\xi}_j}\delta \dot{\xi}_j + \frac{\delta L}{\delta \xi_j}\delta \xi_j\right). \tag{A.3}$$

It may be shown that through Hamilton's principle, the following variational indicator may be derived,

$$V.I. = \int_{t_1}^{t_2} \left[\delta\left(T^* - V\right) + \sum_{j=1}^{n} \Xi_j \delta\xi_j \right] dt \tag{A.4}$$

where admissible variations are represented by the n independent $\delta\xi_j$. These admissible variations vanish at times t_1 and t_2, but are otherwise arbitrary functions time in the interval from t_1 to t_2. Substituting (A.3) into (A.4), integrating, and using the agreement that the $\delta\xi_j$ vanish at $t=t_1$ and $t=t_2$, n equations are left,

$$\begin{aligned} &\frac{d}{dt}\left(\frac{\delta L}{\delta\dot{\xi}_j}\right) - \frac{\delta L}{\delta\xi_j} = \Xi_j \,. \\ &j = 1, 2, \ldots, n \end{aligned} \tag{A.5}$$

To derive the equations of motion for the tractor semi-trailer, the following steps are required:

1. Establish a complete set of independent generalized coordinates ξ_j.
2. Identify generalized nonconservative forces Ξ_j (if any).
3. Construct the Lagrangian (A.1).
4. Substitute in Lagrange's equations (A.5).

For simplicity, only the stiffness terms are included in the derivation. This is due to the spring and damping elements being in parallel, therefore making the derivation of the damping elements the same as the derivation of the spring elements. Positive deflection is assumed to be down for heave and nose up for pitch. Refer to Figures A.1 and A.2 for a visual representation of the degrees of freedom and the dimensions of the model. Appendix C contains the numeric parameters for the model.

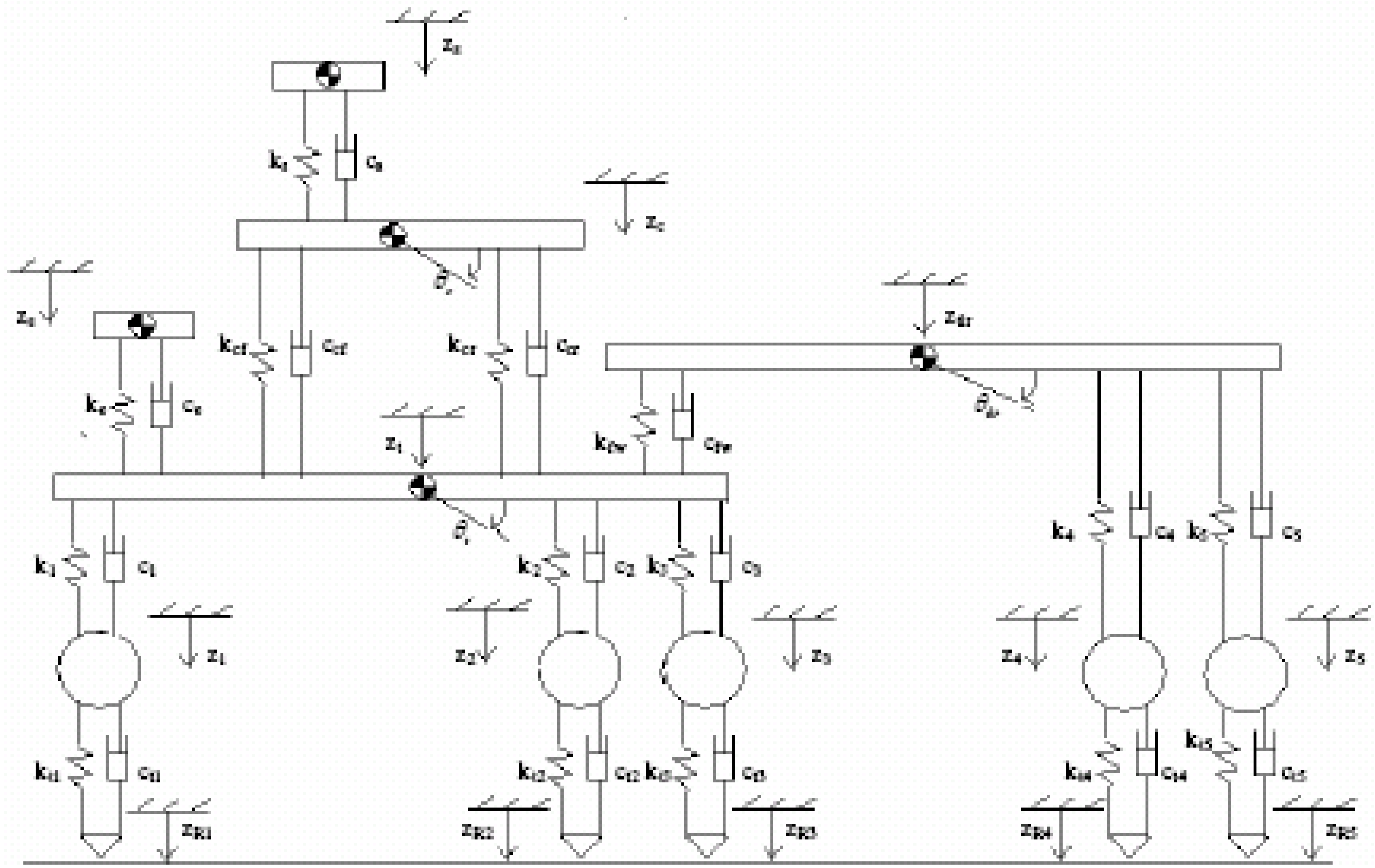


Figure A.1: Fifteen Degree-of-Freedom System Model

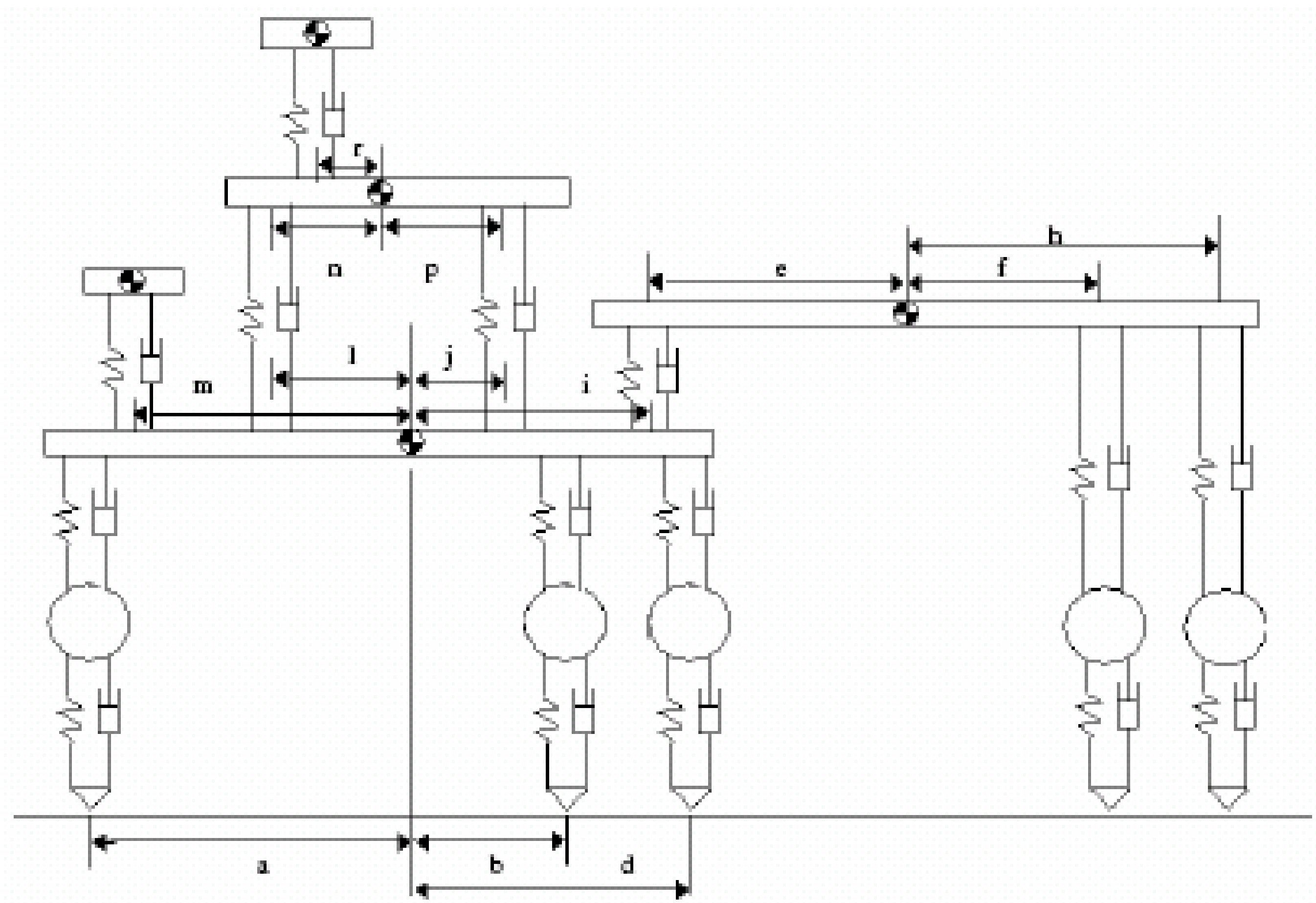


Figure A.2: Dimensions of Tractor Semi-Trailer Model

Equations of Motion for the Driver's Seat

The set of independent generalized coordinates for the vertical displacement of the driver's seat is

$$\xi_j = [z_s] \tag{A.6}$$

And the generalized nonconservative forces are defined as

$$\Xi_j = 0\,. \tag{A.7}$$

To obtain the Lagrangian, the kinetic coenergy and potential energy are defined as below:

$$T^* = \frac{1}{2} m_s \dot{z}_s^{\,2} \tag{A.8}$$

$$V = \frac{1}{2} k_s \left(z_s - z_c + r\theta_c \right)^2 . \tag{A.9}$$

This gives,

$$L = T^* - V = \frac{1}{2} m_s \dot{z}_s^{\,2} - \frac{1}{2} k_s \left(z_s - z_c + r\theta_c \right)^2 . \tag{A.10}$$

Substituting into Lagrange's equation gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{z}_s} \right) - \frac{\delta L}{\delta z_s} = 0 \tag{A.11}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$\frac{d}{dt}\left(m_s \dot{z}_s \right) - \left\{ \frac{\delta}{\delta z_s}\left(-\frac{1}{2} k_s \left[z_s^{\,2} - 2 z_c z_s + 2 r\theta_c z_s \right] \right) \right\} = 0 \tag{A.12}$$

or,

$$[m_s]\ddot{z}_s + [k_s] z_s + [-k_s] z_c + [r k_s]\theta_c = 0 \tag{A.13}$$

Equations of Motion for the Cab

The set of independent generalized coordinates for the motion of the cab is

$$\xi_j = \left[z_c, \theta_c \right] \tag{A.14}$$

And the generalized nonconservative forces are defined as

$$\Xi_j = 0\,. \tag{A.15}$$

To obtain the Lagrangian, the kinetic coenergy and potential energy are defined as below:

$$T^* = \frac{1}{2} m_c \dot{z}_c^{\,2} + \frac{1}{2} I_c \dot{\theta}_c^{\,2} \tag{A.16}$$

$$\begin{aligned} V &= \frac{1}{2} k_s \left(z_c - z_s - r\theta_c \right)^2 + \frac{1}{2} k_{cf} \left(z_c - n\theta_c - z_t + l\theta_t - f_t(a-l) q_t \right)^2 \\ &+ \frac{1}{2} k_{cr} \left(z_c + p\theta_c - z_t - j\theta_t - f_t(a+j) q_t \right)^2 \end{aligned} .$$

(A.17)

This gives,

$$\begin{aligned} L = T^* - V &= \frac{1}{2} m_c \dot{z}_c^{\,2} + \frac{1}{2} I_c \dot{\theta}_c - \frac{1}{2} k_s \left(z_c - z_s - r\theta_c \right)^2 \\ &- \frac{1}{2} k_{cf} \left(z_c - n\theta_c - z_t + l\theta_t - f_t(a-l) q_t \right)^2 \\ &- \frac{1}{2} k_{cr} \left(z_c + p\theta_c - z_t - j\theta_t - f_t(a+j) q_t \right)^2 \end{aligned} . \tag{A.18}$$

Substituting into Lagrange's equation with the generalized coordinate chosen to obtain the equation for vertical displacement of the cab gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{z}_c} \right) - \frac{\delta L}{\delta z_c} = 0 \tag{A.19}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$\frac{d}{dt}\left(m_c\dot{z}_c\right)-\left\{\frac{\delta}{\delta z_c}\begin{pmatrix}-\frac{1}{2}k_s\left[z_c{}^2-2z_sz_c-2r\theta_cz_c\right]\\-\frac{1}{2}k_{cf}\left[z_c{}^2-2n\theta_cz_c-2z_tz_c+2l\theta_tz_c-2f_t(a-l)q_tz_c\right]\\-\frac{1}{2}k_{cr}\left[z_c{}^2+2p\theta_cz_c-2z_tz_c-2j\theta_tz_c-2f_t(a+j)q_tz_c\right]\end{pmatrix}\right\}=0 \tag{A.20}$$

or,

$$\begin{aligned}&\left[-k_s\right]z_s+\left[m_c\right]\ddot{z}_c+\left[k_s+k_{cf}+k_{cr}\right]z_c+\left[-rk_s-nk_{cf}+pk_{cr}\right]\theta_c\\&+\left[-k_{cf}-k_{cr}\right]z_t+\left[lk_{cf}-jk_{cr}\right]\theta_t+\left[-k_{cf}f_t(a-l)-k_{cr}f_t(a+j)\right]q_t=0\end{aligned} \tag{A.21}$$

Substituting the Lagrangian into Lagrange's equation with the generalized coordinate chosen to obtain the equation for pitch of the cab gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta\dot{\theta}_c}\right)-\frac{\delta L}{\delta\theta_c}=0 \tag{A.22}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$\frac{d}{dt}\left(I_c\dot{\theta}_c\right)-\left\{\frac{\delta}{\delta\theta_c}\begin{pmatrix}-\frac{1}{2}k_s\left[-2rz_c\theta_c+2rz_s\theta_c+r^2\theta_c{}^2\right]\\-\frac{1}{2}k_{cf}\left[-2nz_c\theta_c+n^2\theta_c{}^2+2nz_t\theta_c-2nl\theta_t\theta_c+2nf_t(a-l)q_t\theta_c\right]\\-\frac{1}{2}k_{cr}\left[2pz_c\theta_c+p^2\theta_c{}^2-2pz_t\theta_c-2pj\theta_t\theta_c-2pf_t(a+j)q_t\theta_c\right]\end{pmatrix}\right\}=0 \tag{A.23}$$

or,

$$\begin{aligned}&\left[rk_s\right]z_s+\left[-rk_s-nk_{cf}+pk_{cr}\right]z_c+\left[I_c\right]\ddot{\theta}_c+\left[r^2k_s+n^2k_{cf}+p^2k_{cr}\right]\theta_c\\&+\left[nk_{cf}-pk_{cr}\right]z_t+\left[-nlk_{cf}-pjk_{cr}\right]\theta_t+\left[nk_{cf}f_t(a-l)-pk_{cr}f_t(a+j)\right]q_t=0\end{aligned} \tag{A.24}$$

Equations of Motion for the Engine

The set of independent generalized coordinates for the vertical displacement of the engine is

$$\xi_j = [z_e] \tag{A.25}$$

And the generalized nonconservative forces are defined as

$$\Xi_j = 0\,. \tag{A.26}$$

To obtain the Lagrangian, the kinetic coenergy and potential energy are defined as below:

$$T^* = \frac{1}{2} m_e \dot{z}_e^{\,2} \tag{A.27}$$

$$V = \frac{1}{2} k_e \left(z_e - z_t + m\theta_t - f_t(a-m)q_t \right)^2 . \tag{A.28}$$

This gives,

$$L = T^* - V = \frac{1}{2} m_e \dot{z}_e^{\,2} - \frac{1}{2} k_e \left(z_e - z_t + m\theta_t - f_t(a-m)q_t \right)^2 . \tag{A.29}$$

Substituting into Lagrange's equation gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{z}_e} \right) - \frac{\delta L}{\delta z_e} = 0 \tag{A.30}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$\frac{d}{dt}\left(m_e \dot{z}_e \right) - \left\{ \frac{\delta}{\delta z_e} \left(-\frac{1}{2} k_e \left[z_e^{\,2} - 2 z_t z_e + 2m\theta_t z_e - 2 f_t(a-m) q_t z_e \right] \right) \right\} = 0 \tag{A.31}$$

or,

$$[m_e]\ddot{z}_e + [k_e] z_e + [-k_e] z_t + [mk_e]\theta_t + [-k_e f_t(a-m)] q_t = 0 \tag{A.32}$$

Equations of Motion for the Tractor Frame

The set of independent generalized coordinates for the motion of the tractor frame is

$$\xi_j = \left[z_t, \theta_t, q_t \right] \tag{A.33}$$

And the generalized nonconservative forces are defined as

$$\Xi_j = 0 . \tag{A.34}$$

To obtain the Lagrangian, the kinetic coenergy and potential energy are defined as below:

$$T^* = \int_0^{a+d} \frac{1}{2} \rho A \left[\dot{z}_t - \left(a - x \right) \dot{\theta}_t + f_t(x) \dot{q}_t \right]^2 dx \tag{A.35}$$

$$\begin{aligned}
V &= \frac{1}{2} EI \int_0^{a+d} \left[f_t''(x) \right]^2 dx q_t^{\,2} + \frac{1}{2} k_e \left(z_t - z_e - m\theta_t + f_t(a-m) q_t \right)^2 \\
&+ \frac{1}{2} k_{cf} \left(z_t - z_c + n\theta_c - l\theta_t + f_t(a-l) q_t \right)^2 + \frac{1}{2} k_{cr} \left(z_t - z_c - p\theta_c + j\theta_t + f_t(a+j) q_t \right)^2 \\
&+ \frac{1}{2} k_{fw} \left(z_t - z_{tlr} + i\theta_t + e\theta_{tlr} + f_t(a+i) q_t - f_{tlr}(0) q_{tlr} \right)^2 \\
&+ \frac{1}{2} k_1 \left(z_t - a\theta_t + f_t(0) q_t - z_1 \right)^2 + \frac{1}{2} k_2 \left(z_t + b\theta_t + f_t(a+b) q_t - z_2 \right)^2 \\
&+ \frac{1}{2} k_3 \left(z_t + d\theta_t + f_t(a+d) q_t - z_3 \right)^2 .
\end{aligned} \tag{A.36}$$

Constructing the Lagrangian results in,

$$
\begin{aligned}
L = T^* - V = &\int_0^{a+d} \frac{1}{2}\rho A\left[\dot{z}_t - \left(a-x\right)\dot{\theta}_t + f_t(x)\dot{q}_t\right]^2 dx \\
&-\frac{1}{2}EI\int_0^{a+d}\left[f_t''(x)\right]^2 dx q_t^{\,2} - \frac{1}{2}k_e\left(z_t - z_e - m\theta_t + f_t(a-m)q_t\right)^2 \\
&-\frac{1}{2}k_{cf}\left(z_t - z_c + n\theta_c - l\theta_t + f_t(a-l)q_t\right)^2 - \frac{1}{2}k_{cr}\left(z_t - z_c - p\theta_c + j\theta_t + f_t(a+j)q_t\right)^2 \\
&-\frac{1}{2}k_{fw}\left(z_t - z_{tlr} + i\theta_t + e\theta_{tlr} + f_t(a+i)q_t - f_{tlr}(0)q_{tlr}\right)^2 \\
&-\frac{1}{2}k_1\left(z_t - a\theta_t + f_t(0)q_t - z_1\right)^2 - \frac{1}{2}k_2\left(z_t + b\theta_t + f_t(a+b)q_t - z_2\right)^2 \\
&-\frac{1}{2}k_3\left(z_t + d\theta_t + f_t(a+d)q_t - z_3\right)^2 .
\end{aligned}
\tag{A.37}
$$

Substituting into Lagrange's equation with the generalized coordinate chosen to obtain the equation for vertical displacement of the tractor frame gives,

$$
\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{z}_t}\right) - \frac{\delta L}{\delta z_t} = 0 \tag{A.38}
$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$
\begin{aligned}
&\frac{d}{dt}\left\{\frac{\delta}{\delta \dot{z}_t}\left(\int_0^{a+d}\frac{1}{2}\rho A\left[\dot{z}_t^{\,2}+2f_t(x)\dot{q}_t\dot{z}_t-2(a-x)\dot{\theta}_t\dot{z}_t\right]\right)\right\} \\
&-\left\{\frac{\delta}{\delta z_t}\left(\begin{array}{l}
-\frac{1}{2}k_e\left[z_t^{\,2}-2z_ez_t-2m\theta_tz_t+2f_t(a-m)q_tz_t\right] \\
-\frac{1}{2}k_{cf}\left[z_t^{\,2}-2z_cz_t+2n\theta_cz_t-2l\theta_tz_t+2f_t(a-l)q_tz_t\right] \\
-\frac{1}{2}k_{cr}\left[z_t^{\,2}-2z_cz_t-2p\theta_cz_t+2j\theta_tz_t+2f_t(a+j)q_tz_t\right] \\
-\frac{1}{2}k_{fw}\left[z_t^{\,2}-2z_{tlr}z_t+2i\theta_tz_t+2e\theta_{tlr}z_t+2f_t(a+i)q_tz_t-2f_{tlr}(0)q_{tlr}z_t\right] \\
-\frac{1}{2}k_1\left[z_t^{\,2}-2a\theta_tz_t+2f_t(0)q_tz_t-2z_1z_t\right] \\
-\frac{1}{2}k_2\left[z_t^{\,2}+2b\theta_tz_t+2f_t(a+b)q_tz_t-2z_2z_t\right] \\
-\frac{1}{2}k_3\left[z_t^{\,2}+2d\theta_tz_t+2f_t(a+d)q_tz_t-2z_3z_t\right]
\end{array}\right)\right\}=0
\end{aligned}
\tag{A.39}
$$

or,

$$
\begin{aligned}
&\left[-k_{cf}-k_{cr}\right]z_c+\left[nk_{cf}-pk_{cr}\right]\theta_c+\left[-k_e\right]z_e+\left[m_t\right]\ddot{z}_t \\
&+\left[k_e+k_{cf}+k_{cr}+k_{fw}+k_1+k_2+k_3\right]z_t+\left[-\int_0^{a+d}\rho A(a-x)dx\right]\ddot{\theta}_t \\
&+\left[-mk_e-lk_{cf}+jk_{cr}+ik_{fw}-ak_1+bk_2+dk_3\right]\theta_t+\left[\int_0^{a+d}\rho Af_t(x)dx\right]\ddot{q}_t \\
&+\left[\begin{array}{l} k_ef_t(a-m)+k_{cf}f_t(a-l)+k_{cr}f_t(a+j) \\ +k_{fw}f_t(a+i)+k_1f_t(0)+k_2f_t(a+b)+k_3f_t(a+d)\end{array}\right]q_t \\
&+\left[-k_{fw}\right]z_{tlr}+\left[ek_{fw}\right]\theta_{tlr}+\left[-k_{fw}f_{tlr}(0)\right]q_{tlr}+\left[-k_1\right]z_1+\left[-k_2\right]z_2+\left[-k_3\right]z_3=0
\end{aligned}
\tag{A.40}
$$

Substituting the Lagrangian into Lagrange's equation with the generalized coordinate chosen to obtain the equation for pitch of the tractor frame gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\theta}_t}\right)-\frac{\delta L}{\delta \theta_t}=0 \tag{A.41}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$\frac{d}{dt}\left\{\frac{\delta}{\delta\dot{\theta}_t}\left(\int_0^{a+d}\frac{1}{2}\rho A\left[-2(a-x)\dot{z}_t\dot{\theta}_t+(a-x)^2\dot{\theta}_t^2-2(a-x)f_t(x)\dot{q}_t\dot{\theta}_t\right]dx\right)\right\}$$

$$-\left\{\frac{\delta}{\delta\theta_t}\left(\begin{array}{l}
-\frac{1}{2}k_e\left[-2mz_t\theta_t+2mz_e\theta_t+m^2\theta_t^2-2mf_t(a-m)q_t\right]\\
-\frac{1}{2}k_{cf}\left[-2lz_t\theta_t+2lz_c\theta_t-2nl\theta_c\theta_t+l^2\theta_t^2-2lf_t(a-l)q_t\theta_t\right]\\
-\frac{1}{2}k_{cr}\left[2jz_t\theta_t-2jz_c\theta_t-2pj\theta_c\theta_t+j^2\theta_t^2+2jf_t(a+j)q_t\theta_t\right]\\
-\frac{1}{2}k_{fw}\left[\begin{array}{l}2iz_t\theta_t-2iz_{tlr}\theta_t+i^2\theta_t^2+2ei\theta_{tlr}\theta_t+2if_t(a+i)q_t\theta_t\\-2if_{tlr}(0)q_{tlr}\theta_t\end{array}\right]\\
-\frac{1}{2}k_1\left[-2az_t\theta_t+a^2\theta_t^2-2af_t(0)q_t\theta_t+2az_1\theta_t\right]\\
-\frac{1}{2}k_2\left[2bz_t\theta_t+b^2\theta_t^2+2bf_t(a+b)q_t\theta_t+2bz_2\theta_t\right]\\
-\frac{1}{2}k_3\left[2dz_t\theta_t+d^2\theta_t^2+2df_t(a+d)q_t\theta_t+2dz_3\theta_t\right]
\end{array}\right)\right\}=0 \tag{A.42}$$

or,

$$
\begin{aligned}
&\left[lk_{cf} - jk_{cr}\right]z_c + \left[-nlk_{cf} - pjk_{cr}\right]\theta_c + \left[mk_e\right]z_e + \left[-\int_0^{a+d} \rho A(a-x)dx\right]\ddot{z}_t \\
&+\left[-mk_e - lk_{cf} + jk_{cr} + ik_{fw} - ak_1 + bk_2 + dk_3\right]z_t + \left[I_t\right]\ddot{\theta}_t \\
&+\left[m^2k_e + l^2k_{cf} + j^2k_{cr} + i^2k_{fw} + a^2k_1 + b^2k_2 + d^2k_3\right]\theta_t + \left[-\int_0^{a+d} \rho A(a-x)f_t(x)dx\right]\ddot{q}_t \\
&+\left[\begin{array}{l} -mk_e f_t(a-m) - lk_{cf} f_t(a-l) + jk_{cr} f_t(a+j) \\ +ik_{fw} f_t(a+i) - ak_1 f_t(0) + bk_2 f_t(a+b) + dk_3 f_t(a+d) \end{array}\right]q_t \\
&+\left[-ik_{fw}\right]z_{tlr} + \left[eik_{fw}\right]\theta_{tlr} + \left[-ik_{fw} f_{tlr}(0)\right]q_{tlr} + \left[ak_1\right]z_1 + \left[-bk_2\right]z_2 + \left[-dk_3\right]z_3 = 0
\end{aligned}
\tag{A.43}
$$

Substituting the Lagrangian into Lagrange's equation with the generalized coordinate chosen to obtain the equation for beaming of the tractor frame gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}_t}\right) - \frac{\delta L}{\delta q_t} = 0 \tag{A.44}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$
\begin{aligned}
&\frac{d}{dt}\left\{\frac{\delta}{\delta \dot{q}_t}\left(\int_0^{a+d}\frac{1}{2}\rho A\left[2f_t(x)\dot{z}_t\dot{q}_t-2(a-x)f_t(x)\dot{\theta}_t\dot{q}_t+f_t^2(x)\dot{q}_t^{\,2}\right]dx\right)\right\}\\
&-\left\{\frac{\delta}{\delta q_t}\left(\begin{aligned}
&-\frac{1}{2}EI\int_0^{a+d}\left[f_t''(x)\right]^2 dx q_t^{\,2}\\
&-\frac{1}{2}k_e\left[\begin{aligned}&2f_t\left(a-m\right)z_tq_t-2f_t(a-m)z_eq_t-2mf_t(a-m)\theta_tq_t\\&+f_t^2(a-m)q_t^{\,2}\end{aligned}\right]\\
&-\frac{1}{2}k_{cf}\left[\begin{aligned}&2f_t\left(a-l\right)z_tq_t-2f_t(a-l)z_cq_t+2nf_t(a-l)\theta_cq_t\\&-2lf_t(a-l)\theta_tq_t+f_t^2(a-l)q_t^{\,2}\end{aligned}\right]\\
&-\frac{1}{2}k_{cr}\left[\begin{aligned}&2f_t\left(a+j\right)z_tq_t-2f_t(a+j)z_cq_t-2pf_t(a+j)\theta_cq_t\\&+2jf_t(a+j)\theta_tq_t+f_t^2(a+j)q_t^{\,2}\end{aligned}\right]\\
&-\frac{1}{2}k_{fw}\left[\begin{aligned}&2f_t\left(a+i\right)z_tq_t-2f_t(a+i)z_{tlr}q_t+2if_t(a+i)\theta_tq_t\\&+2ef_t(a+i)\theta_{tlr}q_t+f_t^2(a+i)q_t^{\,2}-2f_{tlr}(0)f_t(a+i)q_{tlr}q_t\end{aligned}\right]\\
&-\frac{1}{2}k_1\left[2f_t\left(0\right)z_tq_t-2af_t(0)\theta_tq_t+f_t^2(0)q_t^{\,2}-2f_t(0)z_1q_t\right]\\
&-\frac{1}{2}k_2\left[\begin{aligned}&2f_t\left(a+b\right)z_tq_t+2bf_t(a+b)\theta_tq_t+f_t^2(a+b)q_t^{\,2}\\&-2f_t(a+b)z_2q_t\end{aligned}\right]\\
&-\frac{1}{2}k_3\left[\begin{aligned}&2f_t\left(a+d\right)z_tq_t+2df_t(a+d)\theta_tq_t+f_t^2(a+d)q_t^{\,2}\\&-2f_t(a+d)z_3q_t\end{aligned}\right]
\end{aligned}\right)\right\}=0
\end{aligned}
\tag{A.45}
$$

or,

$$
\begin{aligned}
&\left[-k_{cf} f_t(a-l) - k_{cr} f_t(a+j)\right] z_c + \left[n k_{cf} f_t(a-l) - p k_{cr} f_t(a+j)\right] \theta_c \\
&+\left[-k_e f_t(a-m)\right] z_e + \left[\int_0^{a+d} \rho A f_t(x) dx\right] \ddot{z}_t \\
&+\begin{bmatrix} k_e f_t(a-m) + k_{cf} f_t(a-l) + k_{cr} f_t(a+j) + k_{fw} f_t(a+i) \\ +k_1 f_t(0) + k_2 f_t(a+b) + k_3 f_t(a+d) \end{bmatrix} z_t \\
&+\left[-\int_0^{a+d} \rho A (a-x) f_t(x) dx\right] \ddot{\theta}_t \\
&+\begin{bmatrix} -m k_e f_t(a-m) - l k_{cf} f_t(a-l) + j k_{cr} f_t(a+j) \\ +i k_{fw} f_t(a+i) - a k_1 f_t(0) + b k_2 f_t(a+b) + d k_3 f_t(a+d) \end{bmatrix} \theta_t \\
&+\left[\int_0^{a+d} \rho A \left[f_t(x)\right]^2 dx\right] \ddot{q}_t \\
&+\begin{bmatrix} EI \int_0^{a+d} \left[f_t''(x)\right]^2 dx + k_e f_t^2(a-m) + k_{cf} f_t^2(a-l) + k_{cr} f_t^2(a+j) \\ +k_{fw} f_t^2(a+i) + k_1 f_t^2(0) + k_2 f_t^2(a+b) + k_3 f_t^2(a+d) \end{bmatrix} q_t \\
&+\left[-k_{fw} f_t(a+i)\right] z_{tlr} + \left[e k_{fw} f_t(a+i)\right] \theta_{tlr} + \left[-k_{fw} f_{tlr}(0) f_t(a+i)\right] q_{tlr} \\
&+\left[-k_1 f_t(0)\right] z_1 + \left[-k_2 f_t(a+b)\right] z_2 + \left[-k_3 f_t(a+d)\right] z_3 = 0
\end{aligned}
$$

(A.46)

Equations of Motion for the Trailer

The set of independent generalized coordinates for the motion of the trailer is

$$\xi_j = \left[z_{tlr}, \theta_{tlr}, q_{tlr} \right] \tag{A.47}$$

And the generalized nonconservative forces are defined as

$$\Xi_j = 0\,. \tag{A.48}$$

To obtain the Lagrangian, the kinetic coenergy and potential energy are defined as below:

$$T^* = \int_0^{e+h} \frac{1}{2}\rho A\left[\dot{z}_{tlr} - (e-x)\dot{\theta}_{tlr} + f_{tlr}(x)\dot{q}_{tlr} \right]^2 dx \tag{A.49}$$

$$\begin{aligned} V = &\frac{1}{2}EI\int_0^{e+h}\left[f''_{tlr}(x)\right]^2 dx q_{tlr}{}^2 + \frac{1}{2}k_{fw}\left(-z_t + z_{tlr} - i\theta_t - e\theta_{tlr} - f_t(a+i)q_t + f_{tlr}(0)q_{tlr}\right)^2 \\ &+\frac{1}{2}k_4\left(z_{tlr} + f\theta_{tlr} + f_{tlr}(e+f)q_{tlr} - z_4\right)^2 + \frac{1}{2}k_5\left(z_{tlr} + h\theta_{tlr} + f_{tlr}(e+h)q_{tlr} - z_5\right)^2 \end{aligned}\,. \tag{A.50}$$

Constructing the Lagrangian results in,

$$\begin{aligned} L = T^* - V = &\int_0^{e+h} \frac{1}{2}\rho A\left[\dot{z}_{tlr} - (e-x)\dot{\theta}_{tlr} + f_{tlr}(x)\dot{q}_{tlr} \right]^2 dx \\ &-\frac{1}{2}EI\int_0^{e+h}\left[f''_{tlr}(x)\right]^2 dx q_{tlr}{}^2 - \frac{1}{2}k_{fw}\left(-z_t + z_{tlr} - i\theta_t - e\theta_{tlr} - f_t(a+i)q_t + f_{tlr}(0)q_{tlr}\right)^2\,. \\ &-\frac{1}{2}k_4\left(z_{tlr} + f\theta_{tlr} + f_{tlr}(e+f)q_{tlr} - z_4\right)^2 - \frac{1}{2}k_5\left(z_{tlr} + h\theta_{tlr} + f_{tlr}(e+h)q_{tlr} - z_5\right)^2 \end{aligned} \tag{A.51}$$

Substituting into Lagrange's equation with the generalized coordinate chosen to obtain the equation for vertical displacement of the trailer gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{z}_{tlr}}\right)-\frac{\delta L}{\delta z_{tlr}}=0 \tag{A.52}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$\frac{d}{dt}\left\{\frac{\delta}{\delta \dot{z}_{tlr}}\left(\int_0^{e+h}\frac{1}{2}\rho A\left[\dot{z}_{tlr}{}^2-2(e-x)\dot{\theta}_{tlr}\dot{z}_{tlr}+2f_{tlr}(x)\dot{q}_{tlr}\dot{z}_{tlr}\right]dx\right)\right\}$$
$$-\left\{\frac{\delta}{\delta z_{tlr}}\left(\begin{array}{l}-\frac{1}{2}k_{fw}\begin{bmatrix}-2z_t z_{tlr}+z_{tlr}{}^2-2i\theta_t z_{tlr}-2e\theta_{tlr}z_{tlr}-2f_t(a+i)q_t z_{tlr}\\ +2f_{tlr}(0)q_{tlr}z_{tlr}\end{bmatrix}\\ -\frac{1}{2}k_4\left[z_{tlr}{}^2+2f\theta_{tlr}z_{tlr}+2f_{tlr}(e+f)q_{tlr}z_{tlr}-2z_4 z_{tlr}\right]\\ -\frac{1}{2}k_5\left[z_{tlr}{}^2+2h\theta_{tlr}z_{tlr}+2f_{tlr}(e+h)q_{tlr}z_{tlr}-2z_5 z_{tlr}\right]\end{array}\right)\right\}=0 \tag{A.53}$$

or,

$$\begin{aligned}&\left[-k_{fw}\right]z_t+\left[-ik_{fw}\right]\theta_t+\left[-k_{fw}f_t(a+i)\right]q_t+\left[m_{tlr}\right]\ddot{z}_{tlr}+\left[k_{fw}+k_4+k_5\right]z_{tlr}\\&+\left[-\int_0^{e+h}\rho A(e-x)dx\right]\ddot{\theta}_{tlr}+\left[-ek_{fw}+fk_4+hk_5\right]\theta_{tlr}+\left[\int_0^{e+h}\rho Af_{tlr}(x)dx\right]\ddot{q}_{tlr}\\&+\left[k_{fw}f_{tlr}(0)+k_4f_{tlr}(e+f)+k_5f_{tlr}(e+h)\right]q_{tlr}+\left[-k_4\right]z_4+\left[-k_5\right]z_5=0\end{aligned} \tag{A.54}$$

Substituting the Lagrangian into Lagrange's equation with the generalized coordinate chosen to obtain the equation for pitch of the trailer gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\theta}_{tlr}}\right)-\frac{\delta L}{\delta \theta_{tlr}}=0 \tag{A.55}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$\frac{d}{dt}\left\{\frac{\delta}{\delta\dot{\theta}_{tlr}}\left(\int_0^{e+h}\frac{1}{2}\rho A\left[-2(e-x)\dot{z}_{tlr}\dot{\theta}_{tlr}+(e-x)^2\dot{\theta}_{tlr}{}^2-2(e-x)f_{tlr}(x)\dot{q}_{tlr}\dot{\theta}_{tlr}\right]dx\right)\right\}$$

$$-\left\{\frac{\delta}{\delta\theta_{tlr}}\left(\begin{array}{l}-\frac{1}{2}k_{fw}\begin{bmatrix}2ez_t\theta_{tlr}-2ez_{tlr}\theta_{tlr}+2ei\theta_t\theta_{tlr}+e^2\theta_{tlr}{}^2+2ef_t(a+i)q_t\theta_{tlr}\\-2ef_{tlr}(0)q_{tlr}\theta_{tlr}\end{bmatrix}\\-\frac{1}{2}k_4\left[2fz_{tlr}\theta_{tlr}+f^2\theta_{tlr}{}^2+2ff_{tlr}(e+f)q_{tlr}\theta_{tlr}-2fz_4\theta_{tlr}\right]\\-\frac{1}{2}k_5\left[2hz_{tlr}\theta_{tlr}+h^2\theta_{tlr}{}^2+2hf_{tlr}(e+h)q_{tlr}\theta_{tlr}-2hz_5\theta_{tlr}\right]\end{array}\right)\right\}=0$$

(A.56)

or,

$$\begin{aligned}&\left[ek_{fw}\right]z_t+\left[eik_{fw}\right]\theta_t+\left[ek_{fw}f_t(a+i)\right]q_t+\left[-\int_0^{e+h}\rho A(e-x)dx\right]\ddot{z}_{tlr}\\&+\left[-ek_{fw}+fk_4+hk_5\right]z_{tlr}+\left[I_{tlr}\right]\ddot{\theta}_{tlr}+\left[e^2k_{fw}+f^2k_4+h^2k_5\right]\theta_{tlr}\\&+\left[-\int_0^{e+h}\rho A(e-x)f_{tlr}(x)dx\right]\ddot{q}_{tlr}+\left[-ek_{fw}f_{tlr}(0)+fk_4f_{tlr}(e+f)+hk_5f_{tlr}(e+h)\right]q_{tlr}\\&+\left[-fk_4\right]z_4+\left[-hk_5\right]z_5=0\end{aligned}$$

(A.57)

Substituting the Lagrangian into Lagrange's equation with the generalized coordinate chosen to obtain the equation for beaming of the trailer gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta\dot{q}_{tlr}}\right)-\frac{\delta L}{\delta q_{tlr}}=0 \qquad \text{(A.58)}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$\frac{d}{dt}\left\{\frac{\delta}{\delta \dot{q}_{tlr}}\left(\int_0^{e+h}\frac{1}{2}\rho A\left[2f_{tlr}(x)\dot{z}_{tlr}\dot{q}_{tlr}-2(e-x)f_{tlr}(x)\dot{\theta}_{tlr}\dot{q}_{tlr}+\left(f_{tlr}(x)\right)^2\dot{q}_{tlr}{}^2\right]dx\right)\right\}$$

$$-\left\{\frac{\delta}{\delta q_{tlr}}\left(\begin{array}{l}-\frac{1}{2}EI\int_0^{e+h}\left[f_{tlr}''(x)\right]^2dxq_{tlr}{}^2\\ -\frac{1}{2}k_{fw}\begin{bmatrix}-2f_{tlr}(0)z_tq_{tlr}+2f_{tlr}(0)z_{tlr}q_{tlr}-2if_{tlr}(0)\theta_tq_{tlr}\\ -2ef_{tlr}(0)\theta_{tlr}q_{tlr}-2f_t(a+i)f_{tlr}(0)q_tq_{tlr}+f_{tlr}{}^2(0)q_{tlr}{}^2\end{bmatrix}\\ -\frac{1}{2}k_4\begin{bmatrix}2f_{tlr}(e+f)z_{tlr}q_{tlr}+2ff_{tlr}(e+f)\theta_{tlr}q_{tlr}+f_{tlr}{}^2(e+f)q_{tlr}{}^2\\ -2f_{tlr}(e+f)z_4q_{tlr}\end{bmatrix}\\ -\frac{1}{2}k_5\begin{bmatrix}2f_{tlr}(e+h)z_{tlr}q_{tlr}+2hf_{tlr}(e+h)\theta_{tlr}q_{tlr}+f_{tlr}{}^2(e+h)q_{tlr}{}^2\\ -2f_{tlr}(e+h)z_5q_{tlr}\end{bmatrix}\end{array}\right)\right\}=0$$

(A.59)

or,

$$\begin{aligned}&\left[-k_{fw}f_{tlr}(0)\right]z_t+\left[-ik_{fw}f_{tlr}(0)\right]\theta_t+\left[-k_{fw}f_t(a+i)f_{tlr}(0)\right]q_t+\left[\int_0^{e+h}\rho Af_{tlr}(x)dx\right]\ddot{z}_{tlr}\\ &+\left[k_{fw}f_{tlr}(0)+k_4f_{tlr}(e+f)+k_5f_{tlr}(e+h)\right]z_{tlr}+\left[-\int_0^{e+h}\rho A(e-x)f_{tlr}(x)dx\right]\ddot{\theta}_{tlr}\\ &+\left[-ek_{fw}f_{tlr}(0)+fk_4f_{tlr}(e+f)+hk_5f_{tlr}(e+h)\right]\theta_{tlr}+\left[\int_0^{e+h}\rho A\left(f_{tlr}(x)\right)^2dx\right]\ddot{q}_{tlr}\\ &+\left[EI\int_0^{e+h}\left[f_{tlr}''(x)\right]^2dx+k_{fw}f_{tlr}{}^2(0)+k_4f_{tlr}{}^2(e+f)+k_5f_{tlr}{}^2(e+h)\right]q_{tlr}\\ &+\left[-k_4f_{tlr}(e+f)\right]z_4+\left[-k_5f_{tlr}(e+h)\right]z_5=0\end{aligned}$$

(A.60)

Equation of Motion for Axle #1

The set of independent generalized coordinates for the vertical displacement of axle #1 is

$$\xi_j = \left[z_1\right] \tag{A.61}$$

And the generalized nonconservative forces are defined as

$$\Xi_j = \left[k_{t1}\right] z_{r1} . \tag{A.62}$$

To obtain the Lagrangian, the kinetic coenergy and potential energy are defined as below:

$$T^* = \frac{1}{2} m_1 \dot{z}_1^{\,2} \tag{A.63}$$

$$V = \frac{1}{2} k_1 \left(z_1 - z_t + a\theta_t - f_t(0) q_t \right)^2 + \frac{1}{2} k_{t1} (z_1)^2 . \tag{A.64}$$

This gives,

$$L = T^* - V = \frac{1}{2} m_1 \dot{z}_1^{\,2} - \frac{1}{2} k_1 \left(z_1 - z_t + a\theta_t - f_t(0) q_t \right)^2 - \frac{1}{2} k_{t1} (z_1)^2 . \tag{A.65}$$

Substituting into Lagrange's equation gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{z}_1} \right) - \frac{\delta L}{\delta z_1} = \left[k_{t1}\right] z_{r1} \tag{A.66}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$\frac{d}{dt}\left(m_1 \dot{z}_1 \right) - \left\{ \frac{\delta}{\delta z_1} \left(-\frac{1}{2} k_1 \left[z_1^{\,2} - 2 z_t z_1 + 2 a\theta_t z_1 - 2 f_t(0) q_t z_1 \right] - \frac{1}{2} k_{t1} \left(z_1^{\,2} \right) \right) \right\} = \left[k_{t1}\right] z_{r1} \tag{A.67}$$

or,

$$\left[-k_1\right] z_t + \left[a k_1\right] \theta_t + \left[-k_1 f_t(0)\right] q_t + \left[m_1\right] \ddot{z}_1 + \left[k_1 + k_{t1}\right] z_1 = \left[k_{t1}\right] z_{r1} \tag{A.68}$$

Equation of Motion for Axle #2

The set of independent generalized coordinates for the vertical displacement of axle #2 is

$$\xi_j = [z_2] \tag{A.69}$$

And the generalized nonconservative forces are defined as

$$\Xi_j = [k_{t2}] z_{r2} . \tag{A.70}$$

To obtain the Lagrangian, the kinetic coenergy and potential energy are defined as below:

$$T^* = \frac{1}{2} m_2 \dot{z}_2{}^2 \tag{A.71}$$

$$V = \frac{1}{2} k_2 \left(z_2 - z_t - b\theta_t - f_t(a+b) q_t \right)^2 + \frac{1}{2} k_{t2} (z_2)^2 . \tag{A.72}$$

This gives,

$$L = T^* - V = \frac{1}{2} m_2 \dot{z}_2{}^2 - \frac{1}{2} k_2 \left(z_2 - z_t - b\theta_t - f_t(a+b) q_t \right)^2 - \frac{1}{2} k_{t2} (z_2)^2 . \tag{A.73}$$

Substituting into Lagrange's equation gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{z}_2} \right) - \frac{\delta L}{\delta z_2} = [k_{t2}] z_{r2} \tag{A.74}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$\frac{d}{dt}\left(m_2 \dot{z}_2 \right) - \left\{ \frac{\delta}{\delta z_2} \begin{pmatrix} -\frac{1}{2} k_2 \left[z_2{}^2 - 2 z_t z_2 - 2 b \theta_t z_2 - 2 f_t (a+b) q_t z_2 \right] \\ -\frac{1}{2} k_{t2} \left(z_2{}^2 \right) \end{pmatrix} \right\} = [k_{t2}] z_{r2} \tag{A.75}$$

or,

$$[-k_2] z_t + [-b k_2] \theta_t + [-k_2 f_t (a+b)] q_t + [m_2] \ddot{z}_2 + [k_2 + k_{t2}] z_2 = [k_{t2}] z_{r2} \tag{A.76}$$

Equation of Motion for Axle #3

The set of independent generalized coordinates for the vertical displacement of axle #3 is

$$\xi_j = [z_3] \tag{A.77}$$

And the generalized nonconservative forces are defined as

$$\Xi_j = [k_{t3}]z_{r3}\,. \tag{A.78}$$

To obtain the Lagrangian, the kinetic coenergy and potential energy are defined as below:

$$T^* = \frac{1}{2}m_3\dot{z}_3^{\,2} \tag{A.79}$$

$$V = \frac{1}{2}k_3\left(z_3 - z_t - d\theta_t - f_t(a+d)q_t\right)^2 + \frac{1}{2}k_{t3}(z_3)^2\,. \tag{A.80}$$

This gives,

$$L = T^* - V = \frac{1}{2}m_3\dot{z}_3^{\,2} - \frac{1}{2}k_3\left(z_3 - z_t - d\theta_t - f_t(a+d)q_t\right)^2 - \frac{1}{2}k_{t3}(z_3)^2\,. \tag{A.81}$$

Substituting into Lagrange’s equation gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{z}_3}\right) - \frac{\delta L}{\delta z_3} = [k_{t3}]z_{r3} \tag{A.82}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$\frac{d}{dt}\left(m_3\dot{z}_3\right) - \left\{\frac{\delta}{\delta z_3}\left(\begin{array}{l} -\frac{1}{2}k_3\left[z_3^{\,2} - 2z_t z_3 - 2d\theta_t z_3 - 2f_t(a+d)q_t z_3\right] \\ -\frac{1}{2}k_{t3}\left(z_3^{\,2}\right) \end{array}\right)\right\} = [k_{t3}]z_{r3} \tag{A.83}$$

or,

$$[-k_3]z_t + [-dk_3]\theta_t + [-k_3 f_t(a+d)]q_t + [m_3]\ddot{z}_3 + [k_3 + k_{t3}]z_3 = [k_{t3}]z_{r3} \tag{A.84}$$

Equation of Motion for Axle #4

The set of independent generalized coordinates for the vertical displacement of axle #4 is

$$\xi_j = [z_4] \tag{A.85}$$

And the generalized nonconservative forces are defined as

$$\Xi_j = [k_{t4}] z_{r4} . \tag{A.86}$$

To obtain the Lagrangian, the kinetic coenergy and potential energy are defined as below:

$$T^* = \frac{1}{2} m_4 \dot{z}_4^{\,2} \tag{A.87}$$

$$V = \frac{1}{2} k_4 \left(z_4 - z_{tlr} - f\theta_{tlr} - f_{tlr}(e+f) q_{tlr} \right)^2 + \frac{1}{2} k_{t4} (z_4)^2 . \tag{A.88}$$

This gives,

$$L = T^* - V = \frac{1}{2} m_4 \dot{z}_4^{\,2} - \frac{1}{2} k_4 \left(z_4 - z_{tlr} - f\theta_{tlr} - f_{tlr}(e+f) q_{tlr} \right)^2 - \frac{1}{2} k_{t4} (z_4)^2 . \tag{A.89}$$

Substituting into Lagrange's equation gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{z}_4} \right) - \frac{\delta L}{\delta z_4} = [k_{t4}] z_{r4} \tag{A.90}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$\frac{d}{dt}(m_4 \dot{z}_4) - \left\{ \frac{\delta}{\delta z_4} \begin{pmatrix} -\frac{1}{2} k_4 \left[z_4^{\,2} - 2 z_{tlr} z_4 - 2 f \theta_{tlr} z_4 - 2 f_{tlr} (e+f) q_{tlr} z_4 \right] \\ -\frac{1}{2} k_{t4} \left(z_4^{\,2} \right) \end{pmatrix} \right\} = [k_{t4}] z_{r4} \tag{A.91}$$

or,

$$\left[-k_4\right]z_{tlr}+\left[-fk_4\right]\theta_{tlr}+\left[-k_4 f_{tlr}(e+f)\right]q_{tlr}+\left[m_4\right]\ddot{z}_4+\left[k_4+k_{t4}\right]z_4=\left[k_{t4}\right]z_{r4} \tag{A.92}$$

Equation of Motion for Axle #5

The set of independent generalized coordinates for the vertical displacement of axle #5 is

$$\xi_j=\left[z_5\right] \tag{A.93}$$

And the generalized nonconservative forces are defined as

$$\Xi_j=\left[k_{t5}\right]z_{r5}\,. \tag{A.94}$$

To obtain the Lagrangian, the kinetic coenergy and potential energy are defined as below:

$$T^*=\frac{1}{2}m_5\dot{z}_5^{\,2} \tag{A.95}$$

$$V=\frac{1}{2}k_5\left(z_5-z_{tlr}-h\theta_{tlr}-f_{tlr}(e+h)q_{tlr}\right)^2+\frac{1}{2}k_{t5}(z_5)^2\,. \tag{A.96}$$

This gives,

$$L=T^*-V=\frac{1}{2}m_5\dot{z}_5^{\,2}-\frac{1}{2}k_5\left(z_5-z_{tlr}-h\theta_{tlr}-f_{tlr}(e+h)q_{tlr}\right)^2-\frac{1}{2}k_{t5}(z_5)^2\,. \tag{A.97}$$

Substituting into Lagrange’s equation gives,

$$\frac{d}{dt}\left(\frac{\delta L}{\delta\dot{z}_5}\right)-\frac{\delta L}{\delta z_5}=\left[k_{t5}\right]z_{r5} \tag{A.98}$$

Simplifying and removing terms that do not include the generalized coordinate yields:

$$\frac{d}{dt}\left(m_5\dot{z}_5\right)-\left\{\frac{\delta}{\delta z_5}\begin{pmatrix}-\frac{1}{2}k_5\left[z_5{}^2-2z_{tlr}z_5-2h\theta_{tlr}z_5-2f_{tlr}(e+h)q_{tlr}z_5\right]\\ -\frac{1}{2}k_{t5}\left(z_5{}^2\right)\end{pmatrix}\right\}=\left[k_{t5}\right]z_{r5}$$

(A.99)

or,

$$\left[-k_5\right]z_{tlr}+\left[-hk_5\right]\theta_{tlr}+\left[-k_5f_{tlr}(e+h)\right]q_{tlr}+\left[m_5\right]\ddot{z}_5+\left[k_5+k_{t5}\right]z_5=\left[k_{t5}\right]z_{r5}$$

(A.100)

The damping terms may now be inserted into the equations of motion derived in the previous pages. The final form of the equation for each of the DOFs are listed below.

Driver's Seat Vertical Displacement

$$\left[m_s\right]\ddot{z}_s + \left[c_s\right]\dot{z}_s + \left[k_s\right]z_s + \left[-c_s\right]\dot{z}_c + \left[-k_s\right]z_c + \left[rc_s\right]\dot{\theta}_c + \left[rk_s\right]\theta_c = 0 \quad \text{(A.101)}$$

Cab Vertical Displacement

$$\begin{aligned}
&\left[-c_s\right]\dot{z}_s + \left[-k_s\right]z_s + \left[m_c\right]\ddot{z}_c + \left[c_s + c_{cf} + c_{cr}\right]\dot{z}_c + \left[k_s + k_{cf} + k_{cr}\right]z_c \\
&+\left[-rc_s - nc_{cf} + pc_{cr}\right]\dot{\theta}_c + \left[-rk_s - nk_{cf} + pk_{cr}\right]\theta_c + \left[-c_{cf} - c_{cr}\right]\dot{z}_t \\
&+\left[-k_{cf} - k_{cr}\right]z_t + \left[lc_{cf} - jc_{cr}\right]\dot{\theta}_t + \left[lk_{cf} - jk_{cr}\right]\theta_t \\
&+\left[-c_{cf} f_t(a-l) - c_{cr} f_t(a+j)\right]\dot{q}_t + \left[-k_{cf} f_t(a-l) - k_{cr} f_t(a+j)\right]q_t = 0
\end{aligned} \quad \text{(A.102)}$$

Cab Pitch

$$\begin{aligned}
&\left[rc_s\right]\dot{z}_s + \left[rk_s\right]z_s + \left[-rc_s - nc_{cf} + pc_{cr}\right]\dot{z}_c + \left[-rk_s - nk_{cf} + pk_{cr}\right]z_c + \left[I_c\right]\ddot{\theta}_c \\
&+\left[r^2c_s + n^2c_{cf} + p^2c_{cr}\right]\dot{\theta}_c + \left[r^2k_s + n^2k_{cf} + p^2k_{cr}\right]\theta_c + \left[nc_{cf} - pc_{cr}\right]\dot{z}_t \\
&+\left[nk_{cf} - pk_{cr}\right]z_t + \left[-nlc_{cf} - pjc_{cr}\right]\dot{\theta}_t + \left[-nlk_{cf} - pjk_{cr}\right]\theta_t \\
&+\left[nc_{cf} f_t(a-l) - pc_{cr} f_t(a+j)\right]\dot{q}_t + \left[nk_{cf} f_t(a-l) - pk_{cr} f_t(a+j)\right]q_t = 0
\end{aligned} \quad \text{(A.103)}$$

Engine Vertical Displacement

$$\begin{aligned}
&\left[m_e\right]\ddot{z}_e + \left[c_e\right]\dot{z}_e + \left[k_e\right]z_e + \left[-c_e\right]\dot{z}_t + \left[-k_e\right]z_t + \left[mc_e\right]\dot{\theta}_t + \left[mk_e\right]\theta_t \\
&+\left[-c_e f_t(a-m)\right]\dot{q}_t + \left[-k_e f_t(a-m)\right]q_t = 0
\end{aligned} \quad \text{(A.104)}$$

Tractor Frame Vertical Displacement

$$
\begin{aligned}
&\left[-c_{cf}-c_{cr}\right]\dot{z}_c+\left[-k_{cf}-k_{cr}\right]z_c+\left[nc_{cf}-pc_{cr}\right]\dot{\theta}_c+\left[nk_{cf}-pk_{cr}\right]\theta_c \\
&+\left[-c_e\right]\dot{z}_e+\left[-k_e\right]z_e+\left[m_t\right]\ddot{z}_t+\left[c_e+c_{cf}+c_{cr}+c_{fw}+c_1+c_2+c_3\right]\dot{z}_t \\
&+\left[k_e+k_{cf}+k_{cr}+k_{fw}+k_1+k_2+k_3\right]z_t+\left[-\int_0^{a+d}\rho A(a-x)dx\right]\ddot{\theta}_t \\
&+\left[-mc_e-lc_{cf}+jc_{cr}+ic_{fw}-ac_1+bc_2+dc_3\right]\dot{\theta}_t \\
&+\left[-mk_e-lk_{cf}+jk_{cr}+ik_{fw}-ak_1+bk_2+dk_3\right]\theta_t+\left[\int_0^{a+d}\rho Af_t(x)dx\right]\ddot{q}_t \\
&+\begin{bmatrix} c_e f_t(a-m)+c_{cf}f_t(a-l)+c_{cr}f_t(a+j) \\ +c_{fw}f_t(a+i)+c_1 f_t(0)+c_2 f_t(a+b)+c_3 f_t(a+d)\end{bmatrix}\dot{q}_t \\
&+\begin{bmatrix} k_e f_t(a-m)+k_{cf}f_t(a-l)+k_{cr}f_t(a+j) \\ +k_{fw}f_t(a+i)+k_1 f_t(0)+k_2 f_t(a+b)+k_3 f_t(a+d)\end{bmatrix}q_t \\
&+\left[-c_{fw}\right]\dot{z}_{tlr}+\left[-k_{fw}\right]z_{tlr}+\left[ec_{fw}\right]\dot{\theta}_{tlr}+\left[ek_{fw}\right]\theta_{tlr}+\left[-c_{fw}f_{tlr}(0)\right]\dot{q}_{tlr} \\
&+\left[-k_{fw}f_{tlr}(0)\right]q_{tlr}+\left[-c_1\right]\dot{z}_1+\left[-k_1\right]z_1+\left[-c_2\right]\dot{z}_2+\left[-k_2\right]z_2 \\
&+\left[-c_3\right]\dot{z}_3+\left[-k_3\right]z_3=0
\end{aligned}
\tag{A.105}
$$

Tractor Frame Pitch

$$
\begin{aligned}
&\left[lc_{cf} - jc_{cr}\right]\dot{z}_c + \left[lk_{cf} - jk_{cr}\right]z_c + \left[-nlc_{cf} - pjc_{cr}\right]\dot{\theta}_c + \left[-nlk_{cf} - pjk_{cr}\right]\theta_c \\
&+\left[mc_e\right]\dot{z}_e + \left[mk_e\right]z_e + \left[-\int_0^{a+d} \rho A(a-x)dx\right]\ddot{z}_t \\
&+\left[-mc_e - lc_{cf} + jc_{cr} + ic_{fw} - ac_1 + bc_2 + dc_3\right]\dot{z}_t \\
&+\left[-mk_e - lk_{cf} + jk_{cr} + ik_{fw} - ak_1 + bk_2 + dk_3\right]z_t + \left[I_t\right]\ddot{\theta}_t \\
&+\left[m^2c_e + l^2c_{cf} + j^2c_{cr} + i^2c_{fw} + a^2c_1 + b^2c_2 + d^2c_3\right]\dot{\theta}_t \\
&+\left[m^2k_e + l^2k_{cf} + j^2k_{cr} + i^2k_{fw} + a^2k_1 + b^2k_2 + d^2k_3\right]\theta_t \\
&+\left[-\int_0^{a+d} \rho A(a-x)f_t(x)dx\right]\ddot{q}_t \\
&+\left[\begin{array}{l} -mc_e f_t(a-m) - lc_{cf} f_t(a-l) + jc_{cr} f_t(a+j) \\ +ic_{fw} f_t(a+i) - ac_1 f_t(0) + bc_2 f_t(a+b) + dc_3 f_t(a+d) \end{array}\right]\dot{q}_t \\
&+\left[\begin{array}{l} -mk_e f_t(a-m) - lk_{cf} f_t(a-l) + jk_{cr} f_t(a+j) \\ +ik_{fw} f_t(a+i) - ak_1 f_t(0) + bk_2 f_t(a+b) + dk_3 f_t(a+d) \end{array}\right]q_t \\
&+\left[-ic_{fw}\right]\dot{z}_{tlr} + \left[-ik_{fw}\right]z_{tlr} + \left[eic_{fw}\right]\dot{\theta}_{tlr} + \left[eik_{fw}\right]\theta_{tlr} + \left[-ic_{fw} f_{tlr}(0)\right]\dot{q}_{tlr} \\
&+\left[-ik_{fw} f_{tlr}(0)\right]q_{tlr} + \left[ac_1\right]\dot{z}_1 + \left[ak_1\right]z_1 + \left[-bc_2\right]\dot{z}_2 + \left[-bk_2\right]z_2 \\
&+\left[-dc_3\right]\dot{z}_3 + \left[-dk_3\right]z_3 = 0
\end{aligned}
\tag{A.106}
$$

Tractor Frame Beaming

$$
\begin{aligned}
&\left[-c_{cf} f_t(a-l) - c_{cr} f_t(a+j)\right]\dot{z}_c + \left[-k_{cf} f_t(a-l) - k_{cr} f_t(a+j)\right] z_c \\
&+\left[nc_{cf} f_t(a-l) - pc_{cr} f_t(a+j)\right]\dot{\theta}_c + \left[nk_{cf} f_t(a-l) - pk_{cr} f_t(a+j)\right]\theta_c \\
&+\left[-c_e f_t(a-m)\right]\dot{z}_e + \left[-k_e f_t(a-m)\right] z_e + \left[\int_0^{a+d} \rho A f_t(x)dx\right]\ddot{z}_t \\
&+\begin{bmatrix} c_e f_t(a-m) + c_{cf} f_t(a-l) + c_{cr} f_t(a+j) + c_{fw} f_t(a+i) \\ +c_1 f_t(0) + c_2 f_t(a+b) + c_3 f_t(a+d) \end{bmatrix}\dot{z}_t \\
&+\begin{bmatrix} k_e f_t(a-m) + k_{cf} f_t(a-l) + k_{cr} f_t(a+j) + k_{fw} f_t(a+i) \\ +k_1 f_t(0) + k_2 f_t(a+b) + k_3 f_t(a+d) \end{bmatrix} z_t \\
&+\left[-\int_0^{a+d} \rho A(a-x) f_t(x)dx\right]\ddot{\theta}_t \\
&+\begin{bmatrix} -mc_e f_t(a-m) - lc_{cf} f_t(a-l) + jc_{cr} f_t(a+j) \\ +ic_{fw} f_t(a+i) - ac_1 f_t(0) + bc_2 f_t(a+b) + dc_3 f_t(a+d) \end{bmatrix}\dot{\theta}_t \\
&+\begin{bmatrix} -mk_e f_t(a-m) - lk_{cf} f_t(a-l) + jk_{cr} f_t(a+j) \\ +ik_{fw} f_t(a+i) - ak_1 f_t(0) + bk_2 f_t(a+b) + dk_3 f_t(a+d) \end{bmatrix}\theta_t \\
&+\left[\int_0^{a+d} \rho A\left[f_t(x)\right]^2 dx\right]\ddot{q}_t \\
&+\begin{bmatrix} c_e f_t^2(a-m) + c_{cf} f_t^2(a-l) + c_{cr} f_t^2(a+j) \\ +c_{fw} f_t^2(a+i) + c_1 f_t^2(0) + c_2 f_t^2(a+b) + c_3 f_t^2(a+d) \end{bmatrix}\dot{q}_t \\
&+\begin{bmatrix} EI\displaystyle\int_0^{a+d}\left[f_t''(x)\right]^2 dx + k_e f_t^2(a-m) + k_{cf} f_t^2(a-l) + k_{cr} f_t^2(a+j) \\ +k_{fw} f_t^2(a+i) + k_1 f_t^2(0) + k_2 f_t^2(a+b) + k_3 f_t^2(a+d) \end{bmatrix} q_t \\
&+\left[-c_{fw} f_t(a+i)\right]\dot{z}_{tlr} + \left[-k_{fw} f_t(a+i)\right] z_{tlr} + \left[ec_{fw} f_t(a+i)\right]\dot{\theta}_{tlr} \\
&+\left[ek_{fw} f_t(a+i)\right]\theta_{tlr} + \left[-c_{fw} f_{tlr}(0) f_t(a+i)\right]\dot{q}_{tlr} + \left[-k_{fw} f_{tlr}(0) f_t(a+i)\right] q_{tlr} \\
&+\left[-c_1 f_t(0)\right]\dot{z}_1 + \left[-k_1 f_t(0)\right] z_1 + \left[-c_2 f_t(a+b)\right]\dot{z}_2 + \left[-k_2 f_t(a+b)\right] z_2 \\
&+\left[-c_3 f_t(a+d)\right]\dot{z}_3 + \left[-k_3 f_t(a+d)\right] z_3 = 0
\end{aligned}
$$

(A.107)

Trailer Vertical Displacement

$$
\begin{aligned}
&\left[-c_{fw}\right]\dot{z}_t+\left[-k_{fw}\right]z_t+\left[-ic_{fw}\right]\dot{\theta}_t+\left[-ik_{fw}\right]\theta_t+\left[-c_{fw}f_t(a+i)\right]\dot{q}_t\\
&+\left[-k_{fw}f_t(a+i)\right]q_t+\left[m_{tlr}\right]\ddot{z}_{tlr}+\left[c_{fw}+c_4+c_5\right]\dot{z}_{tlr}+\left[k_{fw}+k_4+k_5\right]z_{tlr}\\
&+\left[-\int_0^{e+h}\rho A(e-x)dx\right]\ddot{\theta}_{tlr}+\left[-ec_{fw}+fc_4+hc_5\right]\dot{\theta}_{tlr}\\
&+\left[-ek_{fw}+fk_4+hk_5\right]\theta_{tlr}+\left[\int_0^{e+h}\rho Af_{tlr}(x)dx\right]\ddot{q}_{tlr}\\
&+\left[c_{fw}f_{tlr}(0)+c_4f_{tlr}(e+f)+c_5f_{tlr}(e+h)\right]\dot{q}_{tlr}\\
&+\left[k_{fw}f_{tlr}(0)+k_4f_{tlr}(e+f)+k_5f_{tlr}(e+h)\right]q_{tlr}+\left[-c_4\right]\dot{z}_4+\left[-k_4\right]z_4\\
&+\left[-c_5\right]\dot{z}_5+\left[-k_5\right]z_5=0
\end{aligned}
\tag{A.108}
$$

Trailer Pitch

$$
\begin{aligned}
&\left[ec_{fw}\right]\dot{z}_t+\left[ek_{fw}\right]z_t+\left[eic_{fw}\right]\dot{\theta}_t+\left[eik_{fw}\right]\theta_t+\left[ec_{fw}f_t(a+i)\right]\dot{q}_t\\
&+\left[ek_{fw}f_t(a+i)\right]q_t+\left[-\int_0^{e+h}\rho A(e-x)dx\right]\ddot{z}_{tlr}+\left[-ec_{fw}+fc_4+hc_5\right]\dot{z}_{tlr}\\
&+\left[-ek_{fw}+fk_4+hk_5\right]z_{tlr}+\left[I_{tlr}\right]\ddot{\theta}_{tlr}+\left[e^2c_{fw}+f^2c_4+h^2c_5\right]\dot{\theta}_{tlr}\\
&+\left[e^2k_{fw}+f^2k_4+h^2k_5\right]\theta_{tlr}+\left[-\int_0^{e+h}\rho A(e-x)f_{tlr}(x)dx\right]\ddot{q}_{tlr}\\
&+\left[-ec_{fw}f_{tlr}(0)+fc_4f_{tlr}(e+f)+hc_5f_{tlr}(e+h)\right]\dot{q}_{tlr}\\
&+\left[-ek_{fw}f_{tlr}(0)+fk_4f_{tlr}(e+f)+hk_5f_{tlr}(e+h)\right]q_{tlr}\\
&+\left[-fc_4\right]\dot{z}_4+\left[-fk_4\right]z_4+\left[-hc_5\right]\dot{z}_5+\left[-hk_5\right]z_5=0
\end{aligned}
\tag{A.109}
$$

Trailer Beaming

$$
\begin{aligned}
&\left[-c_{fw}f_{tlr}(0)\right]\dot{z}_t+\left[-k_{fw}f_{tlr}(0)\right]z_t+\left[-ic_{fw}f_{tlr}(0)\right]\dot{\theta}_t+\left[-ik_{fw}f_{tlr}(0)\right]\theta_t\\
&+\left[-c_{fw}f_t(a+i)f_{tlr}(0)\right]\dot{q}_t+\left[-k_{fw}f_t(a+i)f_{tlr}(0)\right]q_t\\
&+\left[\int_0^{e+h}\rho A f_{tlr}(x)dx\right]\ddot{z}_{tlr}+\left[c_{fw}f_{tlr}(0)+c_4f_{tlr}(e+f)+c_5f_{tlr}(e+h)\right]\dot{z}_{tlr}\\
&+\left[k_{fw}f_{tlr}(0)+k_4f_{tlr}(e+f)+k_5f_{tlr}(e+h)\right]z_{tlr}\\
&+\left[-\int_0^{e+h}\rho A(e-x)f_{tlr}(x)dx\right]\ddot{\theta}_{tlr}\\
&+\left[-ec_{fw}f_{tlr}(0)+fc_4f_{tlr}(e+f)+hc_5f_{tlr}(e+h)\right]\dot{\theta}_{tlr}\\
&+\left[-ek_{fw}f_{tlr}(0)+fk_4f_{tlr}(e+f)+hk_5f_{tlr}(e+h)\right]\theta_{tlr}\\
&+\left[\int_0^{e+h}\rho A\left(f_{tlr}(x)\right)^2dx\right]\ddot{q}_{tlr}\\
&+\left[c_{fw}f_{tlr}{}^2(0)+c_4f_{tlr}{}^2(e+f)+c_5f_{tlr}{}^2(e+h)\right]\dot{q}_{tlr}\\
&+\left[EI\int_0^{e+h}\left[f_{tlr}''(x)\right]dx+k_{fw}f_{tlr}{}^2(0)+k_4f_{tlr}{}^2(e+f)+k_5f_{tlr}{}^2(e+h)\right]q_{tlr}\\
&+\left[-c_4f_{tlr}(e+f)\right]\dot{z}_4+\left[-k_4f_{tlr}(e+f)\right]z_4\\
&+\left[-c_5f_{tlr}(e+h)\right]\dot{z}_5+\left[-k_5f_{tlr}(e+h)\right]z_5=0
\end{aligned}
\tag{A.110}
$$

Vertical Displacement of Axle #1

$$
\begin{aligned}
&\left[-c_1\right]\dot{z}_t+\left[-k_1\right]z_t+\left[ac_1\right]\dot{\theta}_t+\left[ak_1\right]\theta_t+\left[-c_1f_t(0)\right]\dot{q}_t+\left[-k_1f_t(0)\right]q_t\\
&+\left[m_1\right]\ddot{z}_1+\left[c_1+c_{t1}\right]\dot{z}_1+\left[k_1+k_{t1}\right]z_1=\left[c_{t1}\right]\dot{z}_{r1}+\left[k_{t1}\right]z_{r1}
\end{aligned}
\tag{A.111}
$$

Vertical Displacement of Axle #2

$$
\begin{aligned}
&\left[-c_2\right]\dot{z}_t+\left[-k_2\right]z_t+\left[-bc_2\right]\dot{\theta}_t+\left[-bk_2\right]\theta_t+\left[-c_2f_t(a+b)\right]\dot{q}_t+\left[-k_2f_t(a+b)\right]q_t\\
&+\left[m_2\right]\ddot{z}_2+\left[c_2+c_{t2}\right]\dot{z}_2+\left[k_2+k_{t2}\right]z_2=\left[c_{t2}\right]\dot{z}_{r2}+\left[k_{t2}\right]z_{r2}
\end{aligned}
\tag{A.112}
$$

Vertical Displacement of Axle #3

$$\left[-c_3\right]\dot{z}_t+\left[-k_3\right]z_t+\left[-dc_3\right]\dot{\theta}_t+\left[-dk_3\right]\theta_t+\left[-c_3 f_t(a+d)\right]\dot{q}_t+\left[-k_3 f_t(a+d)\right]q_t$$
$$+\left[m_3\right]\ddot{z}_3+\left[c_3+c_{t3}\right]\dot{z}_3+\left[k_3+k_{t3}\right]z_3=\left[c_{t3}\right]\dot{z}_{r3}+\left[k_{t3}\right]z_{r3}$$

(A.113)

Vertical Displacement of Axle #4

$$\left[-c_4\right]\dot{z}_{tlr}+\left[-k_4\right]z_{tlr}+\left[-fc_4\right]\dot{\theta}_{tlr}+\left[-fk_4\right]\theta_{tlr}+\left[-c_4 f_{tlr}(e+f)\right]\dot{q}_{tlr}+\left[-k_4 f_{tlr}(e+f)\right]q_{tlr}$$
$$+\left[m_4\right]\ddot{z}_4+\left[c_4+c_{t4}\right]\dot{z}_4+\left[k_4+k_{t4}\right]z_4=\left[c_{t4}\right]\dot{z}_{r4}+\left[k_{t4}\right]z_{r4}$$

(A.114)

Vertical Displacement of Axle #5

$$\left[-c_5\right]\dot{z}_{tlr}+\left[-k_5\right]z_{tlr}+\left[-hc_5\right]\dot{\theta}_{tlr}+\left[-hk_5\right]\theta_{tlr}+\left[-c_5 f_{tlr}(e+h)\right]\dot{q}_{tlr}+\left[-k_5 f_{tlr}(e+h)\right]q_{tlr}$$
$$+\left[m_5\right]\ddot{z}_5+\left[c_5+c_{t5}\right]\dot{z}_5+\left[k_5+k_{t5}\right]z_5=\left[c_{t5}\right]\dot{z}_{r5}+\left[k_{t5}\right]z_{r5}$$

(A.115)

Appendix B: Tractor and trailer beaming equations

To include the effects that flexible frames have on the tractor semi-trailer ride dynamics, both the tractor and trailer frames are modeled as Euler-Bernoulli flexible beams. The details for both frame bending modes are discussed in Chapter 2. This appendix serves to display the derivation process used to obtain the mode shape equations used for the tractor and trailer frames. When a conventional fifth wheel connection is used, the tractor and trailer frames are modeled as “free-pinned” and “pinned-free”, respectively. However, when a fifth wheel suspension system is present, both are modeled as “free-free” Euler-Bernoulli beams. The mode shapes for all three beam types are derived in this appendix.

“Free-Pinned” Mode Shape Equation

When a conventional fifth wheel connection is used, the tractor frame is modeled as a “free-pinned” Euler-Bernoulli beam, with the “free” end located at the front of the tractor and the “pinned” end located at the fifth wheel connection (Figure B.1). The general form of the spatial equation for a uniform beam derived in Chapter 2 is,

$$X\left(x_t\right) = C_1 \cos \beta x_t + C_2 \sin \beta x_t + C_3 \cosh \beta x_t + C_4 \sinh \beta x_t . \tag{B.1}$$

Boundary conditions are applied to Equation B.1 to solve for the constants C_1 through C_4. The first boundary conditions states that the bending moment at the free end is equal to zero. Taking the second derivative of Equation B.1 results in,

$$X''(x_t) = \beta^2 (-C_1 \cos\beta x_t - C_2 \sin\beta x_t + C_3 \cosh\beta x_t + C_4 \sinh\beta x_t). \quad \text{(B.2)}$$

Setting Equation B.2 equal to zero results in,

$$\begin{aligned} &X''(0) = \beta^2 (-C_1 + C_3) = 0 \\ &or \\ &C_1 = C_3. \end{aligned} \quad \text{(B.3)}$$

Substituting this back into Equation B.1 gives,

$$X(x_t) = C_1 (\cos\beta x_t + \cosh\beta x_t) + C_2 \sin\beta x_t + C_4 \sinh\beta x_t. \quad \text{(B.4)}$$

The second boundary condition states that the shear force at the free end is equal to zero. Taking the third derivative of Equation B.4 results in,

$$X'''(x_t) = \beta^3 \left[-C_1 (\sin\beta x_t - \sinh\beta x_t) - C_2 \cos\beta x_t + C_4 \cosh\beta x_t\right]. \quad \text{(B.5)}$$

Setting Equation B.5 equal to zero results in,

$$\begin{aligned} &X'''(0) = \beta^3 [-C_2 + C_4] = 0 \\ &or \\ &C_4 = C_2. \end{aligned} \quad \text{(B.6)}$$

Substituting this back into Equation B.4 gives,

$$X(x_t) = C_1 (\cos\beta x_t + \cosh\beta x_t) + C_2 (\sin\beta x_t + \sinh\beta x_t). \quad \text{(B.7)}$$

The third boundary condition states that the displacement at the pinned end is equal to zero. Thus, the original form of the equation can be set equal to zero at the point l along the beam,

$$X(l) = C_1 (\cos\beta l + \cosh\beta l) + C_2 (\sin\beta l + \sinh\beta l) = 0. \quad \text{(B.8)}$$

Finally, the fourth boundary condition states that the bending moment at the "pinned" end is equal to zero. Setting the second derivative of Equation B.7 evaluated at the point l along the beam equal to zero results in,

$$X''(l) = -C_1(\cos\beta l - \cosh\beta l) - C_2(\sin\beta l - \sinh\beta l) = 0. \tag{B.9}$$

Equations B.8 and B.9 can be used to solve for C_2 in terms of C_1. This gives,

$$C_2 = -C_1\left(\frac{\cos\beta l + \cosh\beta l}{\sin\beta l + \sinh\beta l}\right). \tag{B.10}$$

Equation B.10 is then substituted back into Equation B.7 to obtain the mode shape equation in its final form,

$$X(x_t) = C_1\left[\cos\beta x_t + \cosh\beta x_t - \left(\frac{\cos\beta l + \cosh\beta l}{\sin\beta l + \sinh\beta l}\right)(\sin\beta x_t - \sinh\beta x_t)\right]. \tag{B.11}$$

By putting the terms from Equations B.8 and B.9 into matrix form, the constant β can be solved for. Taking the determinant and setting it equal to zero results in,

$$\det\begin{vmatrix}(\cos\beta l + \cosh\beta l) & (\sin\beta l + \sinh\beta l)\\(\cos\beta l - \cosh\beta l) & (\sin\beta l - \sinh\beta l)\end{vmatrix} = 0. \tag{B.12}$$

which simplifies to,

$$\sin\beta l\cosh\beta l - \cos\beta l\sinh\beta l = 0. \tag{B.13}$$

Using trigonometric relationships, Equation B.13 can be further simplified to,

$$\tan\beta l = -\tanh\beta l. \tag{B.14}$$

Solving Equation B.14 for the constant βl results in an infinite number of solutions, each of which correspond to mode shapes of the beam. The first solution represents the rigid body motion, followed by the first mode shape, the second mode shape, and so on. Table B.1 lists the values for the constant βl.

Table B.1: Mode Shape Constants for a "Free-Pinned" Euler-Bernoulli Beam

Mode Shape	$\beta_n l$ **Values**
Rigid Body	$\beta_0 l = 0$
First	$\beta_1 l = 2.36502$
Second	$\beta_2 l = 5.4978$
Third	$\beta_3 l = 8.63938$

"Pinned-Free" Mode Shape Equation

When a conventional fifth wheel connection is used, the trailer frame is modeled as a "pinned-free" Euler-Bernoulli beam, with the "pinned" end located at the fifth wheel connection and the "free" end located at the front of the trailer (Figure B.2). The general form of the spatial equation for a uniform beam derived in Chapter 2 is,

$$X\left(x_{tlr}\right) = C_1 \cos \beta x_{tlr} + C_2 \sin \beta x_{tlr} + C_3 \cosh \beta x_{tlr} + C_4 \sinh \beta x_{tlr}. \quad \text{(B.15)}$$

Boundary conditions are applied to Equation B.15 to solve for the constants C_1 through C_4. The first boundary conditions states that the vertical displacement at the pinned end is equal to zero. Taking the original form of Equation B.15 and setting it equal to zero results in,

$$X(0) = (C_1 + C_3) = 0$$
$$or \tag{B.16}$$
$$C_3 = -C_1.$$

Substituting this back into Equation B.15 gives,

$$X(x_{tlr}) = C_1(\cos\beta x_{tlr} - \cosh\beta x_{tlr}) + C_2\sin\beta x_{tlr} + C_4\sinh\beta x_{tlr}. \tag{B.17}$$

The second boundary condition states that the bending moment at the pinned end is equal to zero. Taking the second derivative of Equation B.17 results in,

$$X''(x_{tlr}) = \beta^2\left[-C_1(\cos\beta x_{tlr} - \cosh\beta x_{tlr}) - C_2\sin\beta x_{tlr} + C_4\sinh\beta x_{tlr}\right]. \tag{B.18}$$

Setting Equation B.18 equal to zero results in,

$$X''(0) = \beta^2\left[C_1(-1-1)\right] = 0$$
$$or \tag{B.19}$$
$$C_1 = 0.$$

Substituting this back into Equation B.17 gives,

$$X(x_{tlr}) = C_2\sin\beta x_{tlr} + C_4\sinh\beta x_{tlr}. \tag{B.20}$$

The third boundary condition states that the bending moment at the free end is equal to zero. Thus, the second derivative of the equation can be set equal to zero at the point l along the beam,

$$X''(l) = -C_2\sin\beta l + C_4\sinh\beta l = 0. \tag{B.21}$$

Finally, the fourth boundary condition states that the shear force at the free end is equal to zero. Setting the third derivative of Equation B.20 evaluated at the point l along the beam equal to zero results in,

$$X'''(l) = -C_2\cos\beta l + C_4\cosh\beta l = 0. \tag{B.22}$$

Equations B.21 and B.22 can be used to solve for C_4 in terms of C_2. This gives,

$$C_4 = C_2 \left(\frac{\sin \beta l}{\sinh \beta l} \right). \tag{B.23}$$

Equation B.23 is then substituted back into Equation B.20 to obtain the mode shape equation in its final form,

$$X\left(x_{tlr}\right) = C_2 \left[\sin \beta x_{tlr} + \left(\frac{\sin \beta l}{\sinh \beta l} \right) \sinh \beta x_{tlr} \right]. \tag{B.24}$$

By putting the terms from Equations B.21 and B.22 into matrix form, the constant β can be solved for. Taking the determinant and setting it equal to zero results in,

$$\det \begin{vmatrix} \left(-\sin \beta l\right) & \left(\sinh \beta l\right) \\ \left(-\cos \beta l\right) & \left(\cosh \beta l\right) \end{vmatrix} = 0. \tag{B.25}$$

which simplifies to,

$$-\sin \beta l \cosh \beta l - \cos \beta l \sinh \beta l = 0. \tag{B.26}$$

Using trigonometric relationships, Equation B.26 can be further simplified to,

$$\tan \beta l = \tanh \beta l. \tag{B.27}$$

Solving Equation B.27 for the constant βl results in an infinite number of solutions, each of which correspond to mode shapes of the beam. The first solution represents the rigid body motion, followed by the first mode shape, the second mode shape, and so on. Table B.2 lists the values for the constant βl.

Table B.2: Mode Shape Constants for a "Pinned-Free" Euler-Bernoulli Beam

Mode Shape	$\beta_n l$ **Values**
Rigid Body	$\beta_0 l = 0$
First	$\beta_1 l = 3.9266$
Second	$\beta_2 l = 7.06858$
Third	$\beta_3 l = 10.2102$

"Free-Free" Mode Shape Equation

When a fifth wheel suspension system is present, both the tractor and trailer frames are modeled as "free-free" Euler-Bernoulli beams (Figure B.3). The general form of the spatial equation for a uniform beam derived in Chapter 2 is,

$$X\left(x_f\right) = C_1 \cos\beta x_f + C_2 \sin\beta x_f + C_3 \cosh\beta x_f + C_4 \sinh\beta x_f. \qquad \text{(B.28)}$$

Boundary conditions are applied to Equation B.28 to solve for the constants C_1 through C_4. The first boundary condition states that the bending moment at the first free end is equal to zero. Taking the second derivative of Equation B.28 results in,

$$X''\left(x_f\right) = \beta^2\left(-C_1 \cos\beta x_f - C_2 \sin\beta x_f + C_3 \cosh\beta x_f + C_4 \sinh\beta x_f\right). \text{(B.29)}$$

Setting Equation B.29 equal to zero results in,

$$\begin{aligned} &X''(0) = \beta^2\left(-C_1 + C_3\right) = 0 \\ &or \\ &C_3 = C_1. \end{aligned} \qquad \text{(B.30)}$$

Substituting this back into Equation B.1 gives,

$$X\left(x_f\right)=C_1\left(\cos\beta x_f+\cosh\beta x_f\right)+C_2\sin\beta x_f+C_4\sinh\beta x_f. \tag{B.31}$$

The second boundary condition states that the shear force at the first free end is also equal to zero. Taking the third derivative of Equation B.31 results in,

$$X'''\left(x_f\right)=\beta^3\left[-C_1\left(\sin\beta x_f-\sinh\beta x_f\right)-C_2\cos\beta x_f+C_4\cosh\beta x_f\right]. \tag{B.32}$$

Setting Equation B.32 equal to zero results in,

$$\begin{aligned}&X'''(0)=\beta^3\left[-C_2+C_4\right]=0\\&or\\&C_4=C_2.\end{aligned} \tag{B.33}$$

Substituting this back into Equation B.31 gives,

$$X\left(x_f\right)=C_1\left(\cos\beta x_f+\cosh\beta x_f\right)+C_2\left(\sin\beta x_f+\sinh\beta x_f\right). \tag{B.34}$$

The third boundary condition states that the bending moment at the second free end is equal to zero. Thus, the second derivative of Equation B.34 can be set equal to zero at the point l along the beam,

$$X''(l)=C_1\left(-\cos\beta l+\cosh\beta l\right)+C_2\left(-\sin\beta l+\sinh\beta l\right)=0. \tag{B.35}$$

Finally, the fourth boundary condition states that the shear force at the second free end is equal to zero. Setting the third derivative of Equation B.34 evaluated at the point l along the beam equal to zero results in,

$$X'''(l)=C_1\left(\sin\beta l+\sinh\beta l\right)+C_2\left(-\cos\beta l+\cosh\beta l\right)=0. \tag{B.36}$$

Equations B.35 and B.36 can be used to solve for C_1 in terms of C_2. This gives,

$$C_1=C_2\left(\frac{\cos\beta l-\cosh\beta l}{\sin\beta l+\sinh\beta l}\right). \tag{B.37}$$

Equation B.37 is then substituted back into Equation B.34 to obtain the mode shape equation in its final form,

$$X(x_t) = C_2\left[\sin\beta x_t + \sinh\beta x_t + \left(\frac{\cos\beta l - \cosh\beta l}{\sin\beta l + \sinh\beta l}\right)(\cos\beta x_t + \cosh\beta x_t)\right]. \tag{B.38}$$

By putting the terms from Equations B.35 and B.36 into matrix form, the constant β can be solved for. Taking the determinant and setting it equal to zero results in,

$$\det\begin{vmatrix} (\cos\beta l - \cosh\beta l) & (\sin\beta l - \sinh\beta l) \\ (\sin\beta l + \sinh\beta l) & (\cos\beta l - \cosh\beta l) \end{vmatrix} = 0. \tag{B.39}$$

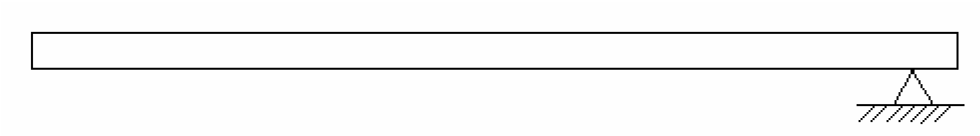
which simplifies to,

$$\cos\beta l \cosh\beta l = 0. \tag{B.40}$$

Solving Equation B.40 for the constant βl results in an infinite number of solutions, each of which correspond to mode shapes of the beam. The first solution represents the rigid body motion, followed by the first mode shape, the second mode shape, and so on. Table B.1 lists the values for the constant βl.

Table B.3: Mode Shape Constants for a "Free-Free" Euler-Bernoulli Beam

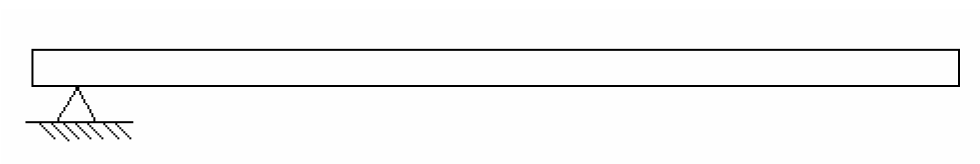
Mode Shape	$\beta_n l$ **Values**
Rigid Body	$\beta_0 l = 0$
First	$\beta_1 l = 4.73004$
Second	$\beta_2 l = 7.85320$
Third	$\beta_3 l = 10.99561$



$$x_t = 0$$
$$EI\frac{\partial^2 \eta}{\partial x^2} = 0$$
$$\frac{\partial}{\partial x}\left(EI\frac{\partial^2 \eta}{\partial x^2}\right) = 0$$

$$x_t = l$$
$$\eta = 0$$
$$EI\frac{\partial^2 \eta}{\partial x^2} = 0$$

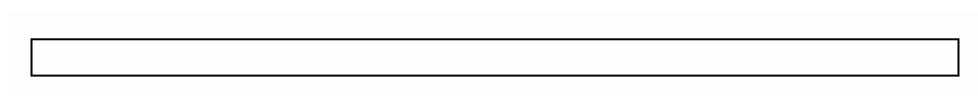
Figure B.1: Boundary Conditions for a "Free-Pinned" Euler-Bernoulli Beam



$$x_{tlr} = 0$$
$$\eta = 0$$
$$EI\frac{\partial^2 \eta}{\partial x^2} = 0$$

$$x_{tlr} = l$$
$$EI\frac{\partial^2 \eta}{\partial x^2} = 0$$
$$\frac{\partial}{\partial x}\left(EI\frac{\partial^2 \eta}{\partial x^2}\right) = 0$$

Figure B.2: Boundary Conditions for a "Pinned-Free" Euler-Bernoulli Beam



$$x_f = 0$$
$$EI\frac{\partial^2 \eta}{\partial x^2} = 0$$
$$\frac{\partial}{\partial x}\left(EI\frac{\partial^2 \eta}{\partial x^2}\right) = 0$$

$$x_f = l$$
$$EI\frac{\partial^2 \eta}{\partial x^2} = 0$$
$$\frac{\partial}{\partial x}\left(EI\frac{\partial^2 \eta}{\partial x^2}\right) = 0$$

Figure B.3: Boundary Conditions for a "Free-Free" Euler-Bernoulli Beam

Appendix C: Vehicle Model Parameters

This appendix outlines the parameters for the nominal cab-over style tractor semi-trailer. These tractor semi-trailer is identical to the one used by Vaduri [3] and Trangsrud [1]. The values have been collected from a number of different sources in an effort to create a model that accurately represents the intended test vehicle, which is a Freightliner Century Class tractor and typical dual axle trailer with a payload. The geometric dimensions and inertial properties were originally provided to Vaduri and Law [17] by both Michelin and Freightliner. These values were obtained either through physical measurements or literature by Ribartis et al [20].

It is assumed that the vehicle described in the following pages is symmetric about the longitudinal centerline of the tractor and trailer. Similarly, it is assumed that the left and right sides of the axles see an identical road profile. These assumptions allow the left and right sides of the axles to be lumped into single masses and suspension elements, which is reflected in the following figures and tables by per-axle values. The same is true for the tires and cab suspension elements.

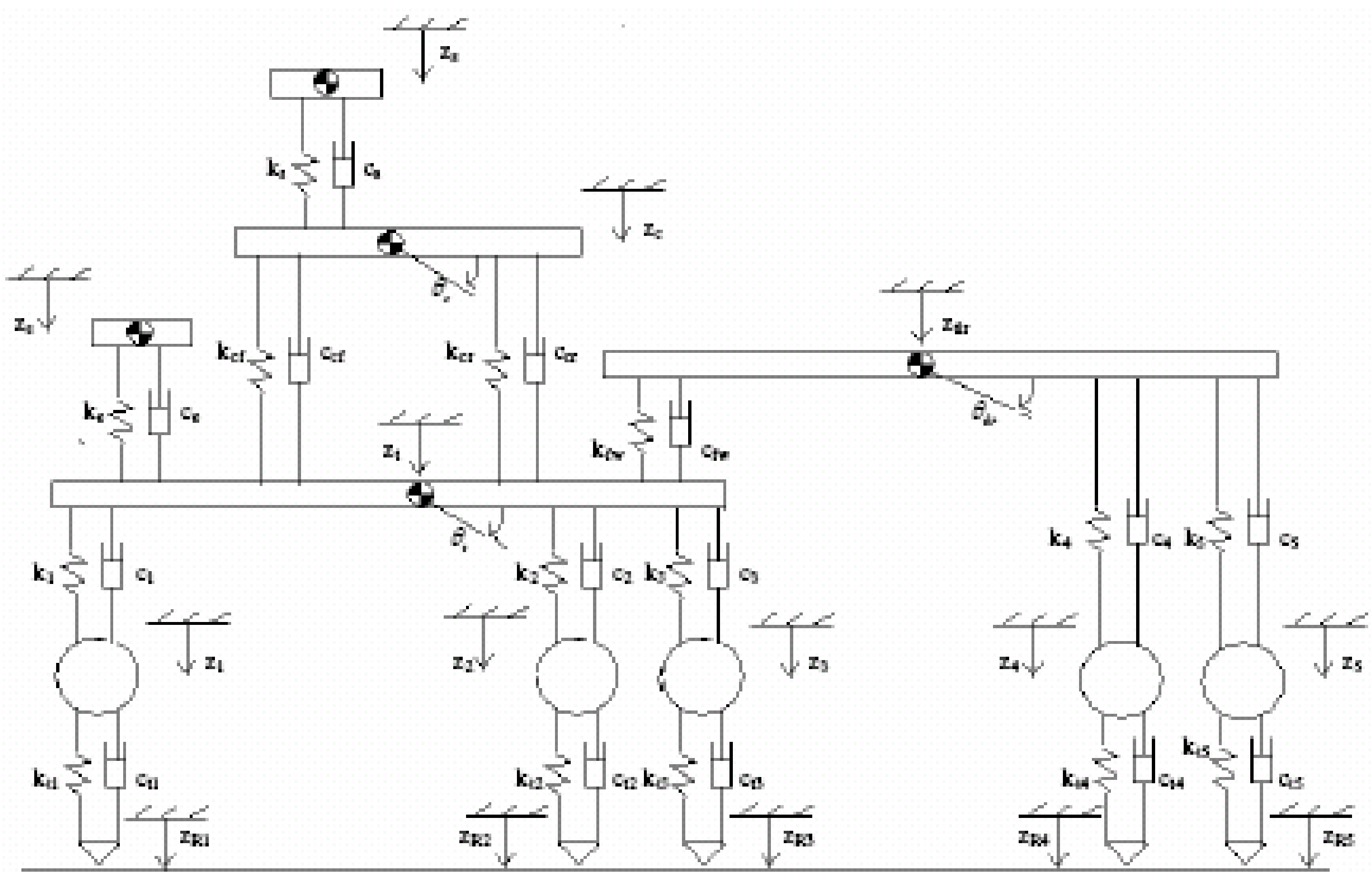


Figure C.1: Fifteen Degree-of-Freedom System Model

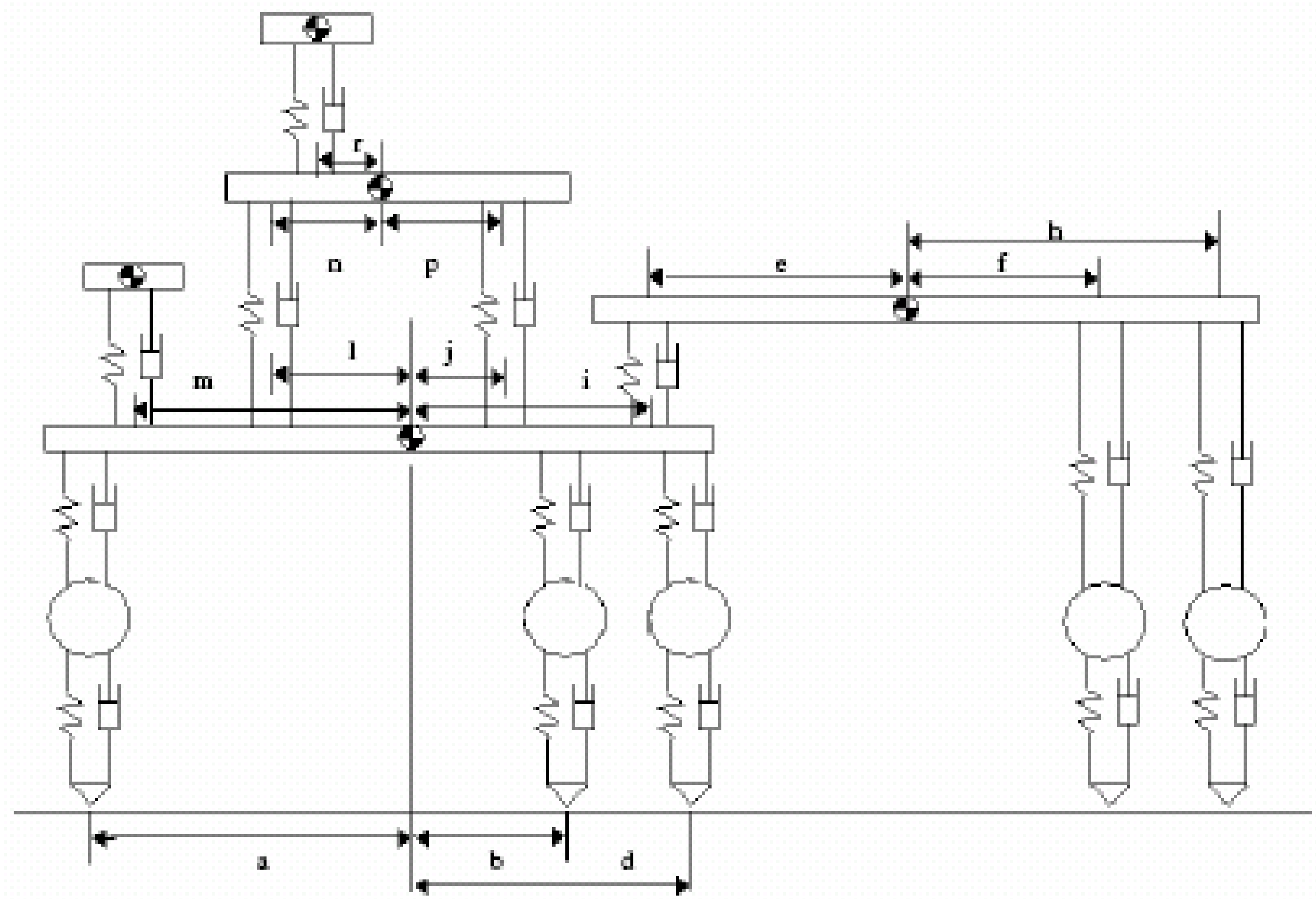


Figure C.2: Dimensions of the Tractor Semi-Trailer Model

Table C.1: Geometric Dimensions of the Tractor Semi-Trailer Model

Symbol	Description	Value	Units
b_a1	Front end of the tractor to the axle #1	1.065	m
b_cf	Front end of the tractor to the cab front	1.470	m
b_e	Front end of the tractor to the engine	2.797	m
b_cr	Front end of the tractor to the cab rear	4.020	m
b_a2	Front end of the tractor to the axle #2	6.035	m
b_fw	Front end of the tractor to the fifth wheel	6.688	m
b_a3	Front end of the tractor to the axle #3	7.340	m
a1	Front end of the tractor to the tractor cg	4.006	m
b_a4	From the fifth wheel to axle #4	8.580	m
b_a5	From the fifth wheel to axle #4	9.780	m
L_t	Length of the tractor	8.200	m
L_tlr	Length of the trailer	9.780	m
e	From the trailer cg to the fifth wheel	5.620	m
f	From the trailer cg to axle #4	2.960	m
h	From the trailer cg to axle #5	4.160	m
a	From the tractor cg to axle #1	2.941	m
b	From the tractor cg to axle #2	2.029	m
d	From the tractor cg to axle #3	3.334	m
l	From the tractor cg to the front of the cab	2.536	m
m	From the tractor cg to the engine	1.209	m
j	From the tractor cg to the rear of the cab	0.014	m
i	From the tractor cg to the fifth wheel	2.682	m
n	From the cab cg to the front of the cab	1.435	m
p	From the cab cg to the rear of the cab	1.115	m
r	From the cab cg to the seat	0.200	m
tc	From the tractor cg to the cab cg	1.101	m
h1	Height of the driver over the cab	1.000	m

Table C.2: Inertial Properties of the Tractor Semi-Trailer Model

Symbol	Description	Value	Units
m_s	Mass of the seat plus 200 lb. driver	106.7	kg
m_c	Mass of the cab	1208	kg
I_c	Moment of inertia of the cab	2100	$kg*m^2$
m_e	Mass of the engine	2000	kg
m_t	Mass of the tractor frame	3783	kg
I_t	Moment of inertia of the tractor frame	46590.9	$kg*m^2$
m_ul	Mass of the unloaded trailer	10800	kg
I_tlr	Moment of inertia of the trailer	200000	$kg*m^2$
m_L	Mass of the trailer load	14000	kg
m_tlr	Mass of the loaded trailer	24800	kg

Table C.3: Suspension Parameters of the Tractor Semi-Trailer Model

Symbol	Description	Value	Units
k1	Steer axle spring coefficient	581300	N/m
k2	#1 drive axle spring coefficient	586900	N/m
k3	#2 drive axle spring coefficient	586900	N/m
k4	#1 trailer axle spring coefficient	1000000	N/m
k5	#2 trailer axle spring coefficient	1000000	N/m
c1	Steer axle damping coefficient	11270	N/(m/s)
c2	#1 drive axle damping coefficient	27500	N/(m/s)
c3	#2 drive axle damping coefficient	27500	N/(m/s)
c4	#1 trailer axle damping coefficient	70000	N/(m/s)
c5	#2 trailer axle damping coefficient	70000	N/(m/s)
ks	Driver's seat spring coefficient (optional)	3403	N/m
cs	Driver's seat damping coefficient (optional)	1140	N/(m/s)
ke	Engine mount spring coefficient	1×10^{10}	N/m
ce	Engine mount damping coefficient	10000	N/(m/s)

Table C.4: Cab Suspension Parameters of the Tractor Semi-Trailer Model

Symbol	Description	Front Only	Rear Only	Front and Rear
kcf	Front spring coefficient	88740 N/m	N/A	86260.5 N/m
kcr	Rear spring coefficient	N/A	65980 N/m	63757.5 N/m
ccf	Front damping coefficient	7062 N/(m/s)	N/A	6864.35 N/(m/s)
ccr	Rear damping coefficient	N/A	8000 N/(m/s)	5073.5 N/(m/s)

Table C.5: Per-Tire Stiffness Values of the Tractor Semi-Trailer Model

Symbol	Description	Position (# of Tires)	Nominal Pressure	Value	Units
kt1	XZA2 275/80R22.5	Steer Axle (2)	80 psi	647.5	kN/m
kt2, kt3	Xone XDA 445/50R22.5	Drive Axle (2)	104 psi	1194.1	kN/m
kt4, kt5	Xone XTA 445/50R22.5	Trailer Axle (2)	104 psi	1194.1	kN/m
kt2, kt3, kt4, kt5	XTE2 LRL 425/65R22.5	Drive or Trailer Axle (2)	110 psi	1169.9	kN/m
kt2, kt3	XDA2 275/80R22.5	Drive Axle (4)	100 psi	894.55	kN/m
kt4, kt5	XT1 275/80R22.5	Trailer Axle (4)	100 psi	894.55	kN/m

Table C.6: Per-Tire Damping Values of the Tractor Semi-Trailer Model

Symbol	Description	Position (# of Tires)	Value	Units
ct1	XZA2 275/80R22.5	Steer Axle (2)	258.5	N/(m/s)
ct2, ct3	Xone XDA 445/50R22.5	Drive Axle (2)	324.15	N/(m/s)
ct4, ct5	Xone XTA 445/50R22.5	Trailer Axle (2)	324.15	N/(m/s)
ct2, ct3, ct4, ct5	XTE2 LRL 425/65R22.5	Drive or Trailer Axle (2)	375.75	N/(m/s)
ct2, ct3	XDA2 275/80R22.5	Drive Axle (4)	261	N/(m/s)
ct4, ct5	XT1 275/80R22.5	Trailer Axle (4)	242.65	N/(m/s)

Figure C.3: Common Fifth Wheel Connection

Appendix D: Normalized Eigenvectors

The eigenvalues and normalized eigenvectors for the nominal vehicle are presented in this appendix. The nominal vehicle is defined as having seat suspension, rear only cab suspension, a loaded trailer, and no fifth wheel suspension. Both the tractor and trailer frame are assumed to have beaming frequencies of 20 Hz. The tires for the nominal vehicle are XZA2 275/80R22.5 tires inflated to 80 psi for the steer axle, Xone XDA 445/50R22.5 tires inflated to 104 psi for each drive axle, and Xone XTA 445/50R22.5 tires inflated to 104 psi for each trailer axle. In all cases, the vehicle is traveling at a velocity of 60 mph over a smooth highway.

For each of the fifteen eigenvalues representing the system, the frequency, damping ratio, and corresponding eigenvector are calculated. In the eigenvector, the magnitude and phase of each of the individual components are calculated. The eigenvector is then normalized about the component with the largest magnitude. This allows for easy determination of which motions are the most dominant at that particular frequency and whether or not that motion is in phase with the dominant component. For easy reference, the degree-of-freedom associated with each particular component is listed beside it in the table.

1. Eigenvalue:-20.647 + 33462i
   Frequency: 5325.7 Hz
   Damping Ratio: 0.00061702

DOF	Magnitude	Phase (deg.)
z_s	4.326e-07	-89.64
z_c	1.623e-03	0.94
θ_c	1.343e-03	-176.26
z_e	2.002e-04	1.99
z_t	1.000	0.00
θ_t	0.040	0.00
η_t	0.569	-179.99
z_{tlr}	0.056	179.93
θ_{tlr}	0.047	-0.09
η_{tlr}	0.167	-0.06
z_1	2.221e-04	89.96
z_2	1.108e-03	-89.99
z_3	1.711e-03	-89.99
z_4	1.282e-04	90.15
z_5	3.136e-04	90.11

2. Eigenvalue:-17.876 + 5008.6i
   Frequency: 797.15 Hz
   Damping Ratio: 0.0035690

DOF	Magnitude	Phase (deg.)
z_s	1.781e-03	-90.10
z_c	1.000	0.00
θ_c	0.825	179.90
z_e	0.186	-0.11
z_t	0.029	0.25
θ_t	5.824e-03	0.04
η_t	0.439	-179.95
z_{tlr}	8.615e-03	0.33
θ_{tlr}	7.276e-03	-179.81
η_{tlr}	0.026	-179.64
z_1	5.171e-03	89.61
z_2	1.038e-03	90.06
z_3	1.793e-03	-89.87
z_4	1.331e-04	-88.20
z_5	3.253e-04	-88.43

3. Eigenvalue:-5.0065 + 2814.3i
   Frequency: 447.91 Hz
   Damping Ratio: 0.0018003

DOF	Magnitude	Phase (deg.)
z_s	1.616e-03	90.04
z_c	0.510	-179.98
θ_c	0.421	0.21
z_e	1.000	0.00
z_t	0.074	0.08
θ_t	0.030	-179.99
η_t	0.396	-179.89
z_{tlr}	1.370e-03	179.62
θ_{tlr}	1.157e-03	-0.63
η_{tlr}	4.132e-03	-0.32
z_1	6.684e-03	89.62
z_2	2.041e-03	90.44
z_3	1.807e-03	-89.48
z_4	3.780e-05	92.50
z_5	9.232e-05	92.09

4. Eigenvalue:-8.6229 + 135.34i
   Frequency: 21.583 Hz
   Damping Ratio: 0.063585

DOF	Magnitude	Phase (deg.)
z_s	3.711e-03	-88.97
z_c	0.056	0.39
θ_c	0.047	175.26
z_e	0.070	-7.00
z_t	0.522	177.92
θ_t	0.014	-22.79
η_t	0.347	-3.22
z_{tlr}	0.063	-152.73
θ_{tlr}	0.077	-4.38
η_{tlr}	1.000	0.00
z_1	0.040	-101.02
z_2	0.147	98.83
z_3	0.242	98.18
z_4	0.444	134.09
z_5	0.905	132.06

5. Eigenvalue:-0.80850 + 80.984i
   Frequency: 12.890 Hz
   Damping Ratio: 0.0099829

DOF	Magnitude	Phase (deg.)
z_s	0.023	-95.80
z_c	0.206	-7.25
θ_c	0.170	-169.37
z_e	0.560	1.20
z_t	0.898	179.67
θ_t	0.244	-1.27
η_t	1.000	0.00
z_{tlr}	0.049	179.83
θ_{tlr}	0.035	2.76
η_{tlr}	0.107	-178.68
z_1	0.370	-61.72
z_2	0.030	145.42
z_3	0.412	137.06
z_4	0.134	-15.59
z_5	0.250	-16.12

6. Eigenvalue:-66.914 + 21.211i
   Frequency: 11.172 Hz
   Damping Ratio: 0.95325

DOF	Magnitude	Phase (deg.)
z_s	5.910e-04	52.63
z_c	3.252e-03	-153.44
θ_c	2.219e-03	-91.89
z_e	0.013	46.38
z_t	0.051	24.75
θ_t	8.321e-03	40.54
η_t	0.017	-169.83
z_{tlr}	0.095	-165.74
θ_{tlr}	0.030	-159.14
η_{tlr}	0.086	-12.80
z_1	6.311e-03	-31.46
z_2	0.022	-155.91
z_3	0.030	-157.65
z_4	0.729	2.72
z_5	1.000	0.00

7. Eigenvalue:-54.921 + 47.320i
   Frequency: 11.538 Hz
   Damping Ratio: 0.75759

DOF	Magnitude	Phase (deg.)
z_s	5.300e-05	-38.56
z_c	3.635e-04	75.32
θ_c	6.575e-04	-170.29
z_e	2.294e-03	-36.79
z_t	7.399e-03	-89.77
θ_t	1.317e-03	-48.80
η_t	3.202e-03	59.38
z_{tlr}	8.675e-03	-168.85
θ_{tlr}	2.442e-03	142.37
η_{tlr}	0.014	167.18
z_1	3.151e-04	-169.55
z_2	4.974e-03	97.66
z_3	6.813e-03	92.92
z_4	1.000	0.00
z_5	0.737	-172.69

8. Eigenvalue:-16.477 + 69.086i
   Frequency: 11.304 Hz
   Damping Ratio: 0.23200

DOF	Magnitude	Phase (deg.)
z_s	2.186e-03	96.66
z_c	0.017	-164.10
θ_c	0.016	24.88
z_e	0.031	-170.30
z_t	0.035	-101.91
θ_t	0.011	72.30
η_t	0.026	114.86
z_{tlr}	1.152e-03	-65.91
θ_{tlr}	8.648e-04	106.40
η_{tlr}	1.553e-03	-26.38
z_1	1.000	0.00
z_2	0.018	176.94
z_3	0.034	-87.28
z_4	3.196e-03	114.19
z_5	5.854e-03	117.15

9. Eigenvalue:-21.954 + 64.320i
Frequency: 10.817 Hz
Damping Ratio: 0.32303

DOF	Magnitude	Phase (deg.)
z_s	1.488e-03	80.40
z_c	0.011	-173.35
θ_c	0.012	21.25
z_e	0.029	179.77
z_t	0.028	-51.19
θ_t	0.011	128.32
η_t	0.037	149.63
z_{tlr}	2.402e-04	127.19
θ_{tlr}	1.836e-04	-64.72
η_{tlr}	2.736e-04	-179.98
z_1	0.056	-42.20
z_2	1.000	0.00
z_3	0.813	-164.23
z_4	6.806e-04	-50.28
z_5	1.237e-03	-45.43

10. Eigenvalue:-23.393 + 62.945i
Frequency: 10.687 Hz
Damping Ratio: 0.34837

DOF	Magnitude	Phase (deg.)
z_s	1.212e-03	172.57
z_c	0.010	-73.50
θ_c	0.016	130.12
z_e	0.049	-80.66
z_t	0.059	-178.71
θ_t	0.023	-93.24
η_t	0.057	-48.59
z_{tlr}	0.017	-124.51
θ_{tlr}	0.013	42.32
η_{tlr}	0.018	-68.09
z_1	0.043	68.99
z_2	0.803	14.39
z_3	1.000	0.00
z_4	0.048	58.40
z_5	0.087	63.87

11. Eigenvalue:-4.7004 + 14.485i
Frequency: 2.4238 Hz
Damping Ratio: 0.30865

DOF	Magnitude	Phase (deg.)
z_s	0.484	-17.67
z_c	0.553	63.07
θ_c	0.469	44.61
z_e	0.629	150.48
z_t	0.962	144.06
θ_t	0.329	138.41
η_t	0.038	171.77
z_{tlr}	0.698	-37.43
θ_{tlr}	0.452	-38.31
η_{tlr}	0.039	-1.10
z_1	0.046	-139.22
z_2	0.363	176.33
z_3	0.453	175.18
z_4	0.794	0.33
z_5	1.000	0.00

12. Eigenvalue:-5.6963 + 1.698i
Frequency: 0.94601 Hz
Damping Ratio: 0.95833

DOF	Magnitude	Phase (deg.)
z_s	1.000	0.00
z_c	0.035	148.69
θ_c	0.022	146.82
z_e	2.849e-03	166.86
z_t	2.186e-03	168.64
θ_t	5.107e-04	-19.43
η_t	2.849e-05	166.16
z_{tlr}	2.868e-04	-172.75
θ_{tlr}	1.010e-04	-1.11
η_{tlr}	2.223e-06	-22.84
z_1	1.062e-03	167.08
z_2	1.802e-04	-178.81
z_3	8.327e-05	-159.80
z_4	8.825e-06	-63.67
z_5	2.841e-05	-88.90

13. Eigenvalue:-5.6689 + 7.5895i
Frequency: 1.5077 Hz
Damping Ratio: 0.59843

DOF	Magnitude	Phase (deg.)
z_s	1.000	0.00
z_c	0.552	76.62
θ_c	0.402	69.86
z_e	0.064	-176.93
z_t	0.056	-176.74
θ_t	5.516e-03	12.35
η_t	1.033e-03	138.78
z_{tlr}	0.019	-147.25
θ_{tlr}	4.911e-03	-21.31
η_{tlr}	3.512e-04	-69.96
z_1	0.021	-168.29
z_2	7.709e-03	-156.71
z_3	6.410e-03	-158.16
z_4	4.069e-03	-65.96
z_5	4.622e-03	-46.99

14. Eigenvalue:-0.89408 + 9.8286i
Frequency: 1.5707 Hz
Damping Ratio: 0.090593

DOF	Magnitude	Phase (deg.)
z_s	0.900	-78.02
z_c	0.993	-21.53
θ_c	0.419	-139.17
z_e	1.000	0.00
z_t	0.670	-1.72
θ_t	0.287	-175.62
η_t	9.778e-03	-158.11
z_{tlr}	0.090	-157.90
θ_{tlr}	8.497e-03	71.63
η_{tlr}	1.848e-03	-148.75
z_1	0.474	89.82
z_2	0.023	-17.25
z_3	0.064	-141.47
z_4	0.026	-148.50
z_5	0.024	-156.53

15. Eigenvalue:-1.7590 + 9.0625i
Frequency: 1.4693 Hz
Damping Ratio: 0.19054

DOF	Magnitude	Phase (deg.)
z_s	1.000	0.00
z_c	0.867	56.15
θ_c	0.467	23.99
z_e	0.631	94.49
z_t	0.781	88.30
θ_t	0.150	66.45
η_t	8.491e-03	89.40
z_{tlr}	0.827	78.67
θ_{tlr}	0.060	-93.68
η_{tlr}	0.015	100.52
z_1	0.131	118.71
z_2	0.221	102.40
z_3	0.259	99.93
z_4	0.212	101.14
z_5	0.185	99.41

Appendix E: Program User's Guide

Overview

Both the frequency domain simulation and parameter variation programs described in this thesis were written in MATLAB and were meant to only run in this tool. The model being simulated is a fifteen degree-of-freedom (15 DOF) tractor semi-trailer and both types of programs require the user to make selections and input values while they are running. This guide is meant to help first time users run the programs and gain understanding about how they work.

Getting Started

Once MATLAB has been started, the user must check to make sure that the proper directory has been selected in the "Current Directory" menu. The directory will be different depending on whether the user is attempting to run the simulation program or one of the parameter variation programs. In the "Command Window", type in the name of the desired program. For the simulation, type in "dof15_freq2" and press enter. To run any of the parameter variation programs, type in either "opt_axleK_freq", "opt_axleC_freq", "opt_tireK_freq", "opt_tireC_freq", "opt_tlr_axlebeam", "opt_beam_freq", or "opt_5wKC_freq" and press "Enter". This will begin running the desired program.

Menus

Both the simulation program and the parameter variation programs contain the same menus that appear in the command window. The only difference in the operation of the two types of programs is the output plots and tables. Also, when running the tire stiffness variation program, "opt_tireK_freq", there will be no tire selection menu because these particular values will be varied by the program. The same holds true for the tire damping variation program, "opt_tireC_freq", the trailer parameter variation program, "opt_tlr_axlebeam", the tractor and trailer beaming frequency program, "opt_beam_freq", and the fifth wheel suspension parameter variation program, "opt_5wKC_freq".

The menus provide the user with different options for the model being simulated and the desired outputs. The options in each menu may be presented in the form of a list using letter numbering, a yes or no response, or a numeric input. In the case of a list, simply type in the lower case letter of the selection and press "Enter". Similarly, for the yes or no response, type in a lower case "y" for yes or "n" for no and press "Enter". Finally, for a numeric input, input the number with no commas in the specified units and press "Enter".

Vehicle Selection Menu

This menu provides the user with the option of selecting which type of vehicle is going to be simulated. Different vehicle have different geometric parameters, inertial parameters, and suspension parameters. Choosing the desired vehicle will call upon the appropriate set of parameters.

Seat Suspension Menu

The vehicle model has the option to implement a seat suspension system for the driver. If a seat suspension system is desired, the appropriate value will be used in the program. If no seat suspension is desired, the seat suspension stiffness will be set to a very high value, which is representative of a rigid connection. Numeric values for the seat suspension can be found in Appendix C.

Cab Suspension Menu

The 15 DOF system provides the user with the option of four different cab suspension orientations. This menu allows the user to choose from a front only cab suspension, a rear only cab suspension, front and rear cab suspension, and no cab suspension. In the cases where a rigid connection is desired, the stiffness values are set very high. Numeric values for the cab suspension can be found in Appendix C.

Trailer Configuration Menu

The program allows the user to choose from simulating a fully laden trailer, or an empty trailer. In the case of a fully laden trailer, the payload is placed at the CG of the trailer.

Fifth Wheel Suspension Menu

This menu allows the user to choose from a pin connection at the fifth wheel or to implement a fifth wheel suspension system. If no fifth wheel suspension system is desired, the stiffness across the fifth wheel is set to a very high value. If the user desires to implement a fifth wheel suspension system, two prompts will appear requesting the user to input values for the stiffness and damping of the fifth wheel suspension system. The values for the stiffness can range from 50,000 N/m to 1,000,000 N/m, and the values for the damping can range from 2,000 N/(m/s) and 40,000 N/(m/s).

Beaming Frequency Menu

The fifteen DOF model incorporates the effects of both tractor and trailer frame beaming in the dynamic response. In both cases, only the bare frames are experiencing any beaming. In this menu, the user is prompted to input a beaming frequency, in Hz, for the tractor and trailer frame. The nominal frequency for both is 20 Hz, but appropriate beaming frequencies can range from 10 Hz to 30 Hz. This frequency is representative of the first bending mode only.

Tire Selection Menus

In this menu, the user is asked to choose from six different tire types for use on the each of the tractor semi-trailer's axles. The first prompt requests the user to choose a tire for the steer axle, followed by prompts for the drive axles and trailer axles. In the case of the drive axles, the user may only choose one tire that will be used on both of the axles. The same is true for the trailer axles. Once the tire type has been selected, a prompt appears asking the user to input the tire pressure to be used in that tire in units of pounds per square inch (psi). The nominal value for that tire is displayed to serve as a reference value.

Vehicle Velocity Menu

This menu provides the user first with the option to select whether they would like to input the velocity of the tractor semi-trailer in units of meters per second (m/s) or mile per hour (mph). Once the user decides on the units, a prompt appears requesting the user to input a numeric value. The nominal value used the case studies in this thesis was 60 mph.

Road Surface PSD Selection Menu

This menu allows the user to select from four different road surfaces for the vehicle to traverse. All of the surfaces are typical to roads that a tractor semi-trailer might encounter in normal operation. The road surfaces are listed along with constants in SI units that were taken from Table 7.1 in Theory of Ground Vehicles by Wong [37]. Both the value for C_{sp} and N are used in the calculation of the road surface PSD.

J Penalty Factor Menu

This is the only menu that appears in the parameter variation programs, but not in the simulation program. This menu allows the user to select values for K1 and K2 which are used in the calculation of the J penalty value. K1 represents the importance of the driver ride comfort in the function, and K2 represents the importance of vertical accelerations experienced at the trailer CG. Both values should add up to one. For example, if the most importance is placed on driver ride comfort, a value of 0.8 should be assigned to K1, and a value of 0.2 should be assigned to K2.

Output Options

There are many different output options available in the simulation program and the parameter variation programs. All of the options have a yes or no response, and may appear in the form of tabular results that will appear in the "Command Window" or graphical results that will appear in separate windows created by MATLAB.

Appendix F: dof15_freq2.m

This program, entitled “dof15_freq2”, performs a simulation of the 15 DOF tractor semi-trailer in the frequency domain. Upon initiation, the program prompts the user for various input parameter, calls upon the appropriate data files, and calls upon the appropriate function files to perform the integrations necessary to calculate displacements of the tractor and trailer frame caused by beaming. It also performs the necessary calculations and displays the desired output information. The first data file called upon is “parameters” which contains the geometric parameters, inertial parameters, and suspension characteristics of the desired tractor semi-trailer. After the beaming frequencies have been chosen, the program calls upon the function files which are labeled, “modeD1_t”, “modeD1_tlr”, “modeD2_t”, “modeD2_tlr”, “modeD3_t”, “modeD3_tlr”, “modeD4_t”, and “modeD4_tlr”. These files form the integrals that calculate the beaming constants used in the simulation. Finally, the program calls upon the data file “TireData3” to calculate the stiffness and damping values depending on which types of tires are chosen and the pressures defined for those tires.

dof15_freq2.m

```
% dof15_freq2.m
% Developed by Ryan Spivey, 4/10/07
%
% This program models the fifteen DOF tractor semi-trailer
% DOFs include - 1)Vertical Disp. of Driver's Seat
%                2)Vertical Disp. of Cab
%                3)Pitch of Cab
%                4)Vertical Disp. of Engine
%                5)Vertical Disp. of Tractor Frame
%                6)Pitch of Tractor Frame
%                7)Beaming of Tractor Frame
%                8)Vertical Disp. of Trailer
%                9)Pitch of Trailer
%               10)Beaming of Trailer
%               11)Vertical Disp. of Axle #1
%               12)Vertical Disp. of Axle #2
%               13)Vertical Disp. of Axle #3
%               14)Vertical Disp. of Axle #4
%               15)Vertical Disp. of Axle #5

clc
clear all
% close all
format short e
format compact

global D1_t D2_t D3_t D4_t D1_tlr D2_tlr D3_tlr D4_tlr
global e a1 kb1 kb2 b_fw L_tlr alpha

disp('  ')
disp('Frequency Response of 15 DOF Tractor Semi-Trailer')
disp('                Roadholding Model                ')
disp(['                    ',date])

parameters;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%  Trailer Configuration  %%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('TRAILER CONFIGURATION')
disp('  ')
disp('Please choose which configuration to use');
disp('a : Loaded Trailer');
disp('b : Unloaded Trailer');
z44 = input('Please give your choice : ', 's');

if z44 == 'a'
    m_tlr = m_tlr;
elseif z44 == 'b'
    m_tlr = m_ul;
```

```
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%  Fifth Wheel Configuration
%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

disp('  ')
disp('Give your choice for the fifth wheel configuration: ')
disp('Note: If a fifth wheel suspension system is chosen, the
beaming of')
disp('      the tractor frame and trailer will be modeled as
free-free.  If')
disp('      no suspension is chosen, the tractor frame and
trailer will be')
disp('      modeled as free-pinned and pinned-free
respectively.')
disp('a : With fifth wheel suspension')
disp('b : Without fifth wheel suspension')
z33 = input('Please give your choice : ', 's');

if z33 == 'a',          % Choice 'a' is with fifth wheel
suspension
    disp('  ')
    kfw = input('Input the fifth wheel spring constant (N/m): ');
    disp('  ')
    cfw = input('Input the fifth wheel damping ratio (N/(m/s)):
');

    % The parameters for the first bending mode of the Tractor
frame
    disp('  ')
    fhz = input('Input the Tractor frequency of beaming (hz) fhz
: ');

    % The parameters for the first bending mode of the Trailer
frame
    disp('  ')
    fhz2 = input('Input the Trailer frequency of beaming (hz) fhz
: ');

    kb1 = 4.73004074;    %Constant for the first bending mode
(free-free)
    alpha = 0.982502;

    z1 = 'cosh(kb1*x1/b_fw) + cos(kb1*x1/b_fw) -
alpha*(sinh(kb1*x1/b_fw)+sin(kb1*x1/b_fw))';
    % free-free beam mode function
    z1dd = '(kb1/b_fw)^2*(cosh(kb1*x1/b_fw) - cos(kb1*x1/b_fw) -
alpha*(sinh(kb1*x1/b_fw)-sin(kb1*x1/b_fw)))';
    % second derivative of free-free beam mode function
```

```
    kb2 = 4.73004074;    %Constant for the first bending mode
(free-free)

    z2 = 'cosh(kb2*x2/L_tlr) + cos(kb2*x2/L_tlr) -
alpha*(sinh(kb2*x2/L_tlr)+sin(kb2*x2/L_tlr))';
    % free-free beam mode function
    z2dd = '(kb2/L_tlr)^2*(cosh(kb2*x2/L_tlr) - cos(kb2*x2/L_tlr)
- alpha*(sinh(kb2*x2/L_tlr)-sin(kb2*x2/L_tlr)))';
    % second derivative of free-free beam mode function

elseif z33 == 'b',          % Choice 'b' is without fifth wheel
suspension
    kfw = 1000000000000;    %(N/m)      fifth wheel spring
constant
    cfw = 1000;             %(N/(m/s))  fifth wheel damping ratio

    % The parameters for the first bending mode of the Tractor
frame
    disp('  ')
    fhz = input('Input the Tractor frequency of beaming (hz) fhz
: ');

    % The parameters for the first bending mode of the Trailer
frame
    disp('  ')
    fhz2 = input('Input the Trailer frequency of beaming (hz) fhz
: ');

    kb1 = 2.36502;      % Constant for the first bending mode
(free-pinned)
    %                     (from Rao pg. 527)

    z1 = '(cos(kb1*x1/b_fw) + (cosh(kb1*x1/b_fw)) -
((cos(kb1)+cosh(kb1))/(sin(kb1)-sinh(kb1)))*(sin(kb1*x1/b_fw)-
sinh(kb1*x1/b_fw)))';
    % free-pinned beam mode function
    z1dd = '((kb1/b_fw)^2)*(-cos(kb1*x1/b_fw) +
(cosh(kb1*x1/b_fw)) - ((cos(kb1)+cosh(kb1))/(sin(kb1)-
sinh(kb1)))*(-sin(kb1*x1/b_fw)-sinh(kb1*x1/b_fw)))';
    % second derivative of free-pinned beam mode function

    kb2 = 3.926602;     % Constant for the first bending mode
(pinned-free)
    %                     (from Rao pg. 527)

    z2 = '(sin(kb2*x2/L_tlr) +
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
    % pinned-free beam mode function
    z2dd = '(kb2/L_tlr)^2*(-sin(kb2*x2/L_tlr) +
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
    % second derivative of pinned-free beam mode function

else disp('Insufficient information regarding fifth wheel
suspension.')
end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%%%%%%%%%%%%%%%%%%  Computation of Integrals
%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

D1_t=['(',z1,')'];                  % Tractor frame beaming
equations to be
D2_t=['((a1-x1).*(',z1,'))'];       % used in the integrals
(string form)
D3_t=['((',z1,').*(',z1,'))'];
D4_t=['((',z1dd,').*(',z1dd,'))'];

D1_tlr=['(',z2,')'];                % Trailer beaming equations
to be
D2_tlr=['((e-x2).*(',z2,'))'];      % used in the integrals
(string form)
D3_tlr=['((',z2,').*(',z2,'))'];
D4_tlr=['((',z2dd,').*(',z2dd,'))'];

I1_t=quadl('modeD1_t',0,b_fw);      % Integrals of functions
defined above
I2_t=quadl('modeD2_t',0,b_fw);      % (along length of tractor
frame)
I3_t=quadl('modeD3_t',0,b_fw);
I4_t=quadl('modeD4_t',0,b_fw);

I1_tlr=quadl('modeD1_tlr',0,L_tlr); % Integrals of functions
defined above
I2_tlr=quadl('modeD2_tlr',0,L_tlr); % (along length of trailer)
I3_tlr=quadl('modeD3_tlr',0,L_tlr);
I4_tlr=quadl('modeD4_tlr',0,L_tlr);

E_a1=modeD1_t(b_a1);        % Disp at axle #1 due to tractor frame
beaming
E_cf=modeD1_t(b_cf);        % Disp at cab front due to tractor
frame beaming
E_e=modeD1_t(b_e);          % Disp at engine due to tractor frame
beaming
E_cr=modeD1_t(b_cr);        % Disp at cab rear due to tractor frame
beaming
E_a2=modeD1_t(b_a2);        % Disp at axle #2 due to tractor frame
beaming
E_fw=modeD1_t(b_fw);        % Disp at fifth wheel due to tractor
frame beaming
E_a3=modeD1_t(b_a3);        % Disp at axle #3 due to tractor frame
beaming
E_0=modeD1_tlr(0);          % Disp at fifth wheel due to trailer
beaming
E_a4=modeD1_tlr(b_a4);      % Disp at axle #4 due to trailer
beaming
E_a5=modeD1_tlr(b_a5);      % Disp at axle #5 due to trailer
beaming
```

```
EI_t = 4*pi^2*fhz^2*(b_fw/kb1)^4*ML_t;       %Tractor frame
flexural rigidity
EI_tlr = 4*pi^2*fhz2^2*(L_tlr/kb2)^4*ML_tlr; %Trailer flexural
rigidity

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%  Vehicle Tire Selection
%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

disp('   ')
disp('STEER AXLE TIRE SELECTION')
TireData3;                  % M-file for tire data
wd1 = wd;                   %(m)        Nominal cross section
width
mt1 = mt;                   %(kg)       Mass of axle #1
P1 = P;                     %(psi)      Tire pressure from
TireData3.m
press1 = press;             %(psi)      Tire pressure array
numtires1 = numtires;       %           Number of tires on axle
Kstiff1 = Kstiff;           %(N/m)      Tire stiffness array
kt1 = KK * numtires1;       %(N/m)      Per-axle Rad Stiffness
ct1 = ct;                   %(N/(m/s))  Per-axle Damping

disp('   ')
disp('DRIVE AXLE TIRE SELECTION')
TireData3;                  % M-file for tire data
wd23 = wd;                  %(m)        Nominal cross section
width
mt2 = mt;                   %(kg)       Mass of axle #2
mt3 = mt;                   %(kg)       Mass of axle #3
P23 = P;                    %(psi)      Tire pressure from
TireData3.m
press23 = press;            %(psi)      Tire Pressure array
numtires23 = numtires;      %           Number of tires on axle
Kstiff23 = Kstiff;          %(N/m)      Tire stiffness array
kt2 = KK * numtires23;      %(N/m)      Per-axle Rad Stiffness
kt3 = KK * numtires23;      %(N/m)      Per-axle Rad Stiffness
ct2 = ct;                   %(N/(m/s))  Per-axle Damping
ct3 = ct;                   %(N/(m/s))  Per-axle Damping

disp('   ')
disp('TRAILER AXLE TIRE SELECTION')
TireData3;                  % M-file for tire data
wd45 = wd;                  %(m)        Nominal cross section
width
mt4 = mt;                   %(kg)       Mass of axle #4
mt5 = mt;                   %(kg)       Mass of axle #5
P45 = P;                    %(psi)      Tire pressure from
TireData3.m
press45 = press;            %(psi)      Tire Pressure array
numtires45 = numtires;      %           Number of tires on axle
Kstiff45 = Kstiff;          %(N/m)      Tire stiffness array
```

```
kt4 = KK * numtires45;      %(N/m)      Per-axle Rad Stiffness
kt5 = KK * numtires45;      %(N/m)      Per-axle Rad Stiffness
ct4 = ct;                   %(N/(m/s))  Per-axle Damping
ct5 = ct;                   %(N/(m/s))  Per-axle Damping

% Adjusted Tire Parameters
% kt1 = 945350;       %N/m
% kt2 = 1671741;      %N/m
% kt3 = 1671741;      %N/m
%
% ct1 = 517;          %N/(m/s)
% ct2 = 648.3;        %N/(m/s)
% ct3 = 648.3;        %N/(m/s)

% J Penalty Parameters
% kt1 = 906500;       %N/m
% kt2 = 2244909;      %N/m
% kt3 = 2244909;      %N/m

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%  Speed of the Vehicle  %%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('VEHICLE VELOCITY')
disp('  ')
disp('Please choose the unit of velocity');
disp('a : Miles per Hour (mph)');
disp('b : Kilometers per Hour (kph)');
vel = input('Input the unit of velocity (a/b):  ', 's');
disp('  ')
vm = input('Input the velocity of the vehicle, vm : ');

if vel == 'a'
    v = 0.4473*vm;              %Velocity conversion from mph to
m/s
elseif vel == 'b'
    v = 0.277778*vm;            %Velocity conversion from kph to
m/s
end

T(1) = 0;               %Time delay between front axle and
remaining axles
T(2) = (a+b)/v;         % Axle #2
T(3) = (a+d)/v;         % Axle #3
T(4) = (a+i+e+f)/v;     % Axle #4
T(5) = (a+i+e+h)/v;     % Axle #5

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%  Road PSD Selection  %%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('ROAD PSD SELECTION')
disp('  ')
```

```
disp('Road PSD Constants, m^2/cyc/m, Ref: Wong, Theory of Ground
Vehicles')
disp('S(W)=Csp/W^N  where W=spatial frequency')
disp('  ')
disp('a : Csp = 4.3e-11,N=3.8     Smooth Runway')
disp('b : Csp = 8.1e-6, N=2.1     Rough Runway')
disp('c : Csp = 4.8e-7, N=2.1     Smooth Highway')
disp('d : Csp = 4.4e-6, N=2.1     Highway with Gravel')
disp('  ')
tabchoice11=input('Input the road surface to be used :   ','s');

if tabchoice11== 'a',              % smooth runway
   Csp = 4.3e-11;
   N=3.8;

   elseif tabchoice11== 'b',       % rough runway
      Csp = 8.1e-6;
      N=2.1;

   elseif tabchoice11 == 'c',      % smooth highway
      Csp = 4.8e-7;
      N=2.1;

   elseif tabchoice11 == 'd',      % highway with gravel
      Csp = 4.4e-6;
      N=2.1;

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%  System Matrices
%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

% THE SYSTEM IS WRITTEN AS (M*S*S+C*S+K)X(S)=(A*S+B)U(S)

%%%%%%%%%%%%%%%%%%%%%%%
%%%%  Mass Matrix  %%%%
%%%%%%%%%%%%%%%%%%%%%%%
M = zeros(15,15);

M(1,1) = m_s;                  % Eqn #1: Vertical Disp of Seat

M(2,2) = m_c;                  % Eqn #2: Vertical Disp of Cab

M(3,3) = I_c;                  % Eqn #3: Pitch of Cab

M(4,4) = m_e;                  % Eqn #4: Vertical Disp of Engine

M(5,5) = m_t;                  % Eqn #5: Vertical Disp of
Tractor Frame
M(5,6) = ML_t*b_fw*(b_fw/2-a1);
```

```
M(5,7) = ML_t*I1_t;

M(6,5) = ML_t*b_fw*(b_fw/2-a1);   % Eqn #6: Pitch of Tractor
Frame
M(6,6) = I_t;
M(6,7) = -ML_t*I2_t;

M(7,5) = ML_t*I1_t;              % Eqn #7: Beaming of Tractor
Frame
M(7,6) = -ML_t*I2_t;
M(7,7) = ML_t*I3_t;

M(8,8) = m_tlr;                  % Eqn #8: Vertical Disp of
Trailer
M(8,9) = -ML_tlr*L_tlr*(e-L_tlr/2);
M(8,10) = ML_tlr*I1_tlr;

M(9,8) = -ML_tlr*L_tlr*(e-L_tlr/2);% Eqn #9: Pitch of Trailer
M(9,9) = I_tlr;
M(9,10) = -ML_tlr*I2_tlr;

M(10,8) = ML_tlr*I1_tlr;         % Eqn #10: Beaming of Trailer
M(10,9) = -ML_tlr*I2_tlr;
M(10,10) = ML_tlr*I3_tlr;

M(11,11) = mt1;                  % Eqn #11: Vertical Disp of Axle
#1

M(12,12) = mt2;                  % Eqn #12: Vertical Disp of Axle
#2

M(13,13) = mt3;                  % Eqn #13: Vertical Disp of Axle
#3

M(14,14) = mt4;                  % Eqn #14: Vertical Disp of Axle
#4

M(15,15) = mt5;                  % Eqn #15: Vertical Disp of Axle
#5

%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%
C = zeros(15,15);

C(1,1) = cs;
C(1,2) = -cs;
C(1,3) = r*cs;

C(2,1) = -cs;
C(2,2) = cs+ccf+ccr;
C(2,3) = -r*cs-n*ccf+p*ccr;
C(2,5) = -ccf-ccr;
C(2,6) = l*ccf-j*ccr;
```

```
C(2,7) = -ccf*E_cf-ccr*E_cr;

C(3,1) = r*cs;
C(3,2) = -r*cs-n*ccf+p*ccr;
C(3,3) = (r^2)*cs+(n^2)*ccf+(p^2)*ccr;
C(3,5) = n*ccf-p*ccr;
C(3,6) = -n*l*ccf-p*j*ccr;
C(3,7) = n*ccf*E_cf-p*ccr*E_cr;

C(4,4) = ce;
C(4,5) = -ce;
C(4,6) = m*ce;
C(4,7) = -ce*E_e;

C(5,2) = -ccf-ccr;
C(5,3) = n*ccf-p*ccr;
C(5,4) = -ce;
C(5,5) = ce+ccf+ccr+cfw+c1+c2+c3;
C(5,6) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(5,7) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(5,8) = -cfw;
C(5,9) = e*cfw;
C(5,10) = -cfw*E_0;
C(5,11) = -c1;
C(5,12) = -c2;
C(5,13) = -c3;

C(6,2) = l*ccf-j*ccr;
C(6,3) = -n*l*ccf-p*j*ccr;
C(6,4) = m*ce;
C(6,5) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(6,6) =
(m^2)*ce+(l^2)*ccf+(j^2)*ccr+(i^2)*cfw+(a^2)*c1+(b^2)*c2+(d^2)*c3
;
C(6,7) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2+...
         d*c3*E_a3;
C(6,8) = -i*cfw;
C(6,9) = e*i*cfw;
C(6,10) = -i*cfw*E_0;
C(6,11) = a*c1;
C(6,12) = -b*c2;
C(6,13) = -d*c3;

C(7,2) = -ccf*E_cf-ccr*E_cr;
C(7,3) = n*ccf*E_cf-p*ccr*E_cr;
C(7,4) = -ce*E_e;
C(7,5) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(7,6) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2 ...
         +d*c3*E_a3;
C(7,7) =
ce*E_e^2+ccf*E_cf^2+ccr*E_cr^2+cfw*E_fw^2+c1*E_a1^2+c2*E_a2^2 ...
         +c3*E_a3^2;
```

```
C(7,8) = -cfw*E_fw;
C(7,9) = e*cfw*E_fw;
C(7,10) = -cfw*E_0*E_fw;
C(7,11) = -c1*E_a1;
C(7,12) = -c2*E_a2;
C(7,13) = -c3*E_a3;

C(8,5) = -cfw;
C(8,6) = -i*cfw;
C(8,7) = -cfw*E_fw;
C(8,8) = cfw+c4+c5;
C(8,9) = -e*cfw+f*c4+h*c5;
C(8,10) = cfw*E_0+c4*E_a4+c5*E_a5;
C(8,14) = -c4;
C(8,15) = -c5;

C(9,5) = e*cfw;
C(9,6) = e*i*cfw;
C(9,7) = e*cfw*E_fw;
C(9,8) = -e*cfw+f*c4+h*c5;
C(9,9) = (e^2)*cfw+(f^2)*c4+(h^2)*c5;
C(9,10) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(9,14) = -f*c4;
C(9,15) = -h*c5;

C(10,5) = -cfw*E_0;
C(10,6) = -i*cfw*E_0;
C(10,7) = -cfw*E_fw*E_0;
C(10,8) = cfw*E_0+c4*E_a4+c5*E_a5;
C(10,9) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(10,10) = cfw*E_0^2+c4*E_a4^2+c5*E_a5^2;
C(10,14) = -c4*E_a4;
C(10,15) = -c5*E_a5;

C(11,5) = -c1;
C(11,6) = a*c1;
C(11,7) = -c1*E_a1;
C(11,11) = c1+ct1;

C(12,5) = -c2;
C(12,6) = -b*c2;
C(12,7) = -c2*E_a2;
C(12,12) = c2+ct2;

C(13,5) = -c3;
C(13,6) = -d*c3;
C(13,7) = -c3*E_a3;
C(13,13) = c3+ct3;

C(14,8) = -c4;
C(14,9) = -f*c4;
C(14,10) = -c4*E_a4;
C(14,14) = c4+ct4;

C(15,8) = -c5;
```

```
C(15,9) = -h*c5;
C(15,10) = -c5*E_a5;
C(15,15) = c5+ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%
K = zeros(15,15);

K(1,1) = ks;
K(1,2) = -ks;
K(1,3) = r*ks;

K(2,1) = -ks;
K(2,2) = ks+kcf+kcr;
K(2,3) = -r*ks-n*kcf+p*kcr;
K(2,5) = -kcf-kcr;
K(2,6) = l*kcf-j*kcr;
K(2,7) = -kcf*E_cf-kcr*E_cr;

K(3,1) = r*ks;
K(3,2) = -r*ks-n*kcf+p*kcr;
K(3,3) = (r^2)*ks+(n^2)*kcf+(p^2)*kcr;
K(3,5) = n*kcf-p*kcr;
K(3,6) = -n*l*kcf-p*j*kcr;
K(3,7) = n*kcf*E_cf-p*kcr*E_cr;

K(4,4) = ke;
K(4,5) = -ke;
K(4,6) = m*ke;
K(4,7) = -ke*E_e;

K(5,2) = -kcf-kcr;
K(5,3) = n*kcf-p*kcr;
K(5,4) = -ke;
K(5,5) = ke+kcf+kcr+kfw+k1+k2+k3;
K(5,6) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(5,7) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(5,8) = -kfw;
K(5,9) = e*kfw;
K(5,10) = -kfw*E_0;
K(5,11) = -k1;
K(5,12) = -k2;
K(5,13) = -k3;

K(6,2) = l*kcf-j*kcr;
K(6,3) = -n*l*kcf-p*j*kcr;
K(6,4) = m*ke;
K(6,5) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(6,6) =
(m^2)*ke+(l^2)*kcf+(j^2)*kcr+(i^2)*kfw+(a^2)*k1+(b^2)*k2+(d^2)*k3
;
K(6,7) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2+...
```

```
        d*k3*E_a3;
K(6,8) = -i*kfw;
K(6,9) = e*i*kfw;
K(6,10) = -i*kfw*E_0;
K(6,11) = a*k1;
K(6,12) = -b*k2;
K(6,13) = -d*k3;

K(7,2) = -kcf*E_cf-kcr*E_cr;
K(7,3) = n*kcf*E_cf-p*kcr*E_cr;
K(7,4) = -ke*E_e;
K(7,5) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(7,6) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2 ...
        +d*k3*E_a3;
K(7,7) =
ke*E_e^2+kcf*E_cf^2+kcr*E_cr^2+kfw*E_fw^2+k1*E_a1^2+k2*E_a2^2 ...
        +k3*E_a3^2+EI_t*I4_t;
K(7,8) = -kfw*E_fw;
K(7,9) = e*kfw*E_fw;
K(7,10) = -kfw*E_0*E_fw;
K(7,11) = -k1*E_a1;
K(7,12) = -k2*E_a2;
K(7,13) = -k3*E_a3;

K(8,5) = -kfw;
K(8,6) = -i*kfw;
K(8,7) = -kfw*E_fw;
K(8,8) = kfw+k4+k5;
K(8,9) = -e*kfw+f*k4+h*k5;
K(8,10) = kfw*E_0+k4*E_a4+k5*E_a5;
K(8,14) = -k4;
K(8,15) = -k5;

K(9,5) = e*kfw;
K(9,6) = e*i*kfw;
K(9,7) = e*kfw*E_fw;
K(9,8) = -e*kfw+f*k4+h*k5;
K(9,9) = (e^2)*kfw+(f^2)*k4+(h^2)*k5;
K(9,10) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(9,14) = -f*k4;
K(9,15) = -h*k5;

K(10,5) = -kfw*E_0;
K(10,6) = -i*kfw*E_0;
K(10,7) = -kfw*E_fw*E_0;
K(10,8) = kfw*E_0+k4*E_a4+k5*E_a5;
K(10,9) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(10,10) = kfw*E_0^2+k4*E_a4^2+k5*E_a5^2+EI_tlr*I4_tlr;
K(10,14) = -k4*E_a4;
K(10,15) = -k5*E_a5;

K(11,5) = -k1;
K(11,6) = a*k1;
K(11,7) = -k1*E_a1;
```

```
K(11,11) = k1+kt1;

K(12,5) = -k2;
K(12,6) = -b*k2;
K(12,7) = -k2*E_a2;
K(12,12) = k2+kt2;

K(13,5) = -k3;
K(13,6) = -d*k3;
K(13,7) = -k3*E_a3;
K(13,13) = k3+kt3;

K(14,8) = -k4;
K(14,9) = -f*k4;
K(14,10) = -k4*E_a4;
K(14,14) = k4+kt4;

K(15,8) = -k5;
K(15,9) = -h*k5;
K(15,10) = -k5*E_a5;
K(15,15) = k5+kt5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A = zeros(15,1);

A(11) = ct1;
A(12) = ct2;
A(13) = ct3;
A(14) = ct4;
A(15) = ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
B = zeros(15,1);

B(11) = kt1;
B(12) = kt2;
B(13) = kt3;
B(14) = kt4;
B(15) = kt5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Calculation of Load on Each Axle  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ST(1,4) = -1;
ST(1,5) = -1;
ST(1,6) = -1;
ST(1,7) = -1;
ST(1,8) = -1;

ST(2,4) = -a;
ST(2,5) = b;
```

```
ST(2,6) = d;
ST(2,7) = i+e+f;
ST(2,8) = i+e+h;

ST(3,7) = e+f;
ST(3,8) = e+h;

ST(4,1) = -k1;
ST(4,2) = -k1*a;
ST(4,4) = 1;

ST(5,1) = -k2;
ST(5,2) = k2*b;
ST(5,5) = 1;

ST(6,1) = -k3;
ST(6,2) = k3*d;
ST(6,6) = 1;

ST(7,1) = -k4;
ST(7,2) = k4*i;
ST(7,3) = k4*(e+f);
ST(7,7) = 1;

ST(8,1) = -k5;
ST(8,2) = k5*i;
ST(8,3) = k5*(e+h);
ST(8,8) = 1;

WT(1,1) = (m_s+m_c+m_e+m_t+m_tlr)*g;
WT(2,1) = m_c*g*tc+m_s*g*(tc+p)+m_e*g*m-m_tlr*g*(i+e);
WT(3,1) = -m_tlr*g*e;
WT(4,1) = 0;
WT(5,1) = 0;
WT(6,1) = 0;
WT(7,1) = 0;
WT(8,1) = 0;
DELTA = inv(ST)*(WT);

Wtire(1) = -DELTA(4,1)+mt1*g;   % Total axle load on steer axle
tires
Wtire(2) = -DELTA(5,1)+mt2*g;   % Total axle load on 1st drive
axle tires
Wtire(3) = -DELTA(6,1)+mt3*g;   % Total axle load on 2nd drive
axle tires
Wtire(4) = -DELTA(7,1)+mt4*g;   % Total axle load on 1st trailer
axle tires
Wtire(5) = -DELTA(8,1)+mt5*g;   % Total axle load on 2nd trailer
axle tires

% System "A" Matrix
AA=[zeros(size(M))   eye(size(M))   % System state variable
matrix
    -inv(M)*K       -inv(M)*C];
```

```
[V,EIGAA]=eig(AA);

for ii= 1:30
    for j2=1:15
        if abs(V(j2,ii))==max(abs(V(1:15,ii))) % Largest value of
ev
            vec(:,ii)=V(1:15,ii)/(V(j2,ii));   % Forming
normalized evs
        end
    end
    EIG(:,ii)=EIGAA(ii,ii);
    mag(ii)=abs(EIG(: ,ii));                                %
Magnitude
    whz(ii)=mag(ii)/(2*pi);                                 %
Frequency
    zeta(ii)=-cos(atan2(imag(EIG(:,ii)),real(EIG(:,ii))));  %
Damping ratio
end

disp('  ')
transfer = input('Print out the system matrices? (y/n): ','s');
if transfer == 'y'
    disp('    ')
    disp('Mass, Damping, and Stiffness Matrices')
    M
    C
    K
    disp('  ')
    disp('System "A" Matrix')
    AA
end

% DISPLAYS EIGENVALUES
ii=1:30;
disp('   ')
optl=input('Do you want eigenvalues, frequencies, and damping?
(y/n): ','s');
if optl=='y'
    disp('  ');
    disp('         EIG VAL              Hz
DAMPING');
    disp([EIG',whz',zeta'])
end

% DISPLAY NORMALIZED EIGENVECTORS
disp('  ')
opt2=input('Do you want normalized eigenvectors? (y/n): ','s');
if opt2=='y'
    i2=1;
    for j3= 1:30
        if i2==31,
        else ii= 1:15;
            if EIG(i2)==real(EIG(i2)),
                disp('  ')
                disp('Normalized eigen vectors');
```

```
                disp('      Eigenvalue              Frequency
...          Damping:');
                disp([EIG(i2),whz(i2),zeta(i2)]);
                disp('     NO            MAG         PHASE');

disp([ii',(abs(vec(:,i2))),(180/pi*angle(vec(:,i2)))]);
                disp('Press "enter" to continue')
                i2=i2+1;
                pause
            else disp(' ')
                disp('Normalized eigen vectors');
                disp('      Eigenvalue              Frequency
...         Damping:');
                disp([EIG(i2),whz(i2),zeta(i2)]);
                disp('     NO            MAG         PHASE
MAG  ...       PHASE');

disp([ii',(abs(vec(:,i2))),(180/pi*angle(vec(:,i2))),(abs(vec(:,i
2+1))),(180/pi*angle(vec(:,i2+1)))]);
                disp('Press "enter" to continue')
                i2=i2+2;
                pause
            end
        end
    end
end

% ISO 2631 FOR REDUCED COMFORT BOUNDARY
% COMFORT BOUNDARIES FOR VERTICAL ACCELERATION
% THE ISO CENTRAL FREQUENCIES (Hz)

wc=[ .1 1 1.25 1.6 2 2.5 3.15 4 5 6.3 8 10 12.5 16 20 25 31.5 40
50];
whzc=[ .1 .125 .16 .2 .25 .315 .4 .5 .63 .8 1 1.25 1.6 2 2.5 3.15
...
    4 5 6.3 8 10 12.5 16 20 25 31.5 40 50];

% 2.5 hr FATIGUE BOUNDARY
fat1=[4.284,1.4,1.25,1.12,1,.9,.8,.71,.71,.71,.71,...
      .9,1.12,1.4,1.8,2.24,2.8,3.55,4.5];
% 2.5hr REDUCED COMFORT BOUNDARY
comf1=fat1/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf2= comf1/2.254;

%-----------------------------------------------------------------
---
% COMFORT BOUNDARIES FOR LONGITUDINAL AND LATERAL ACC
% 2.5hr FATIGUE BOUNDARY
fat2=[0.5,0.5,0.5,0.5,0.5,0.63,0.8,1,1.25,1.6,2,2.5,3.15,4,5,6.3,
8,10,12.5];

% 2.5hr REDUCED COMFORT BOUNDARY
comf3=fat2/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf4= comf3/2.254;
```

```
%-----------------------------------------------------------------
--


whzcr = 2*pi*whzc;      % Calculation of central frequencies in
rad/s
freqlow=0.89*whzcr;     % Lower octave band
freqhigh=1.12*whzcr;    % Upper octave band
freq=[freqlow' whzcr' freqhigh'];

imag=sqrt(-1);

for ii=1:length(whzc);
    for jj=1:3;         % jj=1 is freqlow, jj=2 is center freq
                        % jj=3 is freqhigh
        w = freq(ii,jj);
        s = imag*w;
        dp = sqrt(h1^2+r^2);

        % Time delay array
        time = [0 0 0 0 0 0 0 0 0 0 1 exp(-s*T(2)) exp(-s*T(3))
...
            exp(-s*T(4)) exp(-s*T(5))];

        % TF Matrix
        vectx = (inv(M*s*s+C*s+K)*((A*s+B).*(time.')));

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %%%  Transfer Functions  %%%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%

        z_s=[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]*vectx;     % vert
seat cg
        long=[0 0 -h1 0 0 0 0 0 0 0 0 0 0 0 0]*vectx;  % long
disp of driver
        stroke=[0 0 0 0 1 i E_fw -1 e -E_0 0 0 0 0 0]*vectx;% 5th
wh stroke
        z_tlr=[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]*vectx;   % vert tlr
cg

        %%%%%%%%%%%%%%%%%%%%
        %%%  Magnitudes  %%%
        %%%%%%%%%%%%%%%%%%%%

        % Displacement Transfer Functions
        magcfstroke(ii,jj)=abs(stroke);

        % Acceleration Transter Functions
        magcfA1(ii,jj)=abs(s*s*z_s);  % Mag of trans function,
(m/s*s)/m
        magcfAlong(ii,jj)=abs(s*s*long);
        magcftlr(ii,jj)=abs(s*s*z_tlr);

        %%%%%%%%%%%%%%
```

```
        %%%  PSDs  %%%
        %%%%%%%%%%%%%%

        % Road PSD in m^2/(rad/s)
        rpsd(ii,jj)=Csp*(((2*pi*v)^(N-1))/(w^N));

        % Acceleration PSDs in (m/s^2)^2/(rad/s)
        psdcfA1(ii,jj)=magcfA1(ii,jj)*magcfA1(ii,jj)*rpsd(ii,jj);

psdcfAlong(ii,jj)=magcfAlong(ii,jj)*magcfAlong(ii,jj)*rpsd(ii,jj)
;

psdcftlr(ii,jj)=magcftlr(ii,jj)*magcftlr(ii,jj)*rpsd(ii,jj);

        % 5th Wheel Stroke PSD in m^2/(rad/s)

psdcfstroke(ii,jj)=magcfstroke(ii,jj)*magcfstroke(ii,jj)*rpsd(ii,
jj);
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  RMS CALCULATIONS  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%

for kk=1:length(whzc)
    % Vert. Driver's Seat RMS
    msqy1a(kk)=0.5*(psdcfA1(kk,1)+psdcfA1(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqy1b(kk)=0.5*(psdcfA1(kk,2)+psdcfA1(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqy1(kk)=msqy1a(kk)+msqy1b(kk);
    rmsA1cf(kk)=sqrt(msqy1(kk));
    % Long. Driver RMS
    msqylonga(kk)=0.5*(psdcfAlong(kk,1)+psdcfAlong(kk,2))*...
     (freq(kk,2)-freq(kk,1));
    msqylongb(kk)=0.5*(psdcfAlong(kk,2)+psdcfAlong(kk,3))*...
     (freq(kk,3)-freq(kk,2));
    msqylong(kk)=msqylonga(kk)+msqylongb(kk);
    rmsAlongcf(kk)=sqrt(msqylong(kk));
    % 5th Wheel Stroke RMS
    msqystrokea(kk)=0.5*(psdcfstroke(kk,1)+psdcfstroke(kk,2))*...
     (freq(kk,2)-freq(kk,1));
    msqystrokeb(kk)=0.5*(psdcfstroke(kk,2)+psdcfstroke(kk,3))*...
     (freq(kk,3)-freq(kk,2));
    msqystrokecf(kk)=msqystrokea(kk)+msqystrokeb(kk);
    rmsstrokecf(kk)=sqrt(msqystrokecf(kk));
    % Road Surface RMS
    msqyroada(kk)=0.5*(rpsd(kk,1)+rpsd(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqyroadb(kk)=0.5*(rpsd(kk,2)+rpsd(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqyroadcf(kk)=msqyroada(kk)+msqyroadb(kk);
    rmsroadcf(kk)=sqrt(msqyroadcf(kk));
    % Vert. Trailer CG RMS
```

```
    msqytlra(kk)=0.5*(psdcftlr(kk,1)+psdcftlr(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqytlrb(kk)=0.5*(psdcftlr(kk,2)+psdcftlr(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqytlr(kk)=msqytlra(kk)+msqytlrb(kk);
    rmstlrcf(kk)=sqrt(msqytlr(kk));
end

RMScf = [rmsA1cf',rmsAlongcf',rmstlrcf'];           % Accel. RMS
Matrix

% Calculate weighted rms acceleration from 0.1 to 50 Hz
% at the ISO Center Frequencies .... Wgt are the ISO weights
% Ref: ISO 2631-1:1997(E); V=vertical; L=longitudinal

wcc=[.1,.125,.16,.2,.25,.315,.4,.5,.63,.8,1,1.25,1.6,2,2.5,3.15,4
,5,...
    6.3,8,10,12.5,16,20,25,31.5,40,50];
WgtV=[.0312,.0486,.079,.121,.182,.263,.352,.418,.459,.477,.482,.4
84,...

.494,.531,.631,.804,.967,1.039,1.054,1.036,.988,.902,.768,.636,..
.
    .513,.405,.314,.246];
WgtL=0.001*[62.4,97.3,158,243,365,530,713,853,944,992,1011,1008,9
68,...

890,776,642,512,409,323,253,212,161,125,100,80,63.2,49.4,38.8];

isovert = WgtV.*RMScf(1:28,1)';       % Weighted Vert. Driver RMS
Accel.
isolong = WgtL.*RMScf(1:28,2)';       % Weighted Long. Driver RMS
Accel.
isotlr = WgtV.*RMScf(1:28,3)';        % Weighted Vert. Trailer CG
RMS Accel.

disp('  ')
disp('Would you like to see the driver')
transfer = input('weighted acceleration values? (y/n): ', 's');
if transfer == 'y'
    disp('  ')
    disp('********* DRIVER VERTICAL WEIGHTED ACCELERATION VALUES
********')
    disp('    Freq        RMS acc, CG      WgtV    Wgt*RMSacc ...
(Wgt*RMSacc)^2')
    disp('    Hz          m/s^2                    m/s^2
(m/s^2)^2')
    disp('  ')
    disp([wcc' rmsA1cf(1:28)' WgtV' (WgtV.*rmsA1cf(1:28))' ...
        ((WgtV.*rmsA1cf(1:28)).^2)'])
    disp('  ')
    disp('******* DRIVER LONGITUDINAL WEIGHTED ACCELERATION
VALUES ******')
    disp('    Freq        RMS acc, CG      WgtL    Wgt*RMSacc ...
(Wgt*RMSacc)^2')
```

```
    disp('    Hz          m/s^2                    m/s^2
(m/s^2)^2')
    disp('  ')
    disp([wcc' rmsAlongcf(1:28)' WgtL' (WgtL.*rmsAlongcf(1:28))'
...
        ((WgtL.*rmsAlongcf(1:28)).^2)'])
    disp('  ')
    disp('**** Weighted RMS Acceleration, a0, m/s^2 ****')
    disp('  ')
    disp('a0 < 0.315 m/s^2             Not Uncomfortable')
    disp('0.315 < a0 < 0.63 m/s^2      A Little Uncomfortable')
    disp('0.5   < a0 < 1.0  m/s^2      Fairly Uncomfortable')
    disp('0.8   < a0 < 1.6  m/s^2      Uncomfortable')
    disp('1.25  < a0 < 2.5  m/s^2      Very Uncomfortable')
    disp('a0 > 2 m/s^2                 Extremely Uncomfortable')
    disp('  ')

    term2V=(WgtV.*rmsA1cf(1:28)).^2;
    a0_V_dr=(sum(term2V))^0.5;             % a0 for vert. disp of
driver

    term2L=(WgtL.*rmsAlongcf(1:28)).^2;
    a0_L_dr=(sum(term2L))^0.5;             % a0 for long. disp of
driver

    aV=(a0_L_dr^2 + a0_V_dr^2)^0.5;     % a0 for comb vert and
long disp

    term2tlr=(WgtV.*rmstlrcf(1:28)).^2;
    a0_V_tlr=(sum(term2tlr))^0.5;

    term2S=(rmsstrokecf(1:28)).^2;
    aStroke=(sum(term2S))^0.5;

    disp('************ a0 VALUES FOR DRIVER m/s^2 ***********')
    disp('   Vertical   Longitudinal   Combined    RMS Stroke
(mm)')
    disp('    m/s^2        m/s^2         m/s^2       (0.1-50
Hz)')
    disp([a0_V_dr a0_L_dr aV aStroke*1000])
    disp('**************************************************')
end

% Here the transfer functions are formed again with a smoother
frequency
% vector for closer inspection

omega = logspace(log10(0.1),log10(50),100);     % freq range in
Hz
omrs = 2*pi*omega;                              % freq range in
rad/s

for iii=1:length(omrs)

    rsom=omrs(iii);
```

```
    ss=imag*rsom;

    bbb=[0 0 0 0 0 0 0 0 0 0 1 exp(-ss*T(2)) exp(-ss*T(3)) exp(-
ss*T(4))...
        exp(-ss*T(5))];

    vectxx=inv(M*ss*ss+C*ss+K)*((A*ss+B).*(bbb.'));

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %%%  Transfer Functions  %%%
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%

    z_s2=[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]*vectxx;    % vert seat
cg
    z_c=[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]*vectxx;    % vert cab cg
    z_t=[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]*vectxx;    % vert tractor
cg
    p_t=[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0]*vectxx;    % pitch
tractor cg
    z_tlr=[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]*vectxx;  % vert trailer
cg
    p_tlr=[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]*vectxx;  % pitch
trailer cg
    z_1=[0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]*vectxx;    % vert axle 1
    z_2=[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0]*vectxx;    % vert axle 2
    z_3=[0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]*vectxx;    % vert axle 3
    z_4=[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0]*vectxx;    % vert axle 4
    z_5=[0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]*vectxx;    % vert axle 5
    stroke2=[0 0 0 0 1 i E_fw -1 e -E_0 0 0 0 0 0]*vectxx;% 5th
wh stroke
    long2=[0 0 -h1 0 0 0 0 0 0 0 0 0 0 0 0]*vectxx; % long disp
of driver

    ff(1)=(ct1*ss+kt1)*(z_1-1);               % wheel force
1/road
    ff(2)=(ct2*ss+kt2)*(z_2*exp(ss*T(2))-1);  % wheel force
2/road
    ff(3)=(ct3*ss+kt3)*(z_3*exp(ss*T(3))-1);  % wheel force
3/road
    ff(4)=(ct4*ss+kt4)*(z_4*exp(ss*T(4))-1);  % wheel force
4/road
    ff(5)=(ct5*ss+kt5)*(z_5*exp(ss*T(5))-1);  % wheel force
5/road

    %%%%%%%%%%%%%%%%%%%%
    %%%  Magnitudes  %%%
    %%%%%%%%%%%%%%%%%%%%

    % Displacement Transfer Functions
    mag1(iii)=abs(z_s2);       % Mag of transfer function, (m/m)
    mag2(iii)=abs(z_c);
    mag5(iii)=abs(z_t);
    mag8(iii)=abs(z_tlr);
    mag11(iii)=abs(z_1);
    mag12(iii)=abs(z_2);
```

```
    mag13(iii)=abs(z_3);
    mag14(iii)=abs(z_4);
    mag15(iii)=abs(z_5);
    magstroke(iii)=abs(stroke2);
    maglong(iii)=abs(long2);

    % Acceleration Transter Fuctions
    magA1(iii)=abs(ss*ss*z_s2);  % Mag of trans function,
(m/s*s)/m
    magA2(iii)=abs(ss*ss*z_c);
    magA5(iii)=abs(ss*ss*z_t);
    magA8(iii)=abs(ss*ss*z_tlr);
    magA11(iii)=abs(ss*ss*z_1);
    magA12(iii)=abs(ss*ss*z_2);
    magA13(iii)=abs(ss*ss*z_3);
    magA14(iii)=abs(ss*ss*z_4);
    magA15(iii)=abs(ss*ss*z_5);
    magAlong(iii)=abs(ss*ss*long2);

    % Wheel Force Transfer Functions
    magWF1(iii)=abs(ff(1));   % Mag of TF, N/m
    magWF2(iii)=abs(ff(2));
    magWF3(iii)=abs(ff(3));
    magWF4(iii)=abs(ff(4));
    magWF5(iii)=abs(ff(5));

    %%%%%%%%%%%%%%
    %%%  PSDs  %%%
    %%%%%%%%%%%%%%

    % Road PSD in m*m/(rad/s)
    rpsd2(iii)=Csp*(((2*pi*v)^(N-1))/(rsom^N));

    % Acceleration PSDs in (m/s^2)^2/(rad/s)
    psdA1(iii)=magA1(iii)*magA1(iii)*rpsd2(iii);
    psdA8(iii)=magA8(iii)*magA8(iii)*rpsd2(iii);
    psdAlong(iii)=magAlong(iii)*magAlong(iii)*rpsd2(iii);

    % 5th Wheel Stroke PSD in m^2/(rad/s)
    psdstroke(iii)=magstroke(iii)*magstroke(iii)*rpsd2(iii);

    % Wheel Force PSDs in N^2/(rad/s)
    psdWF1(iii)=magWF1(iii)*magWF1(iii)*rpsd2(iii);
    psdWF2(iii)=magWF2(iii)*magWF2(iii)*rpsd2(iii);
    psdWF3(iii)=magWF3(iii)*magWF3(iii)*rpsd2(iii);
    psdWF4(iii)=magWF4(iii)*magWF4(iii)*rpsd2(iii);
    psdWF5(iii)=magWF5(iii)*magWF5(iii)*rpsd2(iii);
end

MAG =
[mag1,maglong,mag2,mag5,mag8,mag11,mag12,mag13,mag14,mag15];
MAGA =
[magA1,magAlong,magA2,magA5,magA8,magA11,magA12,magA13,magA14,mag
A15];
```

```
PSDWF = [psdWF1,psdWF2,psdWF3,psdWF4,psdWF5];          % WF psd
matrix

%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  RMS CALCULATIONS  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%

for kkk=1:99
    % Wheel Force RMS - Axle 1
    msqyWF1(kkk)=0.5*(psdWF1(kkk)+psdWF1(kkk+1))*(omrs(kkk+1)-
omrs(kkk));
    rmsWF1(kkk)=sqrt(msqyWF1(kkk));
    % Wheel Force RMS - Axle 2
    msqyWF2(kkk)=0.5*(psdWF2(kkk)+psdWF2(kkk+1))*(omrs(kkk+1)-
omrs(kkk));
    rmsWF2(kkk)=sqrt(msqyWF2(kkk));
    % Wheel Force RMS - Axle 3
    msqyWF3(kkk)=0.5*(psdWF3(kkk)+psdWF3(kkk+1))*(omrs(kkk+1)-
omrs(kkk));
    rmsWF3(kkk)=sqrt(msqyWF3(kkk));
    % Wheel Force RMS - Axle 4
    msqyWF4(kkk)=0.5*(psdWF4(kkk)+psdWF4(kkk+1))*(omrs(kkk+1)-
omrs(kkk));
    rmsWF4(kkk)=sqrt(msqyWF4(kkk));
    % Wheel Force RMS - Axle 5
    msqyWF5(kkk)=0.5*(psdWF5(kkk)+psdWF5(kkk+1))*(omrs(kkk+1)-
omrs(kkk));
    rmsWF5(kkk)=sqrt(msqyWF5(kkk));
end

WFRMS = [rmsWF1',rmsWF2',rmsWF3',rmsWF4',rmsWF5'];  % Wheel Force
RMS

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Plotting the Transfer Functions  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
transfer = input('Plot the transfer functions? (y/n): ', 's');
if transfer == 'y'
    disp('Press "enter" to continue')
    clf
    figure(1)
    loglog(omega,mag1)
    title('Vert. Displacement T.F.,Drivers Seat   (1/s*s)')
    xlabel('Frequency, Hz');grid
    figure(2)
    loglog(omega,magA1)
    title('Vert. Acceleration T.F.,Drivers Seat   (1/s*s)')
    xlabel('Frequency, Hz');grid;pause

    disp('Press "enter" to continue')
    figure(1)
    loglog(omega,maglong)
    title('Long. Displacement T.F.,Drivers Seat   (1/s*s)')
    xlabel('Frequency, Hz');grid
```

```
    figure(2)
    loglog(omega,magAlong)
    title('Long. Acceleration T.F.,Drivers Seat   (1/s*s)')
    xlabel('Frequency, Hz');grid;pause

    close all
    figure(1)
    loglog(omega,mag11)
    title('Axle 1 Vert. Displacement TF, (m/m)')
    xlabel('Frequency, Hz');grid;pause;

    clf
    figure(1)
    loglog(omega,mag12)
    title('Axle 2 Vert. Displacement TF, (m/m)')
    xlabel('Frequency, Hz');grid;pause;

    clf
    figure(1)
    loglog(omega,mag13)
    title('Axle 3 Vert. Displacement TF, (m/m)')
    xlabel('Frequency, Hz');grid;pause;

    clf
    figure(1)
    loglog(omega,mag14)
    title('Axle 4 Vert. Displacement TF, (m/m)')
    xlabel('Frequency, Hz');grid;pause;

    clf
    figure(1)
    loglog(omega,mag15)
    title('Axle 5 Vert. Displacement TF, (m/m)')
    xlabel('Frequency, Hz');grid;pause;
end

disp('  ')
disp('Would you like to see the 5th wheel')
transfer = input('stroke transfer function? (y/n): ', 's');
if transfer == 'y'
    disp('Press "enter" to continue')
    clf
    figure(1)
    loglog(omega,magstroke)
    title('Displacement T.F., 5th Wheel Stroke   (m/m)')
    xlabel('Frequency, Hz');grid;pause;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Plotting the RMS Accelerations  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
transfer = input('Plot the RMS Accelerations? (y/n): ', 's');
if transfer == 'y'
```

```
    tile2 = ['RMS Vert Acc of Drivers Seat ,m/s*s  ',
             'RMS Long Acc of Drivers Seat ,m/s*s  '];
    clf
    figure(1)
    disp('Press "enter" to continue')

loglog(wc,comf1,wc,comf2,whzc,RMScf(1:length(whzc),1)),title(tile
2(1,:))
    xlabel('Frequency, Hz');grid;pause;

    disp('Press "enter" to continue')

loglog(wc,comf3,wc,comf4,whzc,RMScf(1:length(whzc),2)),title(tile
2(2,:))
    xlabel('Frequency, Hz');grid;pause;
end

disp('  ')
transfer == input('Plot the 5th wheel RMS stroke? (y/n): ', 's');
if transfer == 'y'
    disp('Press "enter" to continue')
    clf
    figure(1)
    loglog(whzc,rmsstrokecf*1000)
    title('RMS Stroke Across the 5th Wheel  (mm)')
    xlabel('Frequency, Hz');grid;pause;
end

disp('  ')
transfer == input('Plot the Road RMS? (y/n): ', 's');
if transfer == 'y'
    disp('Press "enter" to continue')
    clf
    figure(1)
    loglog(whzc,rmsroadcf*1000)
    title('RMS of Road Surface  (mm)')
    xlabel('Frequency, Hz');grid;pause;
end

disp('  ')
WgtISO = input('Would you like to see the Weighted ISO Values?
(y/n): ', 's');
if WgtISO == 'y'

    figure(1)                               %plot of Weighted ISO
curves

loglog(wc,comf1,'k',wc,comf2,'k',whzc,isovert,'k','LineWidth',1.5
)
    title('Drivers Seat Vertical Weighted ISO Curve')
    %legend('2.5 hour ISO boundary', '8 hour ISO boundary', -1)
    xlabel('Frequency , Hz');
    ylabel('RMS Acceleration, m/s^2');
    grid;

    figure(2)
```

```
    disp('Press "enter" to continue')       %plot of Weighted ISO
curves

loglog(wc,comf3,'k',wc,comf4,'k',whzc,isolong,'k','LineWidth',1.5
)
    title('Drivers Seat Longitudinal Weighted ISO Curve')
    %legend('2.5 hour ISO boundary', '8 hour ISO boundary', -1)
    xlabel('Frequency , Hz');
    ylabel('RMS Acceleration, m/s^2');
    grid;

    pause;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Plotting the Wheel Forces  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Calculate the equivalent stiffness for each of the axles
keq1 = 1/(1/k1+1/kt1);      % N/m
keq2 = 1/(1/k2+1/kt2);      % N/m
keq3 = 1/(1/k3+1/kt3);      % N/m
keq4 = 1/(1/k4+1/kt4);      % N/m
keq5 = 1/(1/k5+1/kt5);      % N/m

% Calculate the total static deflections of each of the axles
stdef1 = Wtire(1)/keq1;     % m
stdef2 = Wtire(2)/keq2;     % m
stdef3 = Wtire(3)/keq3;     % m
stdef4 = Wtire(4)/keq4;     % m
stdef5 = Wtire(5)/keq5;     % m

% Calculate the deflections of the axle suspensions
stdefa1 = Wtire(1)/k1;      % m
stdefa2 = Wtire(2)/k2;      % m
stdefa3 = Wtire(3)/k3;      % m
stdefa4 = Wtire(4)/k4;      % m
stdefa5 = Wtire(5)/k5;      % m

% Calculate the deflections of the tires
stdeft1 = Wtire(1)/kt1;      % m
stdeft2 = Wtire(2)/kt2;      % m
stdeft3 = Wtire(3)/kt3;      % m
stdeft4 = Wtire(4)/kt4;      % m
stdeft5 = Wtire(5)/kt5;      % m

disp('  ')
disp('Would you like to see the')
transfer = input('static loads on the wheels? (y/n): ', 's');
if transfer == 'y'
   disp('  ')
   disp('                     Static Loads on the Wheels
')
   disp('                     Values displayed in Newtons
')
```

```
   disp('  ')
   disp('     Axle 1       Axle 2       Axle 3       Axle 4
Axle 5   ')
   disp([Wtire(1) Wtire(2) Wtire(3) Wtire(4) Wtire(5)])
   disp('  ')
   disp('                    Values displayed in Pounds
')
   disp('  ')
   disp('     Axle 1       Axle 2       Axle 3       Axle 4
Axle 5   ')
   disp([Wtire(1)*0.2248 Wtire(2)*0.2248 Wtire(3)*0.2248
Wtire(4)*0.2248... Wtire(5)*0.2248])
   disp('  ')
   disp('  ')
   disp('                     Total Static Deflection
')
   disp('                    Values displayed in Meters
')
   disp('  ')
   disp('     Axle 1       Axle 2       Axle 3       Axle 4
Axle 5   ')
   disp([stdef1 stdef2 stdef3 stdef4 stdef5])
   disp('  ')
   disp('                    Values displayed in Inches
')
   disp('  ')
   disp('     Axle 1       Axle 2       Axle 3       Axle 4
Axle 5   ')
   disp([stdef1*39.37 stdef2*39.37 stdef3*39.37 stdef4*39.37
stdef5*39.37])
   disp('  ')
   disp('  ')
   disp('                 Static Deflection of Suspension
')
   disp('                    Values displayed in Meters
')
   disp('  ')
   disp('     Axle 1       Axle 2       Axle 3       Axle 4
Axle 5   ')
   disp([stdefa1 stdefa2 stdefa3 stdefa4 stdefa5])
   disp('  ')
   disp('                    Values displayed in Inches
')
   disp('  ')
   disp('     Axle 1       Axle 2       Axle 3       Axle 4
Axle 5   ')
   disp([stdefa1*39.37 stdefa2*39.37 stdefa3*39.37
stdefa4*39.37... stdefa5*39.37])
   disp('  ')
   disp('  ')
   disp('                    Static Deflection of Tires
')
   disp('                    Values displayed in Meters
')
   disp('  ')
   disp('     Axle 1       Axle 2       Axle 3       Axle 4
Axle 5   ')
```

```
   disp([stdeft1 stdeft2 stdeft3 stdeft4 stdeft5])
   disp('  ')
   disp('                    Values displayed in Inches
')
   disp('  ')
   disp('     Axle 1       Axle 2       Axle 3       Axle 4
Axle 5   ')
   disp([stdeft1*39.37 stdeft2*39.37 stdeft3*39.37
stdeft4*39.37... stdeft5*39.37])
   disp('  ')
end

disp('  ')
disp('Would you like to see the')
transfer = input('wheel force transfer functions? (y/n): ', 's');
if transfer == 'y'
    disp('Press "enter" to continue')
    clf
    figure(1)
    loglog(omega,(magWF1./1000),omega,(magWF2./1000),...
        omega,(magWF3./1000),omega,(magWF4./1000),omega,...
        (magWF5./1000)),title('Wheel Forces TF, (N/mm))')
    legend('Axle 1','Axle 2','Axle 3','Axle 4','Axle 5')
    xlabel('Frequency, Hz');grid;pause;
end

% close all
disp('  ')
disp('End of program.')
```

Parameters.m

```
% Tractor Semi-Trailer Parameters
% Developed by Ryan Spivey, 4/10/07
%
% Choose a test vehicle
disp('  ')
disp('VEHICLE SELECTION')
disp('  ')
disp('Please choose a vehicle : ');
disp('a: Ideal Tractor Semi-Trailer');
vehicle = input('Enter your choice : ', 's');

if vehicle == 'a'
    % Inertial Properties
    m_s = 106.7;        %kg      mass of seat
    m_c = 1208;         %kg      mass of cab
    I_c = 2100;         %kg*m^2  M I of cab
    m_e = 2000;         %kg      mass of engine (ESTIMATE)
    m_t = 3783;         %kg      mass of tractor (5783 kg -
engine)
    I_t = 46590.9;      %kg*m^2  M I of tractor
    m_ul = 10800;       %kg      mass of trailer (ESTIMATE)
    I_tlr = 200000;     %kg*m^2  M I of trailer
    m_L = 14000;        %kg      mass of trailer load  (ESTIMATE)
    m_tlr = m_ul+m_L;   %kb      mass of loaded trailer

%     % Original Parameters
%     k1 = 581300;      %N/m      spring const of axle #1
%     k2 = 737600;      %N/m      spring const of axle #2
%     k3 = 436200;      %N/m      spring const of axle #3

    % Nominal Parameters – adjusted to make drive axle
stiffnesses the same
    k1 = 581300;        %N/m     spring const of axle #1
    k2 = 586900;        %N/m     spring const of axle #2
    k3 = 586900;        %N/m     spring const of axle #3
    k4 = 1000000;       %N/m     spring const of axle #4
    k5 = 1000000;       %N/m     spring const of axle #5
    ke = 1e10;          %N/m     spring const of the engine mount
    c1 = 11270;         %N/(m/s) damping const of axle #1
    c2 = 27500;         %N/(m/s) damping const of axle #2
    c3 = 27500;         %N/(m/s) damping const of axle #3
    c4 = 70000;         %N/(m/s) damping const of axle #4
    c5 = 70000;         %N/(m/s) damping const of axle #5
    ce = 10000;         %N/(m/s) damping const of engine mount

    % J Penalty Parameters
%     k1 = 406910;        %N/m     spring const of axle #1
%     k2 = 622114;        %N/m     spring const of axle #2
%     k3 = 622114;        %N/m     spring const of axle #3

%     c1 = 14651;         %N/(m/s) damping const of axle #1
%     c2 = 35750;         %N/(m/s) damping const of axle #2
```

```
%     c3 = 35750;         %N/(m/s) damping const of axle #3

    % Optimized Paramters
%     k1 = 406910;        %N/m     spring const of axle #1
%     k2 = 410830;        %N/m     spring const of axle #2
%     k3 = 410830;        %N/m     spring const of axle #3
%     k4 = 700000;        %N/m     spring const of axle #4
%     k5 = 700000;        %N/m     spring const of axle #5
%     ke = 1e10;          %N/m     spring const of the engine
mount
%     c1 = 14651;         %N/(m/s) damping const of axle #1
%     c2 = 35750;         %N/(m/s) damping const of axle #2
%     c3 = 35750;         %N/(m/s) damping const of axle #3
%     c4 = 70000;         %N/(m/s) damping const of axle #4
%     c5 = 70000;         %N/(m/s) damping const of axle #5
%     ce = 10000;         %N/(m/s) damping const of engine mount

    % Model Dimensions
    b_a1 = 1.065;       %m       Front end of the tractor to axle
#1
    b_cf = 1.470;       %m       Front end of the tractor to cab
front
    b_e = 2.797;        %m       Front end of the tractor to
engine
    b_cr = 4.02;        %m       Front end of the tractor to cab
rear
    b_a2 = 6.035;       %m       Front end of the tractor to axle
#2
    b_fw = 6.688;       %m       Front end of the tractor to 5th
wheel
    b_a3 = 7.34;        %m       Front end of the tractor to axle
#3
    a1 = 4.00607;       %m       Front end of the tractor to
tractor cg

    b_a4 = 8.58;        %m       From the fifth wheel to axle #4
    b_a5 = 9.78;        %m       From the fifth wheel to axle #5

    L_t = 8.2;          %m       Length of Tractor
    L_tlr = 9.78;       %m       Length of Trailer

    e = 5.62;           %m       From the trailer cg to fifth
wheel
    f = 2.96;           %m       From the trailer cg to axle #4
    h = 4.16;           %m       From the trailer cg to axle #5

    a = 2.94107;        %m       From the tractor cg to axle #1
    b = 2.02893;        %m       From the tractor cg to axle #2
    d = 3.33393;        %m       From the tractor cg to axle #3
    l = 2.53607;        %m       From the tractor cg to cab front
    m = 1.209074;       %m       From the tractor cg to engine
    j = 0.013926;       %m       From the tractor cg to cab rear
    i = 2.68193;        %m       From the tractor cg to the fifth
wheel
    n = 1.435;          %m       From the cab cg to cab front
    p = 1.115;          %m       From the cab cg to cab rear
```

```
    r = -0.200;         %m       From the cab cg to seat

    tc = 1.10107;       %m       From the tractor cg to the cab
cg
    h1 = 1.0;           %m       Height of the driver over the
cab
    g = 9.8;            %m/s^2   acceleration due to gravity

    ML_t = m_t/L_t;         %kg/m    Mass per unit length
(Tractor)
    ML_tlr = m_ul/L_tlr;    %kg/m    Mass per unit length
(Trailer)
end

% Seat Suspension Options
disp('  ')
disp('VEHICLE SUSPENSION OPTIONS')
disp('  ')
disp('Give your choice for seat suspension: ')
disp('Note: Without seat suspension gives a very high frequency
mode')
disp('      because the stiffness is set to a high value.')
disp('a : With seat suspension (~0.9 Hz)')
disp('b : Without seat suspension')
z11 = input('Enter your choice : ', 's');

if z11 == 'a',          % Choice 'a' is with seat suspension
    cs = 1140;          % Damping ratio of 0.5
    ks = 3403;          % N/m(spring const of seat suspension)

elseif z11 == 'b',      % Choice 'b' is without seat suspension
    cs = 1329;          % N/(m/s)(damping const of seat
suspension)
    ks = 1e10;          % N/m(spring const of seat suspension)

else disp('Insufficient information regarding seat suspension.')
end

% Cab Suspension Options
disp('  ')
disp('Give your choice for cab suspension: ')
disp('Note: With front or rear or without cab suspension')
disp('      gives a very high frequency mode(s) because the
corresponding')
disp('      stiffness(es)is set to a high value.')
disp('a : With front cab suspension')
disp('b : With rear cab suspension')
disp('c : With front & rear cab suspension')
disp('d : Without cab suspension')
z22 = input('Enter your choice : ', 's');

if z22 == 'a',          % Choice 'a' is front cab suspension
    ccf = 7062;         % N/(m/s)(damping const of front cab
suspension)
```

```
    kcf = 88740;         % N/m(spring const of front cab
suspension)
    ccr = 6430;          % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;          % N/m(spring const of rear cab
suspension)

elseif z22 == 'b',       % Choice 'b' is rear cab suspension
    ccr = 8000;          % Reduced damping
    kcr = 65980;         % N/m(spring const of rear cab
suspension)
    ccf = 13120;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;          % N/m(spring const of front cab
suspension)

elseif z22 == 'c',       % Choice 'c' is front & rear cab
suspension
    ccr = 5073.5;        % N/(m/s)(damping const of rear cab
suspension)
    kcr = 63757.5;       % N/m(spring const of rear cab
suspension)
    ccf = 6864.35;       % N/(m/s)(damping const of front cab
suspension)
    kcf = 86260.5;       % N/m(spring const of front cab
suspension)

elseif z22 == 'd',       % Choice 'd' is without cab suspension
    ccr = 6430;          % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;          % N/m(spring const of rear cab
suspension)
    ccf = 7062;          % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;          % N/m(spring const of front cab
suspension)

else disp('Insufficient information regarding cab suspension.')
end
```

TireData3.m

```
%================================================
%TIRE DATA
%================================================

%************************************************************
**********
%*  TIRE OPTIONS
%************************************************************
**********
% Developed by Ryan Spivey, 4/10/07
%
disp('  ')
disp('Give your choice for the tire type')
disp(' a : XZA2 275/80R22.5 (Steer Axle Design)')
disp(' b : Xone XDA 445/50R22.5 (New Drive Axle Design)')
disp(' c : Xone XTA 445/50R22.5 (New Trailer Axle Design)')
disp(' d : XTE2 LRL 425/65R22.5 (Conventional Wide Base Drive ...
and Trailer Axle Design)')
disp(' e : XDA2 275/80R22.5 (Standard Drive Axle Design)')
disp(' f : XT1 275/80R22.5 (Standard Trailer Axle Design)')
tire = input('Please give your choice : ', 's');

if tire == 'a'     %XZA2 275/80R22.5 (Steer axle tires)
    wd = 0.275;                                   % Cross section
width, m
    mt = 374;                                     % Kg (mass of
the axle)
    press = [8.274 6.895 5.516 4.137 2.758]*14.5; % Tire press,
bar-->psi
    Kstiff = [77.45 72.28 66 58.38 37.2].*9810;   % Per-tire Rad
Stiff
                                                  % kg/mm-->N/m
    numtires = 2;                                 % # of tires
per axle
    ct = 258.5*2;

elseif tire == 'b'      %Xone XDA 445/50R22.5 (New Design)
    wd = 0.445;                                   % Cross section
width, m
    mt = 646;                                     % Kg (mass of
the axle)
    press = [9.2 8.2 7.2 6.2 5.2]*14.5;           % Tire press,
bar-->psi
    Kstiff = [147.6 135 122.1 108.4 93.8].*9810;  % Per-tire Rad
Stiff
                                                  % kg/mm-->N/m
    numtires = 2;                                 % # of tires
per axle
    ct = 324.15*2;

elseif tire == 'c'      %Xone XTA 445/50R22.5 (New Design)
```

```
    wd = 0.445;                                  % Cross section
width, m
    mt = 646;                                    % Kg (mass of
the axle)
    press = [9.2 8.2 7.2 6.2 5.2]*14.5;          % Tire press,
bar-->psi
    Kstiff = [147.6 135 122.1 108.4 93.8].*9810;  % Per-tire Rad
Stiff
                                                 % kg/mm-->N/m
    numtires = 2;                                % # of tires
per axle
    ct = 324.15*2;

elseif tire == 'd'      %XTE2 LRL 425/65R22.5 (Conventional Wide
Base)
    wd = 0.425;                                  % Cross section
width, m
    mt = 646;                                    % Kg (mass of
the axle)
    press = [9 8 7 6 5]*14.5;                    % Tire press,
bar-->psi
    Kstiff = [138 124.8 111.4 95.47 82.31].*9810; % Per-tire Rad
Stiff
                                                 % kg/mm-->N/m
    numtires = 2;                                % # of tires
per axle
    ct = 375.75*2;

elseif tire == 'e'       %XDA2 275/80R22.5 (Standard Drive Axle
Design)
    wd = 0.275;                                  % Cross section
width, m
    mt = 748;                                    % Kg (mass of
the axle)
    press = [9.2 8.2 7.2 6.2 5.2]*14.5;          % Tire press,
bar-->psi
    Kstiff = [111.7 103.3 94.1 84.5 73.7].*9810;  % Per-tire Rad
Stiff
                                                 % kg/mm-->N/m
    numtires = 4;                                % # of tires
per axle
    ct = 261*4;

elseif tire == 'f'      %XT1 275/80R22.5 (Standard Trailer Axle
Design)
    wd = 0.275;                                  % Cross section
width, m
    mt = 648;                                    % Kg (mass of
the axle)
    press = 100;                                 % Tire press,
bar-->psi
    Kstiff = 95.5*9810;                          % Per-tire Rad
Stiff
                                                 % kg/mm-->N/m
    numtires = 4;                                % # of tires
per axle
```

```
    ct = 242.65*4;

else
    disp('Insufficient information regarding tire type.');
end

%*****************************************************************
**********
%*  TIRE STIFFNESS CALCULATIONS
%*****************************************************************
**********

if tire ~= 'f'  % choice "f" does not have the option of diff
tire press
    disp(' ')
    fprintf('Mean Tire Pressure (bar)         %.3e \f (%.3e \f
psi)... \n',(press(3)/14.5),press(3))
    pressure = 1;
    while pressure == 1
        disp(' ')
        P = input('Input the tire pressure(psi) for this axle:
');
        if P >= press(5) & P <= press(1)
            KK = interp1(press,Kstiff,P);              % N/m(Per-
tire stiff)
            pressure = 0;
        elseif P < press(5)
            disp('Tire Pressure Below Minimum Pressure');
        elseif P > press(1)
            disp('Tire Pressure Above Maximum Pressure');
        else
            KK = Kstiff(3);
            pressure = 0;
        end
    end
end
```

Function Files

```
%************************************************************
% modeD1_t.m
%************************************************************

% THIS FUNCTION IS FOR FORMING THE INTEGRAND TO CALCULATE D1_t

function y = modeD1_t(x1)

global D1_t b_fw kb1 alpha;
y = eval(D1_t);


%************************************************************
% modeD1_tlr.m
%************************************************************

% THIS FUNCTION IS FOR FORMING THE INTEGRAND TO CALCULATE D1_tlr

function y = modeD1_tlr(x2)

global D1_tlr L_tlr kb2 alpha;
y = eval(D1_tlr);


%************************************************************
% modeD2_t.m
%************************************************************

% THIS FUNCTION IS FOR FORMING THE INTEGRAND TO CALCULATE D2_t

function y = modeD2_t(x1)

global D2_t b_fw a1 kb1 alpha;
y = eval(D2_t);


%************************************************************
% modeD2_tlr.m
%************************************************************

% THIS FUNCTION IS FOR FORMING THE INTEGRAND TO CALCULATE D2_tlr

function y = modeD2_tlr(x2)

global D2_tlr L_tlr e kb2 alpha;
y = eval(D2_tlr);
```

```
%**************************************************************
% modeD3_t.m
%**************************************************************

% THIS FUNCTION IS FOR FORMING THE INTEGRAND TO CALCULATE D3_t

function y = modeD3_t(x1)

global D3_t b_fw kb1 alpha;
y = eval(D3_t);


%**************************************************************
% modeD3_tlr.m
%**************************************************************

% THIS FUNCTION IS FOR FORMING THE INTEGRAND TO CALCULATE D3_tlr

function y = modeD3_tlr(x2)

global D3_tlr L_tlr kb2 alpha;
y = eval(D3_tlr);


%**************************************************************
% modeD4_t.m
%**************************************************************

% THIS FUNCTION IS FOR FORMING THE INTEGRAND TO CALCULATE D4_t

function y = modeD4_t(x1)

global D4_t b_fw kb1 alpha;
y = eval(D4_t);


%**************************************************************
% modeD4_tlr.m
%**************************************************************

% THIS FUNCTION IS FOR FORMING THE INTEGRAND TO CALCULATE D4_tlr

function y = modeD4_tlr(x2)

global D4_tlr L_tlr kb2 alpha;
y = eval(D4_tlr);
```

Sample Output

Frequency Response of 15 DOF Tractor Semi-Trailer
Roadholding Model
26-Apr-2007

VEHICLE SELECTION

Please choose a vehicle :
a: Ideal Tractor Semi-Trailer
Enter your choice : a

VEHICLE SUSPENSION OPTIONS

Give your choice for seat suspension:
Note: Without seat suspension gives a very high frequency mode
because the stiffness is set to a high value.
a : With seat suspension (~0.9 Hz)
b : Without seat suspension
Enter your choice : a

Give your choice for cab suspension:
Note: With front or rear or without cab suspension
gives a very high frequency mode(s) because the corresponding
stiffness(es)is set to a high value.
a : With front cab suspension
b : With rear cab suspension
c : With front & rear cab suspension
d : Without cab suspension
Enter your choice : b

TRAILER CONFIGURATION

Please choose which configuration to use
a : Loaded Trailer
b : Unloaded Trailer
Please give your choice : a

Give your choice for the fifth wheel configuration:
Note: If a fifth wheel suspension system is chosen, the beaming of
the tractor frame and trailer will be modeled as free-free. If
no suspension is chosen, the tractor frame and trailer will be
modeled as free-pinned and pinned-free respectively.
a : With fifth wheel suspension

b : Without fifth wheel suspension
Please give your choice : b

Input the Tractor frequency of beaming (hz) fhz : 20

Input the Trailer frequency of beaming (hz) fhz : 20

STEER AXLE TIRE SELECTION

Give your choice for the tire type
a : XZA2 275/80R22.5 (Steer Axle Design)
b : Xone XDA 445/50R22.5 (New Drive Axle Design)
c : Xone XTA 445/50R22.5 (New Trailer Axle Design)
d : XTE2 LRL 425/65R22.5 (Conventional Wide Base Drive and Trailer Axle Design)
e : XDA2 275/80R22.5 (Standard Drive Axle Design)
f : XT1 275/80R22.5 (Standard Trailer Axle Design)
Please give your choice : a

Mean Tire Pressure (bar) 5.516e+000 (7.998e+001 psi)

Input the tire pressure(psi) for this axle: 80

DRIVE AXLE TIRE SELECTION

Give your choice for the tire type
a : XZA2 275/80R22.5 (Steer Axle Design)
b : Xone XDA 445/50R22.5 (New Drive Axle Design)
c : Xone XTA 445/50R22.5 (New Trailer Axle Design)
d : XTE2 LRL 425/65R22.5 (Conventional Wide Base Drive and Trailer Axle Design)
e : XDA2 275/80R22.5 (Standard Drive Axle Design)
f : XT1 275/80R22.5 (Standard Trailer Axle Design)
Please give your choice : b

Mean Tire Pressure (bar) 7.200e+000 (1.044e+002 psi)

Input the tire pressure(psi) for this axle: 104

TRAILER AXLE TIRE SELECTION

Give your choice for the tire type
a : XZA2 275/80R22.5 (Steer Axle Design)
b : Xone XDA 445/50R22.5 (New Drive Axle Design)
c : Xone XTA 445/50R22.5 (New Trailer Axle Design)

d : XTE2 LRL 425/65R22.5 (Conventional Wide Base Drive and Trailer Axle Design)
e : XDA2 275/80R22.5 (Standard Drive Axle Design)
f : XT1 275/80R22.5 (Standard Trailer Axle Design)
Please give your choice : c

Mean Tire Pressure (bar)   7.200e+000 (1.044e+002 psi)

Input the tire pressure(psi) for this axle: 104

VEHICLE VELOCITY

Please choose the unit of velocity
a : Miles per Hour (mph)
b : Kilometers per Hour (kph)
Input the unit of velocity (a/b): a

Input the velocity of the vehicle, vm : 60

ROAD PSD SELECTION

Road PSD Constants, m^2/cyc/m, Ref: Wong, Theory of Ground Vehicles
S(W)=Csp/W^N where W=spatial frequency

a : Csp = 4.3e-11,N=3.8   Smooth Runway
b : Csp = 8.1e-6, N=2.1   Rough Runway
c : Csp = 4.8e-7, N=2.1   Smooth Highway
d : Csp = 4.4e-6, N=2.1   Highway with Gravel

Input the road surface to be used :   c

Print out the system matrices? (y/n): n

Do you want eigenvalues, frequencies, and damping? (y/n): y

EIG VAL	Hz	DAMPING
-2.0647e+001 -3.3462e+004i	5.3257e+003	6.1702e-004
-2.0647e+001 +3.3462e+004i	5.3257e+003	6.1702e-004
-1.7876e+001 -5.0086e+003i	7.9715e+002	3.5690e-003
-1.7876e+001 +5.0086e+003i	7.9715e+002	3.5690e-003
-5.0665e+000 -2.8143e+003i	4.4791e+002	1.8003e-003
-5.0665e+000 +2.8143e+003i	4.4791e+002	1.8003e-003
-8.6229e+000 -1.3534e+002i	2.1583e+001	6.3585e-002
-8.6229e+000 +1.3534e+002i	2.1583e+001	6.3585e-002
-8.0850e-001 -8.0984e+001i	1.2890e+001	9.9829e-003
-8.0850e-001 +8.0984e+001i	1.2890e+001	9.9829e-003

```
-6.6914e+001 -2.1211e+001i  1.1172e+001    9.5325e-001
-6.6914e+001 +2.1211e+001i  1.1172e+001    9.5325e-001
-5.4921e+001 -4.7320e+001i  1.1538e+001    7.5759e-001
-5.4921e+001 +4.7320e+001i  1.1538e+001    7.5759e-001
-1.6477e+001 -6.9086e+001i  1.1304e+001    2.3200e-001
-1.6477e+001 +6.9086e+001i  1.1304e+001    2.3200e-001
-2.1954e+001 -6.4320e+001i  1.0817e+001    3.2303e-001
-2.1954e+001 +6.4320e+001i  1.0817e+001    3.2303e-001
-2.3393e+001 -6.2945e+001i  1.0687e+001    3.4837e-001
-2.3393e+001 +6.2945e+001i  1.0687e+001    3.4837e-001
-4.7004e+000 -1.4485e+001i  2.4238e+000    3.0865e-001
-4.7004e+000 +1.4485e+001i  2.4238e+000    3.0865e-001
-5.6963e+000 -1.6980e+000i  9.4601e-001    9.5833e-001
-5.6963e+000 +1.6980e+000i  9.4601e-001    9.5833e-001
-5.6689e+000 -7.5895e+000i  1.5077e+000    5.9843e-001
-5.6689e+000 +7.5895e+000i  1.5077e+000    5.9843e-001
-8.9408e-001 -9.8286e+000i  1.5707e+000    9.0593e-002
-8.9408e-001 +9.8286e+000i  1.5707e+000    9.0593e-002
-1.7590e+000 -9.0625e+000i  1.4693e+000    1.9054e-001
-1.7590e+000 +9.0625e+000i  1.4693e+000    1.9054e-001
```

Do you want normalized eigenvectors? (y/n): n

Would you like to see the driver
weighted acceleration values? (y/n): n

Plot the transfer functions? (y/n): n

Would you like to see the 5th wheel
stroke transfer function? (y/n): n

Plot the RMS Accelerations? (y/n): n

Plot the 5th wheel RMS stroke? (y/n): n

Plot the Road RMS? (y/n): n

Would you like to see the Weighted ISO Values? (y/n): n

Would you like to see the
static loads on the wheels? (y/n): n

Would you like to see the
RMS stroke for the axles? (y/n): y

RMS Stroke for the axles

Values displayed in Millimeters

Axle 1	Axle 2	Axle 3	Axle 4	Axle 5
1.4818e+001	1.4465e+001	1.4649e+001	1.4421e+001	1.4650e+001

Would you like to see the
wheel force transfer functions? (y/n): n

End of program.

Appendix G: opt_axleK_freq.m

This parameter variation program varies the stiffness of the steer axle suspension and the stiffness of the first and second drive axle suspensions combined. Each of the drive axle suspensions on the tractor are assumed to have the same value, so they were combined into one value that was varied, and the individual axle suspension values were assumed to be equal to exactly half of that value. The steer axle was varied from 406,910 N/m to 755,690 N/m in increments of 17,439 N/m. This forms a vector with a length of 21 values that ranges from 30% below to 30% above the nominal value for the steer axle stiffness. Each drive axle was varied from 410,830 N/m to 762,970 N/m in increments of 17,607 N/m. Like the steer axle, this forms a vector with a length of 21 values that ranges from 30% below to 30% above the nominal value for the drive axle stiffness.

The desired output values from this program were the ISO combined weighted acceleration at the driver's seat, the ISO vertical weighted acceleration at the trailer center-of-gravity (CG), and a value called the J penalty, which weighs the importance of the driver ride comfort versus trailer acceleration using weights assigned to them by the user. The program finds the minimum values for each of these outputs, and displays them in tabular form along with the corresponding stiffness values for the steer and drive axle suspensions. Also, the program plots the output information on surface plots to study trends in the information.

opt_axleK_freq.m

```
% opt_axleK_freq.m
% Developed by Ryan Spivey, 4/10/07
%
% Varies axle stiffness using weighted RMS acceleration in the
% frequecncy domain
%
% Incorporates model from dof15_freq2.m
%
% DOFs include - 1)Vertical Disp. of Driver's Seat
%                2)Vertical Disp. of Cab
%                3)Pitch of Cab
%                4)Vertical Disp. of Engine
%                5)Vertical Disp. of Tractor Frame
%                6)Pitch of Tractor Frame
%                7)Beaming of Tractor Frame
%                8)Vertical Disp. of Trailer
%                9)Pitch of Trailer
%               10)Beaming of Trailer
%               11)Vertical Disp. of Axle #1
%               12)Vertical Disp. of Axle #2
%               13)Vertical Disp. of Axle #3
%               14)Vertical Disp. of Axle #4
%               15)Vertical Disp. of Axle #5

clc
clear all
% close all
format short e
format compact

global D1_t D2_t D3_t D4_t D1_tlr D2_tlr D3_tlr D4_tlr
global e a1 kb1 kb2 b_fw L_tlr alpha

disp('  ')
disp('Axle Stiffness Parameter Variation in the Frequency
Domain')
disp('                     Roadholding Model
')
disp(['                        ',date])

% Choose a test vehicle
disp('  ')
disp('VEHICLE SELECTION')
disp('  ')
disp('Please choose a vehicle : ');
disp('a: Ideal Tractor Semi-Trailer');
vehicle = input('Enter your choice : ', 's');

if vehicle == 'a'
    % Inertial Properties
    m_s = 106.7;        %kg      mass of seat
    m_c = 1208;         %kg      mass of cab
```

```
    I_c = 2100;         %kg*m^2  M I of cab
    m_e = 2000;         %kg      mass of engine (ESTIMATE)
    m_t = 3783;         %kg      mass of tractor (5783 kg -
engine)
    I_t = 46590.9;      %kg*m^2  M I of tractor
    m_ul = 10800;       %kg      mass of trailer (ESTIMATE)
    I_tlr = 200000;     %kg*m^2  M I of trailer
    m_L = 14000;        %kg      mass of trailer load  (ESTIMATE)
    m_tlr = m_ul+m_L;   %kb      mass of loaded trailer

    % Suspension Parameters
    c1 = 11270;         %N/(m/s) damping const of axle #1
    c2 = 27500;         %N/(m/s) damping const of axle #2
    c3 = 27500;         %N/(m/s) damping const of axle #3
    c4 = 70000;         %N/(m/s) damping const of axle #4
    c5 = 70000;         %N/(m/s) damping const of axle #5
    ce = 10000;         %N/(m/s) damping const of engine mount
    k4 = 1000000;       %N/m     spring const of axle #4
    k5 = 1000000;       %N/m     spring const of axle #5
    ke = 1e10;          %N/m     spring const of the engine mount

    % Model Dimensions
    b_a1 = 1.065;       %m       Front end of the tractor to axle
#1
    b_cf = 1.470;       %m       Front end of the tractor to cab
front
    b_e = 2.797;        %m       Front end of the tractor to
engine
    b_cr = 4.02;        %m       Front end of the tractor to cab
rear
    b_a2 = 6.035;       %m       Front end of the tractor to axle
#2
    b_fw = 6.688;       %m       Front end of the tractor to 5th
wheel
    b_a3 = 7.34;        %m       Front end of the tractor to axle
#3
    a1 = 4.00607;       %m       Front end of the tractor to
tractor cg

    b_a4 = 8.58;        %m       From the fifth wheel to axle #4
    b_a5 = 9.78;        %m       From the fifth wheel to axle #5

    L_t = 8.2;          %m       Length of Tractor
    L_tlr = 9.78;       %m       Length of Trailer

    e = 5.62;           %m       From the trailer cg to fifth
wheel
    f = 2.96;           %m       From the trailer cg to axle #4
    h = 4.16;           %m       From the trailer cg to axle #5

    a = 2.94107;        %m       From the tractor cg to axle #1
    b = 2.02893;        %m       From the tractor cg to axle #2
    d = 3.33393;        %m       From the tractor cg to axle #3
    l = 2.53607;        %m       From the tractor cg to cab front
    m = 1.209074;       %m       From the tractor cg to engine
    j = 0.013926;       %m       From the tractor cg to cab rear
```

```
    i = 2.68193;        %m       From the tractor cg to the fifth
wheel
    n = 1.435;          %m       From the cab cg to cab front
    p = 1.115;          %m       From the cab cg to cab rear
    r = -0.200;         %m       From the cab cg to seat

    tc = 1.10107;       %m       From the tractor cg to the cab
cg
    h1 = 1.0;           %m       Height of the driver over the
cab
    g = 9.8;            %m/s^2   acceleration due to gravity

    ML_t = m_t/L_t;         %kg/m    Mass per unit length
(Tractor)
    ML_tlr = m_ul/L_tlr;    %kg/m    Mass per unit length
(Trailer)
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%  Fifth Wheel Configuration
%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

disp('  ')
disp('Give your choice for the fifth wheel configuration: ')
disp('Note: If a fifth wheel suspension system is chosen, the
beaming of')
disp('      the tractor frame and trailer will be modeled as
free-free.  If')
disp('      no suspension is chosen, the tractor frame and
trailer will be')
disp('      modeled as free-pinned and pinned-free
respectively.')
disp('a : With fifth wheel suspension')
disp('b : Without fifth wheel suspension')
z33 = input('Please give your choice : ', 's');

if z33 == 'a',          % Choice 'a' is with fifth wheel
suspension
    disp('  ')
    kfw = input('Input the fifth wheel spring constant (N/m): ');
    disp('  ')
    cfw = input('Input the fifth wheel damping ratio (N/(m/s)):
');

    % The parameters for the first bending mode of the Tractor
frame
    disp('  ')
    fhz = input('Input the Tractor frequency of beaming (hz) fhz
: ');

    % The parameters for the first bending mode of the Trailer
frame
```

```
    disp('  ')
    fhz2 = input('Input the Trailer frequency of beaming (hz) fhz
: ');

    kb1 = 4.73004074;    %Constant for the first bending mode
(free-free)
    alpha = 0.982502;

    z1 = 'cosh(kb1*x1/b_fw) + cos(kb1*x1/b_fw) -...
alpha*(sinh(kb1*x1/b_fw)+sin(kb1*x1/b_fw))';
    % free-free beam mode function
    z1dd = '(kb1/b_fw)^2*(cosh(kb1*x1/b_fw) - cos(kb1*x1/b_fw) -
... alpha*(sinh(kb1*x1/b_fw)-sin(kb1*x1/b_fw)))';
    % second derivative of free-free beam mode function

    kb2 = 4.73004074;    %Constant for the first bending mode
(free-free)

    z2 = 'cosh(kb2*x2/L_tlr) + cos(kb2*x2/L_tlr) -...
alpha*(sinh(kb2*x2/L_tlr)+sin(kb2*x2/L_tlr))';
    % free-free beam mode function
    z2dd = '(kb2/L_tlr)^2*(cosh(kb2*x2/L_tlr) - cos(kb2*x2/L_tlr)
-... alpha*(sinh(kb2*x2/L_tlr)-sin(kb2*x2/L_tlr)))';
    % second derivative of free-free beam mode function

elseif z33 == 'b',          % Choice 'b' is without fifth wheel
suspension
    kfw = 1000000000000;    %(N/m)      fifth wheel spring
constant
    cfw = 1000;             %(N/(m/s))  fifth wheel damping ratio

    % The parameters for the first bending mode of the Tractor
frame
    disp('  ')
    fhz = input('Input the Tractor frequency of beaming (hz) fhz
: ');

    % The parameters for the first bending mode of the Trailer
frame
    disp('  ')
    fhz2 = input('Input the Trailer frequency of beaming (hz) fhz
: ');

    kb1 = 2.36502;      % Constant for the first bending mode
(free-pinned)
    %                    (from Rao pg. 527)

    z1 = '(cos(kb1*x1/b_fw) + (cosh(kb1*x1/b_fw)) -...
((cos(kb1)+cosh(kb1))/(sin(kb1)-sinh(kb1)))*(sin(kb1*x1/b_fw)-...
sinh(kb1*x1/b_fw)))';
    % free-pinned beam mode function
    z1dd = '((kb1/b_fw)^2)*(-cos(kb1*x1/b_fw) +
(cosh(kb1*x1/b_fw))...        -((cos(kb1)+cosh(kb1))/(sin(kb1)-
sinh(kb1)))*(-sin(kb1*x1/b_fw)-... sinh(kb1*x1/b_fw)))';
    % second derivative of free-pinned beam mode function
```

```
    kb2 = 3.926602;     % Constant for the first bending mode
(pinned-free)
    %                    (from Rao pg. 527)

    z2 = '(sin(kb2*x2/L_tlr) + ...
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
    % pinned-free beam mode function
    z2dd = '(kb2/L_tlr)^2*(-sin(kb2*x2/L_tlr) + ...
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
    % second derivative of pinned-free beam mode function

else disp('Insufficient information regarding fifth wheel
suspension.')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%%%%%%%%%%%%%%%%%%  Computation of Integrals
%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

D1_t=['(',z1,')'];                  % Tractor frame beaming
equations to be
D2_t=['((a1-x1).*(',z1,'))'];       % used in the integrals
(string form)
D3_t=['((',z1,').*(',z1,'))'];
D4_t=['((',z1dd,').*(',z1dd,'))'];

D1_tlr=['(',z2,')'];                % Trailer beaming equations
to be
D2_tlr=['((e-x2).*(',z2,'))'];      % used in the integrals
(string form)
D3_tlr=['((',z2,').*(',z2,'))'];
D4_tlr=['((',z2dd,').*(',z2dd,'))'];

I1_t=quadl('modeD1_t',0,b_fw);      % Integrals of functions
defined above
I2_t=quadl('modeD2_t',0,b_fw);      % (along length of tractor
frame)
I3_t=quadl('modeD3_t',0,b_fw);
I4_t=quadl('modeD4_t',0,b_fw);

I1_tlr=quadl('modeD1_tlr',0,L_tlr); % Integrals of functions
defined above
I2_tlr=quadl('modeD2_tlr',0,L_tlr); % (along length of trailer)
I3_tlr=quadl('modeD3_tlr',0,L_tlr);
I4_tlr=quadl('modeD4_tlr',0,L_tlr);

E_a1=modeD1_t(b_a1);      % Disp at axle #1 due to tractor frame
beaming
E_cf=modeD1_t(b_cf);      % Disp at cab front due to tractor
frame beaming
```

```
E_e=modeD1_t(b_e);          % Disp at engine due to tractor frame
beaming
E_cr=modeD1_t(b_cr);        % Disp at cab rear due to tractor frame
beaming
E_a2=modeD1_t(b_a2);        % Disp at axle #2 due to tractor frame
beaming
E_fw=modeD1_t(b_fw);        % Disp at fifth wheel due to tractor
frame beaming
E_a3=modeD1_t(b_a3);        % Disp at axle #3 due to tractor frame
beaming
E_0=modeD1_tlr(0);          % Disp at fifth wheel due to trailer
beaming
E_a4=modeD1_tlr(b_a4);      % Disp at axle #4 due to trailer
beaming
E_a5=modeD1_tlr(b_a5);      % Disp at axle #5 due to trailer
beaming

EI_t = 4*pi^2*fhz^2*(b_fw/kb1)^4*ML_t;       %Tractor frame
flexural rigidity
EI_tlr = 4*pi^2*fhz2^2*(L_tlr/kb2)^4*ML_tlr; %Trailer flexural
rigidity

% Seat Suspension Options
disp('  ')
disp('VEHICLE SUSPENSION OPTIONS')
disp('  ')
disp('Give your choice for seat suspension: ')
disp('Note: Without seat suspension gives a very high frequency
mode')
disp('      because the stiffness is set to a high value.')
disp('a : With seat suspension (~0.9 Hz)')
disp('b : Without seat suspension')
z11 = input('Enter your choice : ', 's');

if z11 == 'a',          % Choice 'a' is with seat suspension
    cs = 1140;          % Damping ratio of 0.5
    ks = 3403;          % N/m(spring const of seat suspension)

elseif z11 == 'b',      % Choice 'b' is without seat suspension
    cs = 1329;          % N/(m/s)(damping const of seat
suspension)
    ks = 1e10;          % N/m(spring const of seat suspension)

else disp('Insufficient information regarding seat suspension.')
end

% Cab Suspension Options
disp('  ')
disp('Give your choice for cab suspension: ')
disp('Note: With front or rear or without cab suspension')
disp('      gives a very high frequency mode(s) because the
corresponding')
disp('      stiffness(es)is set to a high value.')
disp('a : With front cab suspension')
disp('b : With rear cab suspension')
disp('c : With front & rear cab suspension')
```

```
disp('d : Without cab suspension')
z22 = input('Enter your choice : ', 's');

if z22 == 'a',          % Choice 'a' is front cab suspension
    ccf = 7062;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 88740;        % N/m(spring const of front cab
suspension)
    ccr = 6430;         % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;         % N/m(spring const of rear cab
suspension)

elseif z22 == 'b',      % Choice 'b' is rear cab suspension
    ccr = 8000;         % Reduced damping
    kcr = 65980;        % N/m(spring const of rear cab
suspension)
    ccf = 13120;        % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;         % N/m(spring const of front cab
suspension)

elseif z22 == 'c',      % Choice 'c' is front & rear cab
suspension
    ccr = 5073.5;       % N/(m/s)(damping const of rear cab
suspension)
    kcr = 63757.5;      % N/m(spring const of rear cab
suspension)
    ccf = 6864.35;      % N/(m/s)(damping const of front cab
suspension)
    kcf = 86260.5;      % N/m(spring const of front cab
suspension)

elseif z22 == 'd',      % Choice 'd' is without cab suspension
    ccr = 6430;         % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;         % N/m(spring const of rear cab
suspension)
    ccf = 7062;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;         % N/m(spring const of front cab
suspension)

else disp('Insufficient information regarding cab suspension.')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%  Vehicle Tire Selection
%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

disp('   ')
disp('STEER AXLE TIRE SELECTION')
```

```
TireData3;                  % M-file for tire data
wd1 = wd;                   %(m)        Nominal cross section
width
mt1 = mt;                   %(kg)       Mass of axle #1
P1 = P;                     %(psi)      Tire pressure from
TireData3.m
press1 = press;             %(psi)      Tire pressure array
numtires1 = numtires;       %           Number of tires on axle
Kstiff1 = Kstiff;           %(N/m)      Tire stiffness array
kt1 = KK * numtires1;       %(N/m)      Per-axle Rad Stiffness
ct1 = ct;                   %(N/(m/s))  Per-axle Damping

disp('   ')
disp('DRIVE AXLE TIRE SELECTION')
TireData3;                  % M-file for tire data
wd23 = wd;                  %(m)        Nominal cross section
width
mt2 = mt;                   %(kg)       Mass of axle #2
mt3 = mt;                   %(kg)       Mass of axle #3
P23 = P;                    %(psi)      Tire pressure from
TireData3.m
press23 = press;            %(psi)      Tire Pressure array
numtires23 = numtires;      %           Number of tires on axle
Kstiff23 = Kstiff;          %(N/m)      Tire stiffness array
kt2 = KK * numtires23;      %(N/m)      Per-axle Rad Stiffness
kt3 = KK * numtires23;      %(N/m)      Per-axle Rad Stiffness
ct2 = ct;                   %(N/(m/s))  Per-axle Damping
ct3 = ct;                   %(N/(m/s))  Per-axle Damping

disp('   ')
disp('TRAILER AXLE TIRE SELECTION')
TireData3;                  % M-file for tire data
wd45 = wd;                  %(m)        Nominal cross section
width
mt4 = mt;                   %(kg)       Mass of axle #4
mt5 = mt;                   %(kg)       Mass of axle #5
P45 = P;                    %(psi)      Tire pressure from
TireData3.m
press45 = press;            %(psi)      Tire Pressure array
numtires45 = numtires;      %           Number of tires on axle
Kstiff45 = Kstiff;          %(N/m)      Tire stiffness array
kt4 = KK * numtires45;      %(N/m)      Per-axle Rad Stiffness
kt5 = KK * numtires45;      %(N/m)      Per-axle Rad Stiffness
ct4 = ct;                   %(N/(m/s))  Per-axle Damping
ct5 = ct;                   %(N/(m/s))  Per-axle Damping

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%  Speed of the Vehicle  %%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('VEHICLE VELOCITY')
disp('  ')
disp('Please choose the unit of velocity');
disp('a : Miles per Hour (mph)');
disp('b : Kilometers per Hour (kph)');
```

```
vel = input('Input the unit of velocity (a/b):  ', 's');
disp('  ')
vm = input('Input the velocity of the vehicle, vm : ');

if vel == 'a'
    v = 0.4473*vm;              %Velocity conversion from mph to
m/s
elseif vel == 'b'
    v = 0.277778*vm;            %Velocity conversion from kph to
m/s
end

T(1) = 0;               %Time delay between front axle and
remaining axles
T(2) = (a+b)/v;         % Axle #2
T(3) = (a+d)/v;         % Axle #3
T(4) = (a+i+e+f)/v;     % Axle #4
T(5) = (a+i+e+h)/v;     % Axle #5

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%  Road PSD Selection  %%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('ROAD PSD SELECTION')
disp('  ')
disp('Road PSD Constants, m^2/cyc/m, Ref: Wong, Theory of Ground
Vehicles')
disp('S(W)=Csp/W^N  where W=spatial frequency')
disp('  ')
disp('a : Csp = 4.3e-11,N=3.8     Smooth Runway')
disp('b : Csp = 8.1e-6, N=2.1     Rough Runway')
disp('c : Csp = 4.8e-7, N=2.1     Smooth Highway')
disp('d : Csp = 4.4e-6, N=2.1     Highway with Gravel')
disp('  ')
tabchoice11=input('Input the road surface to be used :   ','s');

if tabchoice11== 'a',               % smooth runway
   Csp = 4.3e-11;
   N=3.8;

   elseif tabchoice11== 'b',        % rough runway
      Csp = 8.1e-6;
      N=2.1;

   elseif tabchoice11 == 'c',       % smooth highway
      Csp = 4.8e-7;
      N=2.1;

   elseif tabchoice11 == 'd',       % highway with gravel
      Csp = 4.4e-6;
      N=2.1;

end
```

```
disp('  ')
disp('J PENALTY OPTIONS')
disp('  ')
disp('Input the values for K1 and K2 in the J penalty function')
disp('Note: Both values should add up to 1')
disp('  ')
K_1 = input('Input the value for K1 : ');
disp('  ')
K_2 = input('Input the value for K2 : ');

% Start Loop on Axle Stiffness Properties
% Stiffness values will range from 70% to 130% of the nominal
value

for iiii=1:21;
    for jjjj=1:21;
        kf(iiii,jjjj)=17439*iiii;
        kr(iiii,jjjj)=35214*jjjj;

        k1 = 389471+kf(iiii,jjjj);
        k2 = (786446+kr(iiii,jjjj))*0.5;
        k3 = (786446+kr(iiii,jjjj))*0.5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%  System Matrices
%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

% THE SYSTEM IS WRITTEN AS (M*S*S+C*S+K)X(S)=(A*S+B)U(S)

%%%%%%%%%%%%%%%%%%%%%%
%%%%  Mass Matrix  %%%%
%%%%%%%%%%%%%%%%%%%%%%
M = zeros(15,15);

M(1,1) = m_s;                   % Eqn #1: Vertical Disp of Seat

M(2,2) = m_c;                   % Eqn #2: Vertical Disp of Cab

M(3,3) = I_c;                   % Eqn #3: Pitch of Cab

M(4,4) = m_e;                   % Eqn #4: Vertical Disp of Engine

M(5,5) = m_t;                   % Eqn #5: Vertical Disp of
Tractor Frame
M(5,6) = ML_t*b_fw*(b_fw/2-a1);
M(5,7) = ML_t*I1_t;

M(6,5) = ML_t*b_fw*(b_fw/2-a1);   % Eqn #6: Pitch of Tractor
Frame
M(6,6) = I_t;
M(6,7) = -ML_t*I2_t;
```

```
M(7,5) = ML_t*I1_t;              % Eqn #7: Beaming of Tractor
Frame
M(7,6) = -ML_t*I2_t;
M(7,7) = ML_t*I3_t;

M(8,8) = m_tlr;                  % Eqn #8: Vertical Disp of
Trailer
M(8,9) = -ML_tlr*L_tlr*(e-L_tlr/2);
M(8,10) = ML_tlr*I1_tlr;

M(9,8) = -ML_tlr*L_tlr*(e-L_tlr/2);% Eqn #9: Pitch of Trailer
M(9,9) = I_tlr;
M(9,10) = -ML_tlr*I2_tlr;

M(10,8) = ML_tlr*I1_tlr;         % Eqn #10: Beaming of Trailer
M(10,9) = -ML_tlr*I2_tlr;
M(10,10) = ML_tlr*I3_tlr;

M(11,11) = mt1;                  % Eqn #11: Vertical Disp of Axle
#1

M(12,12) = mt2;                  % Eqn #12: Vertical Disp of Axle
#2

M(13,13) = mt3;                  % Eqn #13: Vertical Disp of Axle
#3

M(14,14) = mt4;                  % Eqn #14: Vertical Disp of Axle
#4

M(15,15) = mt5;                  % Eqn #15: Vertical Disp of Axle
#5

%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%
C = zeros(15,15);

C(1,1) = cs;
C(1,2) = -cs;
C(1,3) = r*cs;

C(2,1) = -cs;
C(2,2) = cs+ccf+ccr;
C(2,3) = -r*cs-n*ccf+p*ccr;
C(2,5) = -ccf-ccr;
C(2,6) = l*ccf-j*ccr;
C(2,7) = -ccf*E_cf-ccr*E_cr;

C(3,1) = r*cs;
C(3,2) = -r*cs-n*ccf+p*ccr;
C(3,3) = (r^2)*cs+(n^2)*ccf+(p^2)*ccr;
C(3,5) = n*ccf-p*ccr;
```

```
C(3,6) = -n*l*ccf-p*j*ccr;
C(3,7) = n*ccf*E_cf-p*ccr*E_cr;

C(4,4) = ce;
C(4,5) = -ce;
C(4,6) = m*ce;
C(4,7) = -ce*E_e;

C(5,2) = -ccf-ccr;
C(5,3) = n*ccf-p*ccr;
C(5,4) = -ce;
C(5,5) = ce+ccf+ccr+cfw+c1+c2+c3;
C(5,6) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(5,7) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(5,8) = -cfw;
C(5,9) = e*cfw;
C(5,10) = -cfw*E_0;
C(5,11) = -c1;
C(5,12) = -c2;
C(5,13) = -c3;

C(6,2) = l*ccf-j*ccr;
C(6,3) = -n*l*ccf-p*j*ccr;
C(6,4) = m*ce;
C(6,5) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(6,6) =
(m^2)*ce+(l^2)*ccf+(j^2)*ccr+(i^2)*cfw+(a^2)*c1+(b^2)*c2+(d^2)*c3
;
C(6,7) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2+...
         d*c3*E_a3;
C(6,8) = -i*cfw;
C(6,9) = e*i*cfw;
C(6,10) = -i*cfw*E_0;
C(6,11) = a*c1;
C(6,12) = -b*c2;
C(6,13) = -d*c3;

C(7,2) = -ccf*E_cf-ccr*E_cr;
C(7,3) = n*ccf*E_cf-p*ccr*E_cr;
C(7,4) = -ce*E_e;
C(7,5) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(7,6) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2 ...
         +d*c3*E_a3;
C(7,7) =
ce*E_e^2+ccf*E_cf^2+ccr*E_cr^2+cfw*E_fw^2+c1*E_a1^2+c2*E_a2^2 ...
         +c3*E_a3^2;
C(7,8) = -cfw*E_fw;
C(7,9) = e*cfw*E_fw;
C(7,10) = -cfw*E_0*E_fw;
C(7,11) = -c1*E_a1;
C(7,12) = -c2*E_a2;
C(7,13) = -c3*E_a3;
```

```
C(8,5) = -cfw;
C(8,6) = -i*cfw;
C(8,7) = -cfw*E_fw;
C(8,8) = cfw+c4+c5;
C(8,9) = -e*cfw+f*c4+h*c5;
C(8,10) = cfw*E_0+c4*E_a4+c5*E_a5;
C(8,14) = -c4;
C(8,15) = -c5;

C(9,5) = e*cfw;
C(9,6) = e*i*cfw;
C(9,7) = e*cfw*E_fw;
C(9,8) = -e*cfw+f*c4+h*c5;
C(9,9) = (e^2)*cfw+(f^2)*c4+(h^2)*c5;
C(9,10) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(9,14) = -f*c4;
C(9,15) = -h*c5;

C(10,5) = -cfw*E_0;
C(10,6) = -i*cfw*E_0;
C(10,7) = -cfw*E_fw*E_0;
C(10,8) = cfw*E_0+c4*E_a4+c5*E_a5;
C(10,9) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(10,10) = cfw*E_0^2+c4*E_a4^2+c5*E_a5^2;
C(10,14) = -c4*E_a4;
C(10,15) = -c5*E_a5;

C(11,5) = -c1;
C(11,6) = a*c1;
C(11,7) = -c1*E_a1;
C(11,11) = c1+ct1;

C(12,5) = -c2;
C(12,6) = -b*c2;
C(12,7) = -c2*E_a2;
C(12,12) = c2+ct2;

C(13,5) = -c3;
C(13,6) = -d*c3;
C(13,7) = -c3*E_a3;
C(13,13) = c3+ct3;

C(14,8) = -c4;
C(14,9) = -f*c4;
C(14,10) = -c4*E_a4;
C(14,14) = c4+ct4;

C(15,8) = -c5;
C(15,9) = -h*c5;
C(15,10) = -c5*E_a5;
C(15,15) = c5+ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Stiffness Matrix  %%%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%
K = zeros(15,15);

K(1,1) = ks;
K(1,2) = -ks;
K(1,3) = r*ks;

K(2,1) = -ks;
K(2,2) = ks+kcf+kcr;
K(2,3) = -r*ks-n*kcf+p*kcr;
K(2,5) = -kcf-kcr;
K(2,6) = l*kcf-j*kcr;
K(2,7) = -kcf*E_cf-kcr*E_cr;

K(3,1) = r*ks;
K(3,2) = -r*ks-n*kcf+p*kcr;
K(3,3) = (r^2)*ks+(n^2)*kcf+(p^2)*kcr;
K(3,5) = n*kcf-p*kcr;
K(3,6) = -n*l*kcf-p*j*kcr;
K(3,7) = n*kcf*E_cf-p*kcr*E_cr;

K(4,4) = ke;
K(4,5) = -ke;
K(4,6) = m*ke;
K(4,7) = -ke*E_e;

K(5,2) = -kcf-kcr;
K(5,3) = n*kcf-p*kcr;
K(5,4) = -ke;
K(5,5) = ke+kcf+kcr+kfw+k1+k2+k3;
K(5,6) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(5,7) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(5,8) = -kfw;
K(5,9) = e*kfw;
K(5,10) = -kfw*E_0;
K(5,11) = -k1;
K(5,12) = -k2;
K(5,13) = -k3;

K(6,2) = l*kcf-j*kcr;
K(6,3) = -n*l*kcf-p*j*kcr;
K(6,4) = m*ke;
K(6,5) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(6,6) =
(m^2)*ke+(l^2)*kcf+(j^2)*kcr+(i^2)*kfw+(a^2)*k1+(b^2)*k2+(d^2)*k3
;
K(6,7) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2+...
         d*k3*E_a3;
K(6,8) = -i*kfw;
K(6,9) = e*i*kfw;
K(6,10) = -i*kfw*E_0;
K(6,11) = a*k1;
K(6,12) = -b*k2;
K(6,13) = -d*k3;
```

```
K(7,2) = -kcf*E_cf-kcr*E_cr;
K(7,3) = n*kcf*E_cf-p*kcr*E_cr;
K(7,4) = -ke*E_e;
K(7,5) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(7,6) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2 ...
         +d*k3*E_a3;
K(7,7) =
ke*E_e^2+kcf*E_cf^2+kcr*E_cr^2+kfw*E_fw^2+k1*E_a1^2+k2*E_a2^2 ...
         +k3*E_a3^2+EI_t*I4_t;
K(7,8) = -kfw*E_fw;
K(7,9) = e*kfw*E_fw;
K(7,10) = -kfw*E_0*E_fw;
K(7,11) = -k1*E_a1;
K(7,12) = -k2*E_a2;
K(7,13) = -k3*E_a3;

K(8,5) = -kfw;
K(8,6) = -i*kfw;
K(8,7) = -kfw*E_fw;
K(8,8) = kfw+k4+k5;
K(8,9) = -e*kfw+f*k4+h*k5;
K(8,10) = kfw*E_0+k4*E_a4+k5*E_a5;
K(8,14) = -k4;
K(8,15) = -k5;

K(9,5) = e*kfw;
K(9,6) = e*i*kfw;
K(9,7) = e*kfw*E_fw;
K(9,8) = -e*kfw+f*k4+h*k5;
K(9,9) = (e^2)*kfw+(f^2)*k4+(h^2)*k5;
K(9,10) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(9,14) = -f*k4;
K(9,15) = -h*k5;

K(10,5) = -kfw*E_0;
K(10,6) = -i*kfw*E_0;
K(10,7) = -kfw*E_fw*E_0;
K(10,8) = kfw*E_0+k4*E_a4+k5*E_a5;
K(10,9) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(10,10) = kfw*E_0^2+k4*E_a4^2+k5*E_a5^2+EI_tlr*I4_tlr;
K(10,14) = -k4*E_a4;
K(10,15) = -k5*E_a5;

K(11,5) = -k1;
K(11,6) = a*k1;
K(11,7) = -k1*E_a1;
K(11,11) = k1+kt1;

K(12,5) = -k2;
K(12,6) = -b*k2;
K(12,7) = -k2*E_a2;
K(12,12) = k2+kt2;
```

```
K(13,5) = -k3;
K(13,6) = -d*k3;
K(13,7) = -k3*E_a3;
K(13,13) = k3+kt3;

K(14,8) = -k4;
K(14,9) = -f*k4;
K(14,10) = -k4*E_a4;
K(14,14) = k4+kt4;

K(15,8) = -k5;
K(15,9) = -h*k5;
K(15,10) = -k5*E_a5;
K(15,15) = k5+kt5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A = zeros(15,1);

A(11) = ct1;
A(12) = ct2;
A(13) = ct3;
A(14) = ct4;
A(15) = ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
B = zeros(15,1);

B(11) = kt1;
B(12) = kt2;
B(13) = kt3;
B(14) = kt4;
B(15) = kt5;

% System "A" Matrix
AA=[zeros(size(M))   eye(size(M))   % System state variable
matrix
    -inv(M)*K       -inv(M)*C];

% ISO 2631 FOR REDUCED COMFORT BOUNDARY
% COMFORT BOUNDARIES FOR VERTICAL ACCELERATION
% THE ISO CENTRAL FREQUENCIES (Hz)

wc=[ .1 1 1.25 1.6 2 2.5 3.15 4 5 6.3 8 10 12.5 16 20 25 31.5 40
50];
whzc=[ .1 .125 .16 .2 .25 .315 .4 .5 .63 .8 1 1.25 1.6 2 2.5 3.15
...
    4 5 6.3 8 10 12.5 16 20 25 31.5 40 50];

% 2.5 hr FATIGUE BOUNDARY
fat1=[4.284,1.4,1.25,1.12,1,.9,.8,.71,.71,.71,.71,...
```

```
      .9,1.12,1.4,1.8,2.24,2.8,3.55,4.5];
% 2.5hr REDUCED COMFORT BOUNDARY
comf1=fat1/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf2= comf1/2.254;

%-----------------------------------------------------------------
---
% COMFORT BOUNDARIES FOR LONGITUDINAL AND LATERAL ACC
% 2.5hr FATIGUE BOUNDARY
fat2=[0.5,0.5,0.5,0.5,0.5,0.63,0.8,1,1.25,1.6,2,2.5,3.15,4,5,6.3,
8,10,12.5];

% 2.5hr REDUCED COMFORT BOUNDARY
comf3=fat2/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf4= comf3/2.254;
%-----------------------------------------------------------------
--

whzcr = 2*pi*whzc;      % Calculation of central frequencies in
rad/s
freqlow=0.89*whzcr;     % Lower octave band
freqhigh=1.12*whzcr;    % Upper octave band
freq=[freqlow' whzcr' freqhigh'];

imag=sqrt(-1);

for ii=1:length(whzc);
    for jj=1:3;         % jj=1 is freqlow, jj=2 is center freq
                        % jj=3 is freqhigh
        w = freq(ii,jj);
        s = imag*w;
        dp = sqrt(h1^2+r^2);

        % Time delay array
        time = [0 0 0 0 0 0 0 0 0 0 1 exp(-s*T(2)) exp(-s*T(3))
...
            exp(-s*T(4)) exp(-s*T(5))];

        % TF Matrix
        vectx = (inv(M*s*s+C*s+K)*((A*s+B).*(time.')));

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %%%  Transfer Functions  %%%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%

        z_s=[1 0 0 0 0 0 0 0 0 0 0 0 0 0]*vectx;    % vert seat
cg
        long=[0 0 -h1 0 0 0 0 0 0 0 0 0 0 0]*vectx; % long disp
of driver
        z_tlr=[0 0 0 0 0 0 0 1 0 0 0 0 0 0]*vectx;  % vert
trailer cg

        %%%%%%%%%%%%%%%%%%%%
```

```
        %%%  Magnitudes  %%%
        %%%%%%%%%%%%%%%%%%%%

        % Acceleration Transter Functions
        magcfA1(ii,jj)=abs(s*s*z_s);  % Mag of trans function,
(m/s*s)/m
        magcfAlong(ii,jj)=abs(s*s*long);
        magcftlr(ii,jj)=abs(s*s*z_tlr);

        %%%%%%%%%%%%%%
        %%%  PSDs  %%%
        %%%%%%%%%%%%%%

        % Road PSD in m*m/(rad/s)
        rpsd(ii,jj)=Csp*(((2*pi*v)^(N-1))/(w^N));

        % Acceleration PSDs in (m/s^2)^2/(rad/s)
        psdcfA1(ii,jj)=magcfA1(ii,jj)*magcfA1(ii,jj)*rpsd(ii,jj);

psdcfAlong(ii,jj)=magcfAlong(ii,jj)*magcfAlong(ii,jj)*rpsd(ii,jj)
;

psdcftlr(ii,jj)=magcftlr(ii,jj)*magcftlr(ii,jj)*rpsd(ii,jj);

    end
end

for kk=1:length(whzc)
    % Vert. Driver's Seat RMS
    msqy1a(kk)=0.5*(psdcfA1(kk,1)+psdcfA1(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqy1b(kk)=0.5*(psdcfA1(kk,2)+psdcfA1(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqy1(kk)=msqy1a(kk)+msqy1b(kk);
    rmsA1cf(kk)=sqrt(msqy1(kk));
    % Long. Driver RMS
    msqylonga(kk)=0.5*(psdcfAlong(kk,1)+psdcfAlong(kk,2))*...
     (freq(kk,2)-freq(kk,1));
    msqylongb(kk)=0.5*(psdcfAlong(kk,2)+psdcfAlong(kk,3))*...
     (freq(kk,3)-freq(kk,2));
    msqylong(kk)=msqylonga(kk)+msqylongb(kk);
    rmsAlongcf(kk)=sqrt(msqylong(kk));
    % Vert. Trailer cg RMS
    msqytlra(kk)=0.5*(psdcftlr(kk,1)+psdcftlr(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqytlrb(kk)=0.5*(psdcftlr(kk,2)+psdcftlr(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqytlr(kk)=msqytlra(kk)+msqytlrb(kk);
    rmstlrcf(kk)=sqrt(msqytlr(kk));
end

RMScf = [rmsA1cf',rmsAlongcf',rmstlrcf'];       % Accel. RMS
Matrix

% Calculate weighted rms acceleration from 0.1 to 50 Hz
```

```
% at the ISO Center Frequencies .... Wgt are the ISO weights
% Ref: ISO 2631-1:1997(E); V=vertical; L=longitudinal

wcc=[.1,.125,.16,.2,.25,.315,.4,.5,.63,.8,1,1.25,1.6,2,2.5,3.15,4
,5,...
    6.3,8,10,12.5,16,20,25,31.5,40,50];
WgtV=[.0312,.0486,.079,.121,.182,.263,.352,.418,.459,.477,.482,.4
84,...

.494,.531,.631,.804,.967,1.039,1.054,1.036,.988,.902,.768,.636,..
.
    .513,.405,.314,.246];
WgtL=0.001*[62.4,97.3,158,243,365,530,713,853,944,992,1011,1008,9
68,...

890,776,642,512,409,323,253,212,161,125,100,80,63.2,49.4,38.8];

isovert = WgtV.*RMScf(1:28,1)';      % Weighted Vert. Driver RMS
Accel.
isolong = WgtL.*RMScf(1:28,2)';      % Weighted Long. Driver RMS
Accel.
isotlr = WgtV.*RMScf(1:28,3)';       % Weighted Vert. Trailer RMS
Accel.

term2V=(WgtV.*rmsA1cf(1:28)).^2;
a0_V_dr=(sum(term2V))^0.5;              % a0 for vert. disp of
driver

term2L=(WgtL.*rmsAlongcf(1:28)).^2;
a0_L_dr=(sum(term2L))^0.5;              % a0 for long. disp of
driver

aV=(a0_L_dr^2 + a0_V_dr^2)^0.5;      % a0 for comb vert and long
disp

tlrV=(WgtV.*rmstlrcf(1:28)).^2;
a0_V_tlr=(sum(tlrV))^0.5;               % a0 for vert. disp of
driver

aVV(iiii,jjjj)=aV;               % combined ISO wgt acc, m/s^2
a0_VV_tlr(iiii,jjjj)=a0_V_tlr;
Jpenalty(iiii,jjjj)=K_1*(aVV(iiii,jjjj)/0.44814)+K_2*...
(a0_VV_tlr(iiii,jjjj)/0.3239);

    end         % end of jjjj loop on k2
end             % end of iiii loop on k1

disp('  ')
disp('RESULTS OF PARAMETER VARIATION')
disp('  ')
disp('Minimum aV, m/s^2')
disp(min(aVV(:)))
disp('  ')
[ia,ja]=find(aVV==min(aVV(:)));
disp('Corresponding k1, k2, and k3 values, N/m')
```

```
disp([389471+kf(ia,ja) (786446+kr(ia,ja))*0.5
(786446+kr(ia,ja))*0.5])
disp('  ')

disp('  ')
disp('Minimum a0_V_tlr, m/s^2')
disp(min(a0_VV_tlr(:)))
disp('  ')
[it,jt]=find(a0_VV_tlr==min(a0_VV_tlr(:)));
disp('Corresponding k1, k2, and k3 values, N/m')
disp([389471+kf(it,jt) (786446+kr(it,jt))*0.5
(786446+kr(it,jt))*0.5])
disp('  ')

disp('  ')
disp('Minimum Jpenalty')
disp('J=K1*aV/0.44814 + K2*a0_V_tlr/0.3239')
disp('       K1         K2')
disp([K_1 K_2])
disp('  ')
disp(min(Jpenalty(:)))
disp('  ')
[iJ,jJ]=find(Jpenalty==min(Jpenalty(:)));
disp('Corresponding k1, k2, and k3 values, N/m')
disp([389471+kf(iJ,jJ) (786446+kr(iJ,jJ))*0.5
(786446+kr(iJ,jJ))*0.5])
disp('  ')

figure(1)
surf(389471+kf,(786446+kr)/2,aVV)
xlabel('Steer Axle K, N/m')
ylabel('Single Drive Axle K, N/m')
zlabel('ISO Combined Wgt Acc, m/s^2')
% title('Tractor Suspension Stiffness Variation')

figure(2)
surf((786446+kr)/2,389471+kf,a0_VV_tlr)
ylabel('Steer Axle K, N/m')
xlabel('Single Drive Axle K, N/m')
zlabel('Trailer Wgt Vert Acc, m/s^2')
% title('Tractor Suspension Stiffness Variation')

figure(3)
surf(389471+kf,(786446+kr)/2,Jpenalty)
xlabel('Steer Axle K, N/m')
ylabel('Single Drive Axle K, N/m')
zlabel('Penalty Function')
title(['K1 = ', num2str(K_1),'     K2 = ', num2str(K_2)])
```

Appendix H: opt_axleC_freq.m

This parameter variation program varies the damping of the steer axle suspension and the damping of the first and second drive axle suspensions combined. Each of the drive axle suspensions on the tractor are assumed to have the same value, so they were combined into one value that was varied, and the individual axle suspension values were assumed to be equal to exactly half of that value. The steer axle was varied from 7,889 N/(m/s) to 14,651 N/(m/s) in increments of 338.1 N/(m/s). This forms a vector with a length of 21 values that ranges from 30% below to 30% above the nominal value for the steer axle damping. Each drive axle was varied from 19,250 N/(m/s) to 35,750 N/(m/s) in increments of 825 N/(m/s). Like the steer axle, this forms a vector with a length of 21 values that ranges from 30% below to 30% above the nominal value for the drive axle damping.

The desired output values from this program were the ISO combined weighted acceleration at the driver's seat, the ISO vertical weighted acceleration at the trailer center-of-gravity (CG), and a value called the J penalty, which weighs the importance of the driver ride comfort versus trailer acceleration using weights assigned to them by the user. The program finds the minimum values for each of these outputs, and displays them in tabular form along with the corresponding damping values for the steer and drive axle suspensions. Also, the program plots the output information on surface plots to study trends in the information.

opt_axleC_freq.m

```
% opt_axleC_freq.m
% Developed by Ryan Spivey, 4/10/07
%
% Varies axle damping using weighted RMS acceleration in the
% frequecncy domain
%
% Incorporates model from dof15_freq2.m
%
% DOFs include - 1)Vertical Disp. of Driver's Seat
%                2)Vertical Disp. of Cab
%                3)Pitch of Cab
%                4)Vertical Disp. of Engine
%                5)Vertical Disp. of Tractor Frame
%                6)Pitch of Tractor Frame
%                7)Beaming of Tractor Frame
%                8)Vertical Disp. of Trailer
%                9)Pitch of Trailer
%               10)Beaming of Trailer
%               11)Vertical Disp. of Axle #1
%               12)Vertical Disp. of Axle #2
%               13)Vertical Disp. of Axle #3
%               14)Vertical Disp. of Axle #4
%               15)Vertical Disp. of Axle #5

clc
clear all
close all
format short e
format compact

global D1_t D2_t D3_t D4_t D1_tlr D2_tlr D3_tlr D4_tlr
global e a1 kb1 kb2 b_fw L_tlr alpha

disp('  ')
disp('Axle Damping Parameter Variation in the Frequency Domain')
disp('                  Roadholding Model                    ')
disp(['                      ',date])

% Choose a test vehicle
disp('  ')
disp('VEHICLE SELECTION')
disp('  ')
disp('Please choose a vehicle : ');
disp('a: Ideal Tractor Semi-Trailer');
vehicle = input('Enter your choice : ', 's');

if vehicle == 'a'
    % Inertial Properties
    m_s = 106.7;        %kg      mass of seat
    m_c = 1208;         %kg      mass of cab
    I_c = 2100;         %kg*m^2  M I of cab
    m_e = 2000;         %kg      mass of engine (ESTIMATE)
```

```
    m_t = 3783;          %kg      mass of tractor (5783 kg -
engine)
    I_t = 46590.9;       %kg*m^2  M I of tractor
    m_ul = 10800;        %kg      mass of trailer (ESTIMATE)
    I_tlr = 200000;      %kg*m^2  M I of trailer
    m_L = 14000;         %kg      mass of trailer load  (ESTIMATE)
    m_tlr = m_ul+m_L;    %kb      mass of loaded trailer

    % Suspension Parameters
    c4 = 70000;          %N/(m/s) damping const of axle #4
    c5 = 70000;          %N/(m/s) damping const of axle #5
    ce = 10000;          %N/(m/s) damping const of engine mount
    k1 = 581300;         %N/m     spring const of axle #1
    k2 = 586900;         %N/m     spring const of axle #2
    k3 = 586900;         %N/m     spring const of axle #3
    k4 = 1000000;        %N/m     spring const of axle #4
    k5 = 1000000;        %N/m     spring const of axle #5
    ke = 1e10;           %N/m     spring const of the engine mount

    % Model Dimensions
    b_a1 = 1.065;        %m       Front end of the tractor to axle
#1
    b_cf = 1.470;        %m       Front end of the tractor to cab
front
    b_e = 2.797;         %m       Front end of the tractor to
engine
    b_cr = 4.02;         %m       Front end of the tractor to cab
rear
    b_a2 = 6.035;        %m       Front end of the tractor to axle
#2
    b_fw = 6.688;        %m       Front end of the tractor to 5th
wheel
    b_a3 = 7.34;         %m       Front end of the tractor to axle
#3
    a1 = 4.00607;        %m       Front end of the tractor to
tractor cg

    b_a4 = 8.58;         %m       From the fifth wheel to axle #4
    b_a5 = 9.78;         %m       From the fifth wheel to axle #5

    L_t = 8.2;           %m       Length of Tractor
    L_tlr = 9.78;        %m       Length of Trailer

    e = 5.62;            %m       From the trailer cg to fifth
wheel
    f = 2.96;            %m       From the trailer cg to axle #4
    h = 4.16;            %m       From the trailer cg to axle #5

    a = 2.94107;         %m       From the tractor cg to axle #1
    b = 2.02893;         %m       From the tractor cg to axle #2
    d = 3.33393;         %m       From the tractor cg to axle #3
    l = 2.53607;         %m       From the tractor cg to cab front
    m = 1.209074;        %m       From the tractor cg to engine
    j = 0.013926;        %m       From the tractor cg to cab rear
    i = 2.68193;         %m       From the tractor cg to the fifth
wheel
```

```
    n = 1.435;          %m       From the cab cg to cab front
    p = 1.115;          %m       From the cab cg to cab rear
    r = -0.200;         %m       From the cab cg to seat

    tc = 1.10107;       %m       From the tractor cg to the cab
cg
    h1 = 1.0;           %m       Height of the driver over the
cab
    g = 9.8;            %m/s^2   acceleration due to gravity

    ML_t = m_t/L_t;          %kg/m    Mass per unit length
(Tractor)
    ML_tlr = m_ul/L_tlr;     %kg/m    Mass per unit length
(Trailer)
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%  Fifth Wheel Configuration
%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

disp('  ')
disp('Give your choice for the fifth wheel configuration: ')
disp('Note: If a fifth wheel suspension system is chosen, the
beaming of')
disp('      the tractor frame and trailer will be modeled as
free-free.  If')
disp('      no suspension is chosen, the tractor frame and
trailer will be')
disp('      modeled as free-pinned and pinned-free
respectively.')
disp('a : With fifth wheel suspension')
disp('b : Without fifth wheel suspension')
z33 = input('Please give your choice : ', 's');

if z33 == 'a',          % Choice 'a' is with fifth wheel
suspension
    disp('  ')
    kfw = input('Input the fifth wheel spring constant (N/m): ');
    disp('  ')
    cfw = input('Input the fifth wheel damping ratio (N/(m/s)):
');

    % The parameters for the first bending mode of the Tractor
frame
    disp('  ')
    fhz = input('Input the Tractor frequency of beaming (hz) fhz
: ');

    % The parameters for the first bending mode of the Trailer
frame
    disp('  ')
```

```
    fhz2 = input('Input the Trailer frequency of beaming (hz) fhz
: ');

    kb1 = 4.73004074;    %Constant for the first bending mode
(free-free)
    alpha = 0.982502;

    z1 = 'cosh(kb1*x1/b_fw) + cos(kb1*x1/b_fw) - ...
alpha*(sinh(kb1*x1/b_fw)+sin(kb1*x1/b_fw))';
    % free-free beam mode function
    z1dd = '(kb1/b_fw)^2*(cosh(kb1*x1/b_fw) - cos(kb1*x1/b_fw) -
... alpha*(sinh(kb1*x1/b_fw)-sin(kb1*x1/b_fw)))';
    % second derivative of free-free beam mode function

    kb2 = 4.73004074;    %Constant for the first bending mode
(free-free)

    z2 = 'cosh(kb2*x2/L_tlr) + cos(kb2*x2/L_tlr) - ...
alpha*(sinh(kb2*x2/L_tlr)+sin(kb2*x2/L_tlr))';
    % free-free beam mode function
    z2dd = '(kb2/L_tlr)^2*(cosh(kb2*x2/L_tlr) - cos(kb2*x2/L_tlr)
- ... alpha*(sinh(kb2*x2/L_tlr)-sin(kb2*x2/L_tlr)))';
    % second derivative of free-free beam mode function

elseif z33 == 'b',          % Choice 'b' is without fifth wheel
suspension
    kfw = 1000000000000;    %(N/m)      fifth wheel spring
constant
    cfw = 1000;             %(N/(m/s))  fifth wheel damping ratio

    % The parameters for the first bending mode of the Tractor
frame
    disp('  ')
    fhz = input('Input the Tractor frequency of beaming (hz) fhz
: ');

    % The parameters for the first bending mode of the Trailer
frame
    disp('  ')
    fhz2 = input('Input the Trailer frequency of beaming (hz) fhz
: ');

    kb1 = 2.36502;      % Constant for the first bending mode
(free-pinned)
    %                    (from Rao pg. 527)

    z1 = '(cos(kb1*x1/b_fw) + (cosh(kb1*x1/b_fw)) - ...
((cos(kb1)+cosh(kb1))/(sin(kb1)-sinh(kb1)))*(sin(kb1*x1/b_fw)-
...   sinh(kb1*x1/b_fw)))';
    % free-pinned beam mode function
    z1dd = '((kb1/b_fw)^2)*(-cos(kb1*x1/b_fw) +
(cosh(kb1*x1/b_fw)) ...      - ((cos(kb1)+cosh(kb1))/(sin(kb1)-
sinh(kb1)))*(-sin(kb1*x1/b_fw)- ... sinh(kb1*x1/b_fw)))';
    % second derivative of free-pinned beam mode function
```

```
    kb2 = 3.926602;     % Constant for the first bending mode
(pinned-free)
    %                    (from Rao pg. 527)

    z2 = '(sin(kb2*x2/L_tlr) + ...
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
    % pinned-free beam mode function
    z2dd = '(kb2/L_tlr)^2*(-sin(kb2*x2/L_tlr) + ...
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
    % second derivative of pinned-free beam mode function

else disp('Insufficient information regarding fifth wheel
suspension.')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%%%%%%%%%%%%%%%%%%  Computation of Integrals
%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

D1_t=['(',z1,')'];                   % Tractor frame beaming
equations to be
D2_t=['((a1-x1).*(',z1,'))'];        % used in the integrals
(string form)
D3_t=['((',z1,').*(',z1,'))'];
D4_t=['((',z1dd,').*(',z1dd,'))'];

D1_tlr=['(',z2,')'];                 % Trailer beaming equations
to be
D2_tlr=['((e-x2).*(',z2,'))'];       % used in the integrals
(string form)
D3_tlr=['((',z2,').*(',z2,'))'];
D4_tlr=['((',z2dd,').*(',z2dd,'))'];

I1_t=quadl('modeD1_t',0,b_fw);       % Integrals of functions
defined above
I2_t=quadl('modeD2_t',0,b_fw);       % (along length of tractor
frame)
I3_t=quadl('modeD3_t',0,b_fw);
I4_t=quadl('modeD4_t',0,b_fw);

I1_tlr=quadl('modeD1_tlr',0,L_tlr); % Integrals of functions
defined above
I2_tlr=quadl('modeD2_tlr',0,L_tlr); % (along length of trailer)
I3_tlr=quadl('modeD3_tlr',0,L_tlr);
I4_tlr=quadl('modeD4_tlr',0,L_tlr);

E_a1=modeD1_t(b_a1);      % Disp at axle #1 due to tractor frame
beaming
E_cf=modeD1_t(b_cf);      % Disp at cab front due to tractor
frame beaming
E_e=modeD1_t(b_e);        % Disp at engine due to tractor frame
beaming
```

```
E_cr=modeD1_t(b_cr);       % Disp at cab rear due to tractor frame
beaming
E_a2=modeD1_t(b_a2);       % Disp at axle #2 due to tractor frame
beaming
E_fw=modeD1_t(b_fw);       % Disp at fifth wheel due to tractor
frame beaming
E_a3=modeD1_t(b_a3);       % Disp at axle #3 due to tractor frame
beaming
E_0=modeD1_tlr(0);         % Disp at fifth wheel due to trailer
beaming
E_a4=modeD1_tlr(b_a4);     % Disp at axle #4 due to trailer
beaming
E_a5=modeD1_tlr(b_a5);     % Disp at axle #5 due to trailer
beaming

EI_t = 4*pi^2*fhz^2*(b_fw/kb1)^4*ML_t;       %Tractor frame
flexural rigidity
EI_tlr = 4*pi^2*fhz2^2*(L_tlr/kb2)^4*ML_tlr; %Trailer flexural
rigidity

% Seat Suspension Options
disp('  ')
disp('VEHICLE SUSPENSION OPTIONS')
disp('  ')
disp('Give your choice for seat suspension: ')
disp('Note: Without seat suspension gives a very high frequency
mode')
disp('      because the stiffness is set to a high value.')
disp('a : With seat suspension (~0.9 Hz)')
disp('b : Without seat suspension')
z11 = input('Enter your choice : ', 's');

if z11 == 'a',          % Choice 'a' is with seat suspension
    cs = 1140;          % Damping ratio of 0.5
    ks = 3403;          % N/m(spring const of seat suspension)

elseif z11 == 'b',      % Choice 'b' is without seat suspension
    cs = 1329;          % N/(m/s)(damping const of seat
suspension)
    ks = 1e10;          % N/m(spring const of seat suspension)

else disp('Insufficient information regarding seat suspension.')
end

% Cab Suspension Options
disp('  ')
disp('Give your choice for cab suspension: ')
disp('Note: With front or rear or without cab suspension')
disp('      gives a very high frequency mode(s) because the
corresponding')
disp('      stiffness(es)is set to a high value.')
disp('a : With front cab suspension')
disp('b : With rear cab suspension')
disp('c : With front & rear cab suspension')
disp('d : Without cab suspension')
z22 = input('Enter your choice : ', 's');
```

```
if z22 == 'a',          % Choice 'a' is front cab suspension
    ccf = 7062;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 88740;        % N/m(spring const of front cab
suspension)
    ccr = 6430;         % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;         % N/m(spring const of rear cab
suspension)

elseif z22 == 'b',      % Choice 'b' is rear cab suspension
    ccr = 8000;         % Reduced damping
    kcr = 65980;        % N/m(spring const of rear cab
suspension)
    ccf = 13120;        % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;         % N/m(spring const of front cab
suspension)

elseif z22 == 'c',      % Choice 'c' is front & rear cab
suspension
    ccr = 5073.5;       % N/(m/s)(damping const of rear cab
suspension)
    kcr = 63757.5;      % N/m(spring const of rear cab
suspension)
    ccf = 6864.35;      % N/(m/s)(damping const of front cab
suspension)
    kcf = 86260.5;      % N/m(spring const of front cab
suspension)

elseif z22 == 'd',      % Choice 'd' is without cab suspension
    ccr = 6430;         % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;         % N/m(spring const of rear cab
suspension)
    ccf = 7062;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;         % N/m(spring const of front cab
suspension)

else disp('Insufficient information regarding cab suspension.')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%  Vehicle Tire Selection
%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

disp('   ')
disp('STEER AXLE TIRE SELECTION')
TireData3;                  % M-file for tire data
```

```
wd1 = wd;                    %(m)        Nominal cross section
width
mt1 = mt;                    %(kg)       Mass of axle #1
P1 = P;                      %(psi)      Tire pressure from
TireData3.m
press1 = press;              %(psi)      Tire pressure array
numtires1 = numtires;        %           Number of tires on axle
Kstiff1 = Kstiff;            %(N/m)      Tire stiffness array
kt1 = KK * numtires1;        %(N/m)      Per-axle Rad Stiffness
ct1 = ct;                    %(N/(m/s))  Per-axle Damping

disp('   ')
disp('DRIVE AXLE TIRE SELECTION')
TireData3;                   % M-file for tire data
wd23 = wd;                   %(m)        Nominal cross section
width
mt2 = mt;                    %(kg)       Mass of axle #2
mt3 = mt;                    %(kg)       Mass of axle #3
P23 = P;                     %(psi)      Tire pressure from
TireData3.m
press23 = press;             %(psi)      Tire Pressure array
numtires23 = numtires;       %           Number of tires on axle
Kstiff23 = Kstiff;           %(N/m)      Tire stiffness array
kt2 = KK * numtires23;       %(N/m)      Per-axle Rad Stiffness
kt3 = KK * numtires23;       %(N/m)      Per-axle Rad Stiffness
ct2 = ct;                    %(N/(m/s))  Per-axle Damping
ct3 = ct;                    %(N/(m/s))  Per-axle Damping

disp('   ')
disp('TRAILER AXLE TIRE SELECTION')
TireData3;                   % M-file for tire data
wd45 = wd;                   %(m)        Nominal cross section
width
mt4 = mt;                    %(kg)       Mass of axle #4
mt5 = mt;                    %(kg)       Mass of axle #5
P45 = P;                     %(psi)      Tire pressure from
TireData3.m
press45 = press;             %(psi)      Tire Pressure array
numtires45 = numtires;       %           Number of tires on axle
Kstiff45 = Kstiff;           %(N/m)      Tire stiffness array
kt4 = KK * numtires45;       %(N/m)      Per-axle Rad Stiffness
kt5 = KK * numtires45;       %(N/m)      Per-axle Rad Stiffness
ct4 = ct;                    %(N/(m/s))  Per-axle Damping
ct5 = ct;                    %(N/(m/s))  Per-axle Damping

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%  Speed of the Vehicle  %%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('VEHICLE VELOCITY')
disp('  ')
disp('Please choose the unit of velocity');
disp('a : Miles per Hour (mph)');
disp('b : Kilometers per Hour (kph)');
vel = input('Input the unit of velocity (a/b):  ', 's');
```

```
disp('  ')
vm = input('Input the velocity of the vehicle, vm : ');

if vel == 'a'
    v = 0.4473*vm;              %Velocity conversion from mph to
m/s
elseif vel == 'b'
    v = 0.277778*vm;            %Velocity conversion from kph to
m/s
end

T(1) = 0;               %Time delay between front axle and
remaining axles
T(2) = (a+b)/v;         % Axle #2
T(3) = (a+d)/v;         % Axle #3
T(4) = (a+i+e+f)/v;     % Axle #4
T(5) = (a+i+e+h)/v;     % Axle #5

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%  Road PSD Selection  %%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('ROAD PSD SELECTION')
disp('  ')
disp('Road PSD Constants, m^2/cyc/m, Ref: Wong, Theory of Ground
Vehicles')
disp('S(W)=Csp/W^N  where W=spatial frequency')
disp('  ')
disp('a : Csp = 4.3e-11,N=3.8     Smooth Runway')
disp('b : Csp = 8.1e-6, N=2.1     Rough Runway')
disp('c : Csp = 4.8e-7, N=2.1     Smooth Highway')
disp('d : Csp = 4.4e-6, N=2.1     Highway with Gravel')
disp('  ')
tabchoice11=input('Input the road surface to be used :   ','s');

if tabchoice11== 'a',               % smooth runway
   Csp = 4.3e-11;
   N=3.8;

   elseif tabchoice11== 'b',        % rough runway
      Csp = 8.1e-6;
      N=2.1;

   elseif tabchoice11 == 'c',       % smooth highway
      Csp = 4.8e-7;
      N=2.1;

   elseif tabchoice11 == 'd',       % highway with gravel
      Csp = 4.4e-6;
      N=2.1;

end

disp('  ')
```

```
disp('J PENALTY OPTIONS')
disp('  ')
disp('Input the values for K1 and K2 in the J penalty function')
disp('Note: Both values should add up to 1')
disp('  ')
K_1 = input('Input the value for K1 : ');
disp('  ')
K_2 = input('Input the value for K2 : ');

% Start Loop on Axle Damping Properties
% Damping values will range from 70% to 130% of the nominal value

for iiii=1:21;
    for jjjj=1:21;
        cf(iiii,jjjj)=338.1*iiii;
        cr(iiii,jjjj)=1650*jjjj;

        c1 = 7550.9+cf(iiii,jjjj);
        c2 = (36850+cr(iiii,jjjj))*0.5;
        c3 = (36850+cr(iiii,jjjj))*0.5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%  System Matrices
%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

% THE SYSTEM IS WRITTEN AS (M*S*S+C*S+K)X(S)=(A*S+B)U(S)

%%%%%%%%%%%%%%%%%%%%%%%
%%%%  Mass Matrix  %%%%
%%%%%%%%%%%%%%%%%%%%%%%
M = zeros(15,15);

M(1,1) = m_s;                   % Eqn #1: Vertical Disp of Seat

M(2,2) = m_c;                   % Eqn #2: Vertical Disp of Cab

M(3,3) = I_c;                   % Eqn #3: Pitch of Cab

M(4,4) = m_e;                   % Eqn #4: Vertical Disp of Engine

M(5,5) = m_t;                   % Eqn #5: Vertical Disp of
Tractor Frame
M(5,6) = ML_t*b_fw*(b_fw/2-a1);
M(5,7) = ML_t*I1_t;

M(6,5) = ML_t*b_fw*(b_fw/2-a1);   % Eqn #6: Pitch of Tractor
Frame
M(6,6) = I_t;
M(6,7) = -ML_t*I2_t;
```

```
M(7,5) = ML_t*I1_t;             % Eqn #7: Beaming of Tractor
Frame
M(7,6) = -ML_t*I2_t;
M(7,7) = ML_t*I3_t;

M(8,8) = m_tlr;                 % Eqn #8: Vertical Disp of
Trailer
M(8,9) = -ML_tlr*L_tlr*(e-L_tlr/2);
M(8,10) = ML_tlr*I1_tlr;

M(9,8) = -ML_tlr*L_tlr*(e-L_tlr/2);% Eqn #9: Pitch of Trailer
M(9,9) = I_tlr;
M(9,10) = -ML_tlr*I2_tlr;

M(10,8) = ML_tlr*I1_tlr;        % Eqn #10: Beaming of Trailer
M(10,9) = -ML_tlr*I2_tlr;
M(10,10) = ML_tlr*I3_tlr;

M(11,11) = mt1;                 % Eqn #11: Vertical Disp of Axle
#1

M(12,12) = mt2;                 % Eqn #12: Vertical Disp of Axle
#2

M(13,13) = mt3;                 % Eqn #13: Vertical Disp of Axle
#3

M(14,14) = mt4;                 % Eqn #14: Vertical Disp of Axle
#4

M(15,15) = mt5;                 % Eqn #15: Vertical Disp of Axle
#5

%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%
C = zeros(15,15);

C(1,1) = cs;
C(1,2) = -cs;
C(1,3) = r*cs;

C(2,1) = -cs;
C(2,2) = cs+ccf+ccr;
C(2,3) = -r*cs-n*ccf+p*ccr;
C(2,5) = -ccf-ccr;
C(2,6) = l*ccf-j*ccr;
C(2,7) = -ccf*E_cf-ccr*E_cr;

C(3,1) = r*cs;
C(3,2) = -r*cs-n*ccf+p*ccr;
C(3,3) = (r^2)*cs+(n^2)*ccf+(p^2)*ccr;
C(3,5) = n*ccf-p*ccr;
C(3,6) = -n*l*ccf-p*j*ccr;
```

```
C(3,7) = n*ccf*E_cf-p*ccr*E_cr;

C(4,4) = ce;
C(4,5) = -ce;
C(4,6) = m*ce;
C(4,7) = -ce*E_e;

C(5,2) = -ccf-ccr;
C(5,3) = n*ccf-p*ccr;
C(5,4) = -ce;
C(5,5) = ce+ccf+ccr+cfw+c1+c2+c3;
C(5,6) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(5,7) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(5,8) = -cfw;
C(5,9) = e*cfw;
C(5,10) = -cfw*E_0;
C(5,11) = -c1;
C(5,12) = -c2;
C(5,13) = -c3;

C(6,2) = l*ccf-j*ccr;
C(6,3) = -n*l*ccf-p*j*ccr;
C(6,4) = m*ce;
C(6,5) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(6,6) =
(m^2)*ce+(l^2)*ccf+(j^2)*ccr+(i^2)*cfw+(a^2)*c1+(b^2)*c2+(d^2)*c3
;
C(6,7) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2+...
         d*c3*E_a3;
C(6,8) = -i*cfw;
C(6,9) = e*i*cfw;
C(6,10) = -i*cfw*E_0;
C(6,11) = a*c1;
C(6,12) = -b*c2;
C(6,13) = -d*c3;

C(7,2) = -ccf*E_cf-ccr*E_cr;
C(7,3) = n*ccf*E_cf-p*ccr*E_cr;
C(7,4) = -ce*E_e;
C(7,5) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(7,6) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2 ...
         +d*c3*E_a3;
C(7,7) =
ce*E_e^2+ccf*E_cf^2+ccr*E_cr^2+cfw*E_fw^2+c1*E_a1^2+c2*E_a2^2 ...
         +c3*E_a3^2;
C(7,8) = -cfw*E_fw;
C(7,9) = e*cfw*E_fw;
C(7,10) = -cfw*E_0*E_fw;
C(7,11) = -c1*E_a1;
C(7,12) = -c2*E_a2;
C(7,13) = -c3*E_a3;
```

```
C(8,5) = -cfw;
C(8,6) = -i*cfw;
C(8,7) = -cfw*E_fw;
C(8,8) = cfw+c4+c5;
C(8,9) = -e*cfw+f*c4+h*c5;
C(8,10) = cfw*E_0+c4*E_a4+c5*E_a5;
C(8,14) = -c4;
C(8,15) = -c5;

C(9,5) = e*cfw;
C(9,6) = e*i*cfw;
C(9,7) = e*cfw*E_fw;
C(9,8) = -e*cfw+f*c4+h*c5;
C(9,9) = (e^2)*cfw+(f^2)*c4+(h^2)*c5;
C(9,10) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(9,14) = -f*c4;
C(9,15) = -h*c5;

C(10,5) = -cfw*E_0;
C(10,6) = -i*cfw*E_0;
C(10,7) = -cfw*E_fw*E_0;
C(10,8) = cfw*E_0+c4*E_a4+c5*E_a5;
C(10,9) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(10,10) = cfw*E_0^2+c4*E_a4^2+c5*E_a5^2;
C(10,14) = -c4*E_a4;
C(10,15) = -c5*E_a5;

C(11,5) = -c1;
C(11,6) = a*c1;
C(11,7) = -c1*E_a1;
C(11,11) = c1+ct1;

C(12,5) = -c2;
C(12,6) = -b*c2;
C(12,7) = -c2*E_a2;
C(12,12) = c2+ct2;

C(13,5) = -c3;
C(13,6) = -d*c3;
C(13,7) = -c3*E_a3;
C(13,13) = c3+ct3;

C(14,8) = -c4;
C(14,9) = -f*c4;
C(14,10) = -c4*E_a4;
C(14,14) = c4+ct4;

C(15,8) = -c5;
C(15,9) = -h*c5;
C(15,10) = -c5*E_a5;
C(15,15) = c5+ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
K = zeros(15,15);

K(1,1) = ks;
K(1,2) = -ks;
K(1,3) = r*ks;

K(2,1) = -ks;
K(2,2) = ks+kcf+kcr;
K(2,3) = -r*ks-n*kcf+p*kcr;
K(2,5) = -kcf-kcr;
K(2,6) = l*kcf-j*kcr;
K(2,7) = -kcf*E_cf-kcr*E_cr;

K(3,1) = r*ks;
K(3,2) = -r*ks-n*kcf+p*kcr;
K(3,3) = (r^2)*ks+(n^2)*kcf+(p^2)*kcr;
K(3,5) = n*kcf-p*kcr;
K(3,6) = -n*l*kcf-p*j*kcr;
K(3,7) = n*kcf*E_cf-p*kcr*E_cr;

K(4,4) = ke;
K(4,5) = -ke;
K(4,6) = m*ke;
K(4,7) = -ke*E_e;

K(5,2) = -kcf-kcr;
K(5,3) = n*kcf-p*kcr;
K(5,4) = -ke;
K(5,5) = ke+kcf+kcr+kfw+k1+k2+k3;
K(5,6) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(5,7) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(5,8) = -kfw;
K(5,9) = e*kfw;
K(5,10) = -kfw*E_0;
K(5,11) = -k1;
K(5,12) = -k2;
K(5,13) = -k3;

K(6,2) = l*kcf-j*kcr;
K(6,3) = -n*l*kcf-p*j*kcr;
K(6,4) = m*ke;
K(6,5) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(6,6) =
(m^2)*ke+(l^2)*kcf+(j^2)*kcr+(i^2)*kfw+(a^2)*k1+(b^2)*k2+(d^2)*k3
;
K(6,7) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2+...
         d*k3*E_a3;
K(6,8) = -i*kfw;
K(6,9) = e*i*kfw;
K(6,10) = -i*kfw*E_0;
K(6,11) = a*k1;
K(6,12) = -b*k2;
K(6,13) = -d*k3;
```

```
K(7,2) = -kcf*E_cf-kcr*E_cr;
K(7,3) = n*kcf*E_cf-p*kcr*E_cr;
K(7,4) = -ke*E_e;
K(7,5) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(7,6) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2 ...
         +d*k3*E_a3;
K(7,7) =
ke*E_e^2+kcf*E_cf^2+kcr*E_cr^2+kfw*E_fw^2+k1*E_a1^2+k2*E_a2^2 ...
         +k3*E_a3^2+EI_t*I4_t;
K(7,8) = -kfw*E_fw;
K(7,9) = e*kfw*E_fw;
K(7,10) = -kfw*E_0*E_fw;
K(7,11) = -k1*E_a1;
K(7,12) = -k2*E_a2;
K(7,13) = -k3*E_a3;

K(8,5) = -kfw;
K(8,6) = -i*kfw;
K(8,7) = -kfw*E_fw;
K(8,8) = kfw+k4+k5;
K(8,9) = -e*kfw+f*k4+h*k5;
K(8,10) = kfw*E_0+k4*E_a4+k5*E_a5;
K(8,14) = -k4;
K(8,15) = -k5;

K(9,5) = e*kfw;
K(9,6) = e*i*kfw;
K(9,7) = e*kfw*E_fw;
K(9,8) = -e*kfw+f*k4+h*k5;
K(9,9) = (e^2)*kfw+(f^2)*k4+(h^2)*k5;
K(9,10) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(9,14) = -f*k4;
K(9,15) = -h*k5;

K(10,5) = -kfw*E_0;
K(10,6) = -i*kfw*E_0;
K(10,7) = -kfw*E_fw*E_0;
K(10,8) = kfw*E_0+k4*E_a4+k5*E_a5;
K(10,9) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(10,10) = kfw*E_0^2+k4*E_a4^2+k5*E_a5^2+EI_tlr*I4_tlr;
K(10,14) = -k4*E_a4;
K(10,15) = -k5*E_a5;

K(11,5) = -k1;
K(11,6) = a*k1;
K(11,7) = -k1*E_a1;
K(11,11) = k1+kt1;

K(12,5) = -k2;
K(12,6) = -b*k2;
K(12,7) = -k2*E_a2;
K(12,12) = k2+kt2;
```

```
K(13,5) = -k3;
K(13,6) = -d*k3;
K(13,7) = -k3*E_a3;
K(13,13) = k3+kt3;

K(14,8) = -k4;
K(14,9) = -f*k4;
K(14,10) = -k4*E_a4;
K(14,14) = k4+kt4;

K(15,8) = -k5;
K(15,9) = -h*k5;
K(15,10) = -k5*E_a5;
K(15,15) = k5+kt5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A = zeros(15,1);

A(11) = ct1;
A(12) = ct2;
A(13) = ct3;
A(14) = ct4;
A(15) = ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
B = zeros(15,1);

B(11) = kt1;
B(12) = kt2;
B(13) = kt3;
B(14) = kt4;
B(15) = kt5;

% System "A" Matrix
AA=[zeros(size(M))   eye(size(M))   % System state variable
matrix
    -inv(M)*K       -inv(M)*C];

% ISO 2631 FOR REDUCED COMFORT BOUNDARY
% COMFORT BOUNDARIES FOR VERTICAL ACCELERATION
% THE ISO CENTRAL FREQUENCIES (Hz)

wc=[ .1 1 1.25 1.6 2 2.5 3.15 4 5 6.3 8 10 12.5 16 20 25 31.5 40
50];
whzc=[ .1 .125 .16 .2 .25 .315 .4 .5 .63 .8 1 1.25 1.6 2 2.5 3.15
...
    4 5 6.3 8 10 12.5 16 20 25 31.5 40 50];

% 2.5 hr FATIGUE BOUNDARY
fat1=[4.284,1.4,1.25,1.12,1,.9,.8,.71,.71,.71,.71,...
```

```
      .9,1.12,1.4,1.8,2.24,2.8,3.55,4.5];
% 2.5hr REDUCED COMFORT BOUNDARY
comf1=fat1/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf2= comf1/2.254;

%-----------------------------------------------------------------
---
% COMFORT BOUNDARIES FOR LONGITUDINAL AND LATERAL ACC
% 2.5hr FATIGUE BOUNDARY
fat2=[0.5,0.5,0.5,0.5,0.5,0.63,0.8,1,1.25,1.6,2,2.5,3.15,4,5,6.3,
8,10,12.5];

% 2.5hr REDUCED COMFORT BOUNDARY
comf3=fat2/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf4= comf3/2.254;
%-----------------------------------------------------------------
--

whzcr = 2*pi*whzc;      % Calculation of central frequencies in
rad/s
freqlow=0.89*whzcr;     % Lower octave band
freqhigh=1.12*whzcr;    % Upper octave band
freq=[freqlow' whzcr' freqhigh'];

imag=sqrt(-1);

for ii=1:length(whzc);
    for jj=1:3;         % jj=1 is freqlow, jj=2 is center freq
                        % jj=3 is freqhigh
        w = freq(ii,jj);
        s = imag*w;
        dp = sqrt(h1^2+r^2);

        % Time delay array
        time = [0 0 0 0 0 0 0 0 0 0 1 exp(-s*T(2)) exp(-s*T(3))
...
            exp(-s*T(4)) exp(-s*T(5))];

        % TF Matrix
        vectx = (inv(M*s*s+C*s+K)*((A*s+B).*(time.')));

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %%%  Transfer Functions  %%%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%

        z_s=[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]*vectx;    % vert seat
cg
        long=[0 0 -h1 0 0 0 0 0 0 0 0 0 0 0 0]*vectx; % long disp
of driver
        z_tlr=[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]*vectx;  % vert
trailer cg

        %%%%%%%%%%%%%%%%%%%%
```

```
        %%%  Magnitudes  %%%
        %%%%%%%%%%%%%%%%%%%%

        % Acceleration Transter Functions
        magcfA1(ii,jj)=abs(s*s*z_s);  % Mag of trans function,
(m/s*s)/m
        magcfAlong(ii,jj)=abs(s*s*long);
        magcftlr(ii,jj)=abs(s*s*z_tlr);

        %%%%%%%%%%%%%%
        %%%  PSDs  %%%
        %%%%%%%%%%%%%%

        % Road PSD in m*m/(rad/s)
        rpsd(ii,jj)=Csp*(((2*pi*v)^(N-1))/(w^N));

        % Acceleration PSDs in (m/s^2)^2/(rad/s)
        psdcfA1(ii,jj)=magcfA1(ii,jj)*magcfA1(ii,jj)*rpsd(ii,jj);

psdcfAlong(ii,jj)=magcfAlong(ii,jj)*magcfAlong(ii,jj)*rpsd(ii,jj)
;

psdcftlr(ii,jj)=magcftlr(ii,jj)*magcftlr(ii,jj)*rpsd(ii,jj);

    end
end

for kk=1:length(whzc)
    % Vert. Driver's Seat RMS
    msqy1a(kk)=0.5*(psdcfA1(kk,1)+psdcfA1(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqy1b(kk)=0.5*(psdcfA1(kk,2)+psdcfA1(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqy1(kk)=msqy1a(kk)+msqy1b(kk);
    rmsA1cf(kk)=sqrt(msqy1(kk));
    % Long. Driver RMS
    msqylonga(kk)=0.5*(psdcfAlong(kk,1)+psdcfAlong(kk,2))*...
    (freq(kk,2)-freq(kk,1));
    msqylongb(kk)=0.5*(psdcfAlong(kk,2)+psdcfAlong(kk,3))*...
    (freq(kk,3)-freq(kk,2));
    msqylong(kk)=msqylonga(kk)+msqylongb(kk);
    rmsAlongcf(kk)=sqrt(msqylong(kk));
    % Vert. Trailer cg RMS
    msqytlra(kk)=0.5*(psdcftlr(kk,1)+psdcftlr(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqytlrb(kk)=0.5*(psdcftlr(kk,2)+psdcftlr(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqytlr(kk)=msqytlra(kk)+msqytlrb(kk);
    rmstlrcf(kk)=sqrt(msqytlr(kk));
end

RMScf = [rmsA1cf',rmsAlongcf',rmstlrcf'];       % Accel. RMS
Matrix

% Calculate weighted rms acceleration from 0.1 to 50 Hz
```

```
% at the ISO Center Frequencies .... Wgt are the ISO weights
% Ref: ISO 2631-1:1997(E); V=vertical; L=longitudinal

wcc=[.1,.125,.16,.2,.25,.315,.4,.5,.63,.8,1,1.25,1.6,2,2.5,3.15,4
,5,...
    6.3,8,10,12.5,16,20,25,31.5,40,50];
WgtV=[.0312,.0486,.079,.121,.182,.263,.352,.418,.459,.477,.482,.4
84,...

.494,.531,.631,.804,.967,1.039,1.054,1.036,.988,.902,.768,.636,..
.
    .513,.405,.314,.246];
WgtL=0.001*[62.4,97.3,158,243,365,530,713,853,944,992,1011,1008,9
68,...

890,776,642,512,409,323,253,212,161,125,100,80,63.2,49.4,38.8];

isovert = WgtV.*RMScf(1:28,1)';      % Weighted Vert. Driver RMS
Accel.
isolong = WgtL.*RMScf(1:28,2)';      % Weighted Long. Driver RMS
Accel.
isotlr = WgtV.*RMScf(1:28,3)';       % Weighted Vert. Trailer RMS
Accel.

term2V=(WgtV.*rmsA1cf(1:28)).^2;
a0_V_dr=(sum(term2V))^0.5;              % a0 for vert. disp of
driver

term2L=(WgtL.*rmsAlongcf(1:28)).^2;
a0_L_dr=(sum(term2L))^0.5;              % a0 for long. disp of
driver

aV=(a0_L_dr^2 + a0_V_dr^2)^0.5;      % a0 for comb vert and long
disp

tlrV=(WgtV.*rmstlrcf(1:28)).^2;
a0_V_tlr=(sum(tlrV))^0.5;               % a0 for vert. disp of
driver

aVV(iiii,jjjj)=aV;               % combined ISO wgt acc, m/s^2
a0_VV_tlr(iiii,jjjj)=a0_V_tlr;
Jpenalty(iiii,jjjj)=K_1*(aVV(iiii,jjjj)/0.44814)+K_2*...
(a0_VV_tlr(iiii,jjjj)/0.3239);

    end         % end of jjjj loop on c2
end             % end of iiii loop on c1

disp('  ')
disp('RESULTS OF VARIATION')
disp('  ')
disp('Minimum aV, m/s^2')
disp(min(aVV(:)))
disp('  ')
[ia,ja]=find(aVV==min(aVV(:)));
disp('Corresponding c1, c2, and c3 values, N/(m/s)')
```

```
disp([7550.9+cf(ia,ja) (36850+cr(ia,ja))*0.5
(36850+cr(ia,ja))*0.5])
disp('  ')

disp('  ')
disp('Minimum a0_V_tlr, m/s^2')
disp(min(a0_VV_tlr(:)))
disp('  ')
[it,jt]=find(a0_VV_tlr==min(a0_VV_tlr(:)));
disp('Corresponding c1, c2, and c3 values, N/(m/s)')
disp([7550.9+cf(it,jt) (36850+cr(it,jt))*0.5
(36850+cr(it,jt))*0.5])
disp('  ')

disp('  ')
disp('Minimum Jpenalty')
disp('J=K1*aV/0.44814 + K2*a0_V_tlr/0.3239')
disp('      K1         K2')
disp([K_1 K_2])
disp('  ')
disp(min(Jpenalty(:)))
disp('  ')
[iJ,jJ]=find(Jpenalty==min(Jpenalty(:)));
disp('Corresponding c1, c2, and c3 values, N/(m/s)')
disp([7550.9+cf(iJ,jJ) (36850+cr(iJ,jJ))*0.5
(36850+cr(iJ,jJ))*0.5])
disp('  ')

figure(1)
surf(7550.9+cf,(36850+cr)/2,aVV)
xlabel('Steer Axle C, N/(m/s)')
ylabel('Single Drive Axle C, N/(m/s)')
zlabel('ISO Combined Wgt Acc, m/s^2')
% title('Tractor Suspension Damping Parameter Variation')

figure(2)
surf(7550.9+cf,(36850+cr)/2,a0_VV_tlr)
xlabel('Steer Axle C, N/(m/s)')
ylabel('Single Drive Axle C, N/(m/s)')
zlabel('Trailer Wgt Vert Acc, m/s^2')
% title('Tractor Suspension Damping Parameter Variation')

figure(3)
surf(7550.9+cf,(36850+cr)/2,Jpenalty)
xlabel('Steer Axle C, N/(m/s)')
ylabel('Single Drive Axle C, N/(m/s)')
zlabel('Penalty Function')
title(['K1 = ', num2str(K_1),'     K2 = ', num2str(K_2)])
```

Appendix I: opt_tireK_freq.m

This parameter variation program varies the stiffness of the steer axle tires and the stiffness of the first and second drive axle tires combined. Each of the drive axle tires on the tractor are assumed to have the same value, so they were combined into one value that was varied, and the individual drive axle tire values were assumed to be equal to exactly half of that value. The steer tires were varied from 906,500 N/m to 1,683,500 N/m in increments of 38,850 N/m. This forms a vector with a length of 21 values that ranges from 30% below to 30% above the nominal value for the steer tire stiffnesses. The drive axle tires were varied from 1,671,740 N/m to 3,104,660 N/m in increments of 71,646 N/m. Like the steer tires, this forms a vector with a length of 21 values that ranges from 30% below to 30% above the nominal value for the drive tire stiffnesses.

The desired output values from this program were the ISO combined weighted acceleration at the driver's seat, the ISO vertical weighted acceleration at the trailer center-of-gravity (CG), and a value called the J penalty, which weighs the importance of the driver ride comfort versus trailer acceleration using weights assigned to them by the user. The program finds the minimum values for each of these outputs, and displays them in tabular form along with the corresponding stiffness values for the steer and drive tires. Also, the program plots the output information on surface plots to study trends in the information.

opt_tireK_freq.m

```
% opt_tireK_freq.m
% Developed by Ryan Spivey, 4/10/07
%
% Varies tire stiffness using weighted RMS acceleration in the
% frequecncy domain
%
% Incorporates model from dof15_freq2.m
%
% DOFs include - 1)Vertical Disp. of Driver's Seat
%                2)Vertical Disp. of Cab
%                3)Pitch of Cab
%                4)Vertical Disp. of Engine
%                5)Vertical Disp. of Tractor Frame
%                6)Pitch of Tractor Frame
%                7)Beaming of Tractor Frame
%                8)Vertical Disp. of Trailer
%                9)Pitch of Trailer
%               10)Beaming of Trailer
%               11)Vertical Disp. of Axle #1
%               12)Vertical Disp. of Axle #2
%               13)Vertical Disp. of Axle #3
%               14)Vertical Disp. of Axle #4
%               15)Vertical Disp. of Axle #5

clc
clear all
close all
format short e
format compact

global D1_t D2_t D3_t D4_t D1_tlr D2_tlr D3_tlr D4_tlr
global e a1 kb1 kb2 b_fw L_tlr alpha

disp('  ')
disp('Tire Stiffness Parameter Variation in the Frequency
Domain')
disp('                  Roadholding Model
')
disp(['                       ',date])

% Choose a test vehicle
disp('  ')
disp('VEHICLE SELECTION')
disp('  ')
disp('Please choose a vehicle : ');
disp('a: Ideal Tractor Semi-Trailer');
vehicle = input('Enter your choice : ', 's');

if vehicle == 'a'
    % Inertial Properties
    m_s = 106.7;        %kg      mass of seat
    m_c = 1208;         %kg      mass of cab
```

```
    I_c = 2100;          %kg*m^2  M I of cab
    m_e = 2000;          %kg      mass of engine (ESTIMATE)
    m_t = 3783;          %kg      mass of tractor (5783 kg -
engine)
    I_t = 46590.9;       %kg*m^2  M I of tractor
    m_ul = 10800;        %kg      mass of trailer (ESTIMATE)
    I_tlr = 200000;      %kg*m^2  M I of trailer
    m_L = 14000;         %kg      mass of trailer load  (ESTIMATE)
    m_tlr = m_ul+m_L;    %kb      mass of loaded trailer

    % Suspension Parameters
    c1 = 11270;          %N/(m/s) damping const of axle #1
    c2 = 27500;          %N/(m/s) damping const of axle #2
    c3 = 27500;          %N/(m/s) damping const of axle #3
    c4 = 70000;          %N/(m/s) damping const of axle #4
    c5 = 70000;          %N/(m/s) damping const of axle #5
    ce = 10000;          %N/(m/s) damping const of engine mount
    k1 = 581300;         %N/m     spring const of axle #1
    k2 = 586900;         %N/m     spring const of axle #2
    k3 = 586900;         %N/m     spring const of axle #3
    k4 = 1000000;        %N/m     spring const of axle #4
    k5 = 1000000;        %N/m     spring const of axle #5
    ke = 1e10;           %N/m     spring const of the engine mount

    % Model Dimensions
    b_a1 = 1.065;        %m       Front end of the tractor to axle
#1
    b_cf = 1.470;        %m       Front end of the tractor to cab
front
    b_e = 2.797;         %m       Front end of the tractor to
engine
    b_cr = 4.02;         %m       Front end of the tractor to cab
rear
    b_a2 = 6.035;        %m       Front end of the tractor to axle
#2
    b_fw = 6.688;        %m       Front end of the tractor to 5th
wheel
    b_a3 = 7.34;         %m       Front end of the tractor to axle
#3
    a1 = 4.00607;        %m       Front end of the tractor to
tractor cg

    b_a4 = 8.58;         %m       From the fifth wheel to axle #4
    b_a5 = 9.78;         %m       From the fifth wheel to axle #5

    L_t = 8.2;           %m       Length of Tractor
    L_tlr = 9.78;        %m       Length of Trailer

    e = 5.62;            %m       From the trailer cg to fifth
wheel
    f = 2.96;            %m       From the trailer cg to axle #4
    h = 4.16;            %m       From the trailer cg to axle #5

    a = 2.94107;         %m       From the tractor cg to axle #1
    b = 2.02893;         %m       From the tractor cg to axle #2
    d = 3.33393;         %m       From the tractor cg to axle #3
```

```
    l = 2.53607;        %m       From the tractor cg to cab front
    m = 1.209074;       %m       From the tractor cg to engine
    j = 0.013926;       %m       From the tractor cg to cab rear
    i = 2.68193;        %m       From the tractor cg to the fifth
wheel
    n = 1.435;          %m       From the cab cg to cab front
    p = 1.115;          %m       From the cab cg to cab rear
    r = -0.200;         %m       From the cab cg to seat

    tc = 1.10107;       %m       From the tractor cg to the cab
cg
    h1 = 1.0;           %m       Height of the driver over the
cab
    g = 9.8;            %m/s^2   acceleration due to gravity

    ML_t = m_t/L_t;         %kg/m    Mass per unit length
(Tractor)
    ML_tlr = m_ul/L_tlr;    %kg/m    Mass per unit length
(Trailer)
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%  Fifth Wheel Configuration
%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

disp('  ')
disp('Give your choice for the fifth wheel configuration: ')
disp('Note: If a fifth wheel suspension system is chosen, the
beaming of')
disp('      the tractor frame and trailer will be modeled as
free-free.  If')
disp('      no suspension is chosen, the tractor frame and
trailer will be')
disp('      modeled as free-pinned and pinned-free
respectively.')
disp('a : With fifth wheel suspension')
disp('b : Without fifth wheel suspension')
z33 = input('Please give your choice : ', 's');

if z33 == 'a',          % Choice 'a' is with fifth wheel
suspension
    disp('  ')
    kfw = input('Input the fifth wheel spring constant (N/m): ');
    disp('  ')
    cfw = input('Input the fifth wheel damping ratio (N/(m/s)):
');

    % The parameters for the first bending mode of the Tractor
frame
    disp('  ')
    fhz = input('Input the Tractor frequency of beaming (hz) fhz
: ');
```

```
    % The parameters for the first bending mode of the Trailer
frame
    disp('  ')
    fhz2 = input('Input the Trailer frequency of beaming (hz) fhz
: ');

    kb1 = 4.73004074;    %Constant for the first bending mode
(free-free)
    alpha = 0.982502;

    z1 = 'cosh(kb1*x1/b_fw) + cos(kb1*x1/b_fw) - ...
alpha*(sinh(kb1*x1/b_fw)+sin(kb1*x1/b_fw))';
    % free-free beam mode function
    z1dd = '(kb1/b_fw)^2*(cosh(kb1*x1/b_fw) - cos(kb1*x1/b_fw) -
... alpha*(sinh(kb1*x1/b_fw)-sin(kb1*x1/b_fw)))';
    % second derivative of free-free beam mode function

    kb2 = 4.73004074;    %Constant for the first bending mode
(free-free)

    z2 = 'cosh(kb2*x2/L_tlr) + cos(kb2*x2/L_tlr) - ...
alpha*(sinh(kb2*x2/L_tlr)+sin(kb2*x2/L_tlr))';
    % free-free beam mode function
    z2dd = '(kb2/L_tlr)^2*(cosh(kb2*x2/L_tlr) - cos(kb2*x2/L_tlr)
- ... alpha*(sinh(kb2*x2/L_tlr)-sin(kb2*x2/L_tlr)))';
    % second derivative of free-free beam mode function

elseif z33 == 'b',          % Choice 'b' is without fifth wheel
suspension
    kfw = 1000000000000;    %(N/m)      fifth wheel spring
constant
    cfw = 1000;             %(N/(m/s))  fifth wheel damping ratio

    % The parameters for the first bending mode of the Tractor
frame
    disp('  ')
    fhz = input('Input the Tractor frequency of beaming (hz) fhz
: ');

    % The parameters for the first bending mode of the Trailer
frame
    disp('  ')
    fhz2 = input('Input the Trailer frequency of beaming (hz) fhz
: ');

    kb1 = 2.36502;      % Constant for the first bending mode
(free-pinned)
    %                     (from Rao pg. 527)

    z1 = '(cos(kb1*x1/b_fw) + (cosh(kb1*x1/b_fw)) - ...
((cos(kb1)+cosh(kb1))/(sin(kb1)-sinh(kb1)))*(sin(kb1*x1/b_fw)-
...  sinh(kb1*x1/b_fw)))';
    % free-pinned beam mode function
```

```
    z1dd = '((kb1/b_fw)^2)*(-cos(kb1*x1/b_fw) +
(cosh(kb1*x1/b_fw)) - ... ((cos(kb1)+cosh(kb1))/(sin(kb1)-
sinh(kb1)))*(-sin(kb1*x1/b_fw)- ...   sinh(kb1*x1/b_fw)))';
    % second derivative of free-pinned beam mode function

    kb2 = 3.926602;     % Constant for the first bending mode
(pinned-free)
    %                     (from Rao pg. 527)

    z2 = '(sin(kb2*x2/L_tlr) + ...
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
    % pinned-free beam mode function
    z2dd = '(kb2/L_tlr)^2*(-sin(kb2*x2/L_tlr) + ...
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
    % second derivative of pinned-free beam mode function

else disp('Insufficient information regarding fifth wheel
suspension.')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%%%%%%%%%%%%%%%%%%  Computation of Integrals
%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

D1_t=['(',z1,')'];                  % Tractor frame beaming
equations to be
D2_t=['((a1-x1).*(',z1,'))'];       % used in the integrals
(string form)
D3_t=['((',z1,').*(',z1,'))'];
D4_t=['((',z1dd,').*(',z1dd,'))'];

D1_tlr=['(',z2,')'];                % Trailer beaming equations
to be
D2_tlr=['((e-x2).*(',z2,'))'];      % used in the integrals
(string form)
D3_tlr=['((',z2,').*(',z2,'))'];
D4_tlr=['((',z2dd,').*(',z2dd,'))'];

I1_t=quadl('modeD1_t',0,b_fw);      % Integrals of functions
defined above
I2_t=quadl('modeD2_t',0,b_fw);      % (along length of tractor
frame)
I3_t=quadl('modeD3_t',0,b_fw);
I4_t=quadl('modeD4_t',0,b_fw);

I1_tlr=quadl('modeD1_tlr',0,L_tlr); % Integrals of functions
defined above
I2_tlr=quadl('modeD2_tlr',0,L_tlr); % (along length of trailer)
I3_tlr=quadl('modeD3_tlr',0,L_tlr);
I4_tlr=quadl('modeD4_tlr',0,L_tlr);
```

```
E_a1=modeD1_t(b_a1);      % Disp at axle #1 due to tractor frame
beaming
E_cf=modeD1_t(b_cf);      % Disp at cab front due to tractor
frame beaming
E_e=modeD1_t(b_e);        % Disp at engine due to tractor frame
beaming
E_cr=modeD1_t(b_cr);      % Disp at cab rear due to tractor frame
beaming
E_a2=modeD1_t(b_a2);      % Disp at axle #2 due to tractor frame
beaming
E_fw=modeD1_t(b_fw);      % Disp at fifth wheel due to tractor
frame beaming
E_a3=modeD1_t(b_a3);      % Disp at axle #3 due to tractor frame
beaming
E_0=modeD1_tlr(0);        % Disp at fifth wheel due to trailer
beaming
E_a4=modeD1_tlr(b_a4);    % Disp at axle #4 due to trailer
beaming
E_a5=modeD1_tlr(b_a5);    % Disp at axle #5 due to trailer
beaming

EI_t = 4*pi^2*fhz^2*(b_fw/kb1)^4*ML_t;       %Tractor frame
flexural rigidity
EI_tlr = 4*pi^2*fhz2^2*(L_tlr/kb2)^4*ML_tlr; %Trailer flexural
rigidity

% Seat Suspension Options
disp('  ')
disp('VEHICLE SUSPENSION OPTIONS')
disp('  ')
disp('Give your choice for seat suspension: ')
disp('Note: Without seat suspension gives a very high frequency
mode')
disp('      because the stiffness is set to a high value.')
disp('a : With seat suspension (~0.9 Hz)')
disp('b : Without seat suspension')
z11 = input('Enter your choice : ', 's');

if z11 == 'a',          % Choice 'a' is with seat suspension
    cs = 1140;          % Damping ratio of 0.5
    ks = 3403;          % N/m(spring const of seat suspension)

elseif z11 == 'b',      % Choice 'b' is without seat suspension
    cs = 1329;          % N/(m/s)(damping const of seat
suspension)
    ks = 1e10;          % N/m(spring const of seat suspension)

else disp('Insufficient information regarding seat suspension.')
end

% Cab Suspension Options
disp('  ')
disp('Give your choice for cab suspension: ')
disp('Note: With front or rear or without cab suspension')
disp('      gives a very high frequency mode(s) because the
corresponding')
```

```
disp('      stiffness(es)is set to a high value.')
disp('a : With front cab suspension')
disp('b : With rear cab suspension')
disp('c : With front & rear cab suspension')
disp('d : Without cab suspension')
z22 = input('Enter your choice : ', 's');

if z22 == 'a',          % Choice 'a' is front cab suspension
    ccf = 7062;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 88740;        % N/m(spring const of front cab
suspension)
    ccr = 6430;         % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;         % N/m(spring const of rear cab
suspension)

elseif z22 == 'b',      % Choice 'b' is rear cab suspension
    ccr = 8000;         % Reduced damping
    kcr = 65980;        % N/m(spring const of rear cab
suspension)
    ccf = 13120;        % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;         % N/m(spring const of front cab
suspension)

elseif z22 == 'c',      % Choice 'c' is front & rear cab
suspension
    ccr = 5073.5;       % N/(m/s)(damping const of rear cab
suspension)
    kcr = 63757.5;      % N/m(spring const of rear cab
suspension)
    ccf = 6864.35;      % N/(m/s)(damping const of front cab
suspension)
    kcf = 86260.5;      % N/m(spring const of front cab
suspension)

elseif z22 == 'd',      % Choice 'd' is without cab suspension
    ccr = 6430;         % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;         % N/m(spring const of rear cab
suspension)
    ccf = 7062;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;         % N/m(spring const of front cab
suspension)

else disp('Insufficient information regarding cab suspension.')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%  Vehicle Tire Selection
%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
```

```
% Values shown represent Wide Singles

mt1 = 374;       % kg    Mass of Steer Axle
mt2 = 648.3;     % kg    Mass of Drive Axle #1
mt3 = 648.3;     % kg    Mass of Drive Axle #2
mt4 = 648.3;     % kg    Mass of Trailer Axle #1
mt5 = 648.3;     % kg    Mass of Trailer Axle #2

ct1 = 517;       % N/(m/s)   Steer Axle Damping Const.
ct2 = 648.3;     % N/(m/s)   Drive Axle #1 Damping Const.
ct3 = 648.3;     % N/(m/s)   Drive Axle #2 Damping Const.
ct4 = 648.3;     % N/(m/s)   Trailer Axle #1 Damping Const.
ct5 = 648.3;     % N/(m/s)   Trailer Axle #2 Damping Const.

kt4 = 2.3882e6; % N/m   Trailer Axle #1 Spring Const.
kt5 = 2.3882e6; % N/m   Trailer Axle #2 Spring Const.

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%  Speed of the Vehicle  %%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('VEHICLE VELOCITY')
disp('  ')
disp('Please choose the unit of velocity');
disp('a : Miles per Hour (mph)');
disp('b : Kilometers per Hour (kph)');
vel = input('Input the unit of velocity (a/b):  ', 's');
disp('  ')
vm = input('Input the velocity of the vehicle, vm : ');

if vel == 'a'
    v = 0.4473*vm;              %Velocity conversion from mph to
m/s
elseif vel == 'b'
    v = 0.277778*vm;            %Velocity conversion from kph to
m/s
end

T(1) = 0;               %Time delay between front axle and
remaining axles
T(2) = (a+b)/v;         % Axle #2
T(3) = (a+d)/v;         % Axle #3
T(4) = (a+i+e+f)/v;     % Axle #4
T(5) = (a+i+e+h)/v;     % Axle #5

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%  Road PSD Selection  %%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('ROAD PSD SELECTION')
disp('  ')
```

```
disp('Road PSD Constants, m^2/cyc/m, Ref: Wong, Theory of Ground
Vehicles')
disp('S(W)=Csp/W^N  where W=spatial frequency')
disp('  ')
disp('a : Csp = 4.3e-11,N=3.8     Smooth Runway')
disp('b : Csp = 8.1e-6, N=2.1     Rough Runway')
disp('c : Csp = 4.8e-7, N=2.1     Smooth Highway')
disp('d : Csp = 4.4e-6, N=2.1     Highway with Gravel')
disp('  ')
tabchoice11=input('Input the road surface to be used :   ','s');

if tabchoice11== 'a',               % smooth runway
   Csp = 4.3e-11;
   N=3.8;

   elseif tabchoice11== 'b',        % rough runway
      Csp = 8.1e-6;
      N=2.1;

   elseif tabchoice11 == 'c',       % smooth highway
      Csp = 4.8e-7;
      N=2.1;

   elseif tabchoice11 == 'd',       % highway with gravel
      Csp = 4.4e-6;
      N=2.1;

end

disp('  ')
disp('J PENALTY OPTIONS')
disp('  ')
disp('Input the values for K1 and K2 in the J penalty function')
disp('Note: Both values should add up to 1')
disp('  ')
K_1 = input('Input the value for K1 : ');
disp('  ')
K_2 = input('Input the value for K2 : ');

% Start Loop on Tire Stiffness Properties
% Stiffness values will range from 70% to 130% of the nominal
value

for iiii=1:21;
    for jjjj=1:21;
        ktf(iiii,jjjj)=38850*iiii;
        ktr(iiii,jjjj)=143292*jjjj;

        kt1 = 867650+ktf(iiii,jjjj);
        kt2 = (3200190+ktr(iiii,jjjj))*0.5;
        kt3 = (3200190+ktr(iiii,jjjj))*0.5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%  System Matrices
%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

% THE SYSTEM IS WRITTEN AS (M*S*S+C*S+K)X(S)=(A*S+B)U(S)

%%%%%%%%%%%%%%%%%%%%%%%
%%%%  Mass Matrix  %%%%
%%%%%%%%%%%%%%%%%%%%%%%
M = zeros(15,15);

M(1,1) = m_s;                   % Eqn #1: Vertical Disp of Seat

M(2,2) = m_c;                   % Eqn #2: Vertical Disp of Cab

M(3,3) = I_c;                   % Eqn #3: Pitch of Cab

M(4,4) = m_e;                   % Eqn #4: Vertical Disp of Engine

M(5,5) = m_t;                   % Eqn #5: Vertical Disp of
Tractor Frame
M(5,6) = ML_t*b_fw*(b_fw/2-a1);
M(5,7) = ML_t*I1_t;

M(6,5) = ML_t*b_fw*(b_fw/2-a1);   % Eqn #6: Pitch of Tractor
Frame
M(6,6) = I_t;
M(6,7) = -ML_t*I2_t;

M(7,5) = ML_t*I1_t;             % Eqn #7: Beaming of Tractor
Frame
M(7,6) = -ML_t*I2_t;
M(7,7) = ML_t*I3_t;

M(8,8) = m_tlr;                 % Eqn #8: Vertical Disp of
Trailer
M(8,9) = -ML_tlr*L_tlr*(e-L_tlr/2);
M(8,10) = ML_tlr*I1_tlr;

M(9,8) = -ML_tlr*L_tlr*(e-L_tlr/2);% Eqn #9: Pitch of Trailer
M(9,9) = I_tlr;
M(9,10) = -ML_tlr*I2_tlr;

M(10,8) = ML_tlr*I1_tlr;        % Eqn #10: Beaming of Trailer
M(10,9) = -ML_tlr*I2_tlr;
M(10,10) = ML_tlr*I3_tlr;

M(11,11) = mt1;                 % Eqn #11: Vertical Disp of Axle
#1

M(12,12) = mt2;                 % Eqn #12: Vertical Disp of Axle
#2
```

```
M(13,13) = mt3;                 % Eqn #13: Vertical Disp of Axle
#3

M(14,14) = mt4;                 % Eqn #14: Vertical Disp of Axle
#4

M(15,15) = mt5;                 % Eqn #15: Vertical Disp of Axle
#5

%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%
C = zeros(15,15);

C(1,1) = cs;
C(1,2) = -cs;
C(1,3) = r*cs;

C(2,1) = -cs;
C(2,2) = cs+ccf+ccr;
C(2,3) = -r*cs-n*ccf+p*ccr;
C(2,5) = -ccf-ccr;
C(2,6) = l*ccf-j*ccr;
C(2,7) = -ccf*E_cf-ccr*E_cr;

C(3,1) = r*cs;
C(3,2) = -r*cs-n*ccf+p*ccr;
C(3,3) = (r^2)*cs+(n^2)*ccf+(p^2)*ccr;
C(3,5) = n*ccf-p*ccr;
C(3,6) = -n*l*ccf-p*j*ccr;
C(3,7) = n*ccf*E_cf-p*ccr*E_cr;

C(4,4) = ce;
C(4,5) = -ce;
C(4,6) = m*ce;
C(4,7) = -ce*E_e;

C(5,2) = -ccf-ccr;
C(5,3) = n*ccf-p*ccr;
C(5,4) = -ce;
C(5,5) = ce+ccf+ccr+cfw+c1+c2+c3;
C(5,6) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(5,7) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(5,8) = -cfw;
C(5,9) = e*cfw;
C(5,10) = -cfw*E_0;
C(5,11) = -c1;
C(5,12) = -c2;
C(5,13) = -c3;

C(6,2) = l*ccf-j*ccr;
C(6,3) = -n*l*ccf-p*j*ccr;
C(6,4) = m*ce;
C(6,5) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
```

```
C(6,6) =
(m^2)*ce+(l^2)*ccf+(j^2)*ccr+(i^2)*cfw+(a^2)*c1+(b^2)*c2+(d^2)*c3
;
C(6,7) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2+...
         d*c3*E_a3;
C(6,8) = -i*cfw;
C(6,9) = e*i*cfw;
C(6,10) = -i*cfw*E_0;
C(6,11) = a*c1;
C(6,12) = -b*c2;
C(6,13) = -d*c3;

C(7,2) = -ccf*E_cf-ccr*E_cr;
C(7,3) = n*ccf*E_cf-p*ccr*E_cr;
C(7,4) = -ce*E_e;
C(7,5) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(7,6) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2 ...
         +d*c3*E_a3;
C(7,7) =
ce*E_e^2+ccf*E_cf^2+ccr*E_cr^2+cfw*E_fw^2+c1*E_a1^2+c2*E_a2^2 ...
         +c3*E_a3^2;
C(7,8) = -cfw*E_fw;
C(7,9) = e*cfw*E_fw;
C(7,10) = -cfw*E_0*E_fw;
C(7,11) = -c1*E_a1;
C(7,12) = -c2*E_a2;
C(7,13) = -c3*E_a3;

C(8,5) = -cfw;
C(8,6) = -i*cfw;
C(8,7) = -cfw*E_fw;
C(8,8) = cfw+c4+c5;
C(8,9) = -e*cfw+f*c4+h*c5;
C(8,10) = cfw*E_0+c4*E_a4+c5*E_a5;
C(8,14) = -c4;
C(8,15) = -c5;

C(9,5) = e*cfw;
C(9,6) = e*i*cfw;
C(9,7) = e*cfw*E_fw;
C(9,8) = -e*cfw+f*c4+h*c5;
C(9,9) = (e^2)*cfw+(f^2)*c4+(h^2)*c5;
C(9,10) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(9,14) = -f*c4;
C(9,15) = -h*c5;

C(10,5) = -cfw*E_0;
C(10,6) = -i*cfw*E_0;
C(10,7) = -cfw*E_fw*E_0;
C(10,8) = cfw*E_0+c4*E_a4+c5*E_a5;
C(10,9) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(10,10) = cfw*E_0^2+c4*E_a4^2+c5*E_a5^2;
C(10,14) = -c4*E_a4;
```

```
C(10,15) = -c5*E_a5;

C(11,5) = -c1;
C(11,6) = a*c1;
C(11,7) = -c1*E_a1;
C(11,11) = c1+ct1;

C(12,5) = -c2;
C(12,6) = -b*c2;
C(12,7) = -c2*E_a2;
C(12,12) = c2+ct2;

C(13,5) = -c3;
C(13,6) = -d*c3;
C(13,7) = -c3*E_a3;
C(13,13) = c3+ct3;

C(14,8) = -c4;
C(14,9) = -f*c4;
C(14,10) = -c4*E_a4;
C(14,14) = c4+ct4;

C(15,8) = -c5;
C(15,9) = -h*c5;
C(15,10) = -c5*E_a5;
C(15,15) = c5+ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%
K = zeros(15,15);

K(1,1) = ks;
K(1,2) = -ks;
K(1,3) = r*ks;

K(2,1) = -ks;
K(2,2) = ks+kcf+kcr;
K(2,3) = -r*ks-n*kcf+p*kcr;
K(2,5) = -kcf-kcr;
K(2,6) = l*kcf-j*kcr;
K(2,7) = -kcf*E_cf-kcr*E_cr;

K(3,1) = r*ks;
K(3,2) = -r*ks-n*kcf+p*kcr;
K(3,3) = (r^2)*ks+(n^2)*kcf+(p^2)*kcr;
K(3,5) = n*kcf-p*kcr;
K(3,6) = -n*l*kcf-p*j*kcr;
K(3,7) = n*kcf*E_cf-p*kcr*E_cr;

K(4,4) = ke;
K(4,5) = -ke;
K(4,6) = m*ke;
K(4,7) = -ke*E_e;
```

```
K(5,2) = -kcf-kcr;
K(5,3) = n*kcf-p*kcr;
K(5,4) = -ke;
K(5,5) = ke+kcf+kcr+kfw+k1+k2+k3;
K(5,6) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(5,7) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(5,8) = -kfw;
K(5,9) = e*kfw;
K(5,10) = -kfw*E_0;
K(5,11) = -k1;
K(5,12) = -k2;
K(5,13) = -k3;

K(6,2) = l*kcf-j*kcr;
K(6,3) = -n*l*kcf-p*j*kcr;
K(6,4) = m*ke;
K(6,5) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(6,6) =
(m^2)*ke+(l^2)*kcf+(j^2)*kcr+(i^2)*kfw+(a^2)*k1+(b^2)*k2+(d^2)*k3
;
K(6,7) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2+...
         d*k3*E_a3;
K(6,8) = -i*kfw;
K(6,9) = e*i*kfw;
K(6,10) = -i*kfw*E_0;
K(6,11) = a*k1;
K(6,12) = -b*k2;
K(6,13) = -d*k3;

K(7,2) = -kcf*E_cf-kcr*E_cr;
K(7,3) = n*kcf*E_cf-p*kcr*E_cr;
K(7,4) = -ke*E_e;
K(7,5) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(7,6) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2 ...
         +d*k3*E_a3;
K(7,7) =
ke*E_e^2+kcf*E_cf^2+kcr*E_cr^2+kfw*E_fw^2+k1*E_a1^2+k2*E_a2^2 ...
         +k3*E_a3^2+EI_t*I4_t;
K(7,8) = -kfw*E_fw;
K(7,9) = e*kfw*E_fw;
K(7,10) = -kfw*E_0*E_fw;
K(7,11) = -k1*E_a1;
K(7,12) = -k2*E_a2;
K(7,13) = -k3*E_a3;

K(8,5) = -kfw;
K(8,6) = -i*kfw;
K(8,7) = -kfw*E_fw;
K(8,8) = kfw+k4+k5;
K(8,9) = -e*kfw+f*k4+h*k5;
K(8,10) = kfw*E_0+k4*E_a4+k5*E_a5;
K(8,14) = -k4;
```

```
K(8,15) = -k5;

K(9,5) = e*kfw;
K(9,6) = e*i*kfw;
K(9,7) = e*kfw*E_fw;
K(9,8) = -e*kfw+f*k4+h*k5;
K(9,9) = (e^2)*kfw+(f^2)*k4+(h^2)*k5;
K(9,10) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(9,14) = -f*k4;
K(9,15) = -h*k5;

K(10,5) = -kfw*E_0;
K(10,6) = -i*kfw*E_0;
K(10,7) = -kfw*E_fw*E_0;
K(10,8) = kfw*E_0+k4*E_a4+k5*E_a5;
K(10,9) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(10,10) = kfw*E_0^2+k4*E_a4^2+k5*E_a5^2+EI_tlr*I4_tlr;
K(10,14) = -k4*E_a4;
K(10,15) = -k5*E_a5;

K(11,5) = -k1;
K(11,6) = a*k1;
K(11,7) = -k1*E_a1;
K(11,11) = k1+kt1;

K(12,5) = -k2;
K(12,6) = -b*k2;
K(12,7) = -k2*E_a2;
K(12,12) = k2+kt2;

K(13,5) = -k3;
K(13,6) = -d*k3;
K(13,7) = -k3*E_a3;
K(13,13) = k3+kt3;

K(14,8) = -k4;
K(14,9) = -f*k4;
K(14,10) = -k4*E_a4;
K(14,14) = k4+kt4;

K(15,8) = -k5;
K(15,9) = -h*k5;
K(15,10) = -k5*E_a5;
K(15,15) = k5+kt5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A = zeros(15,1);

A(11) = ct1;
A(12) = ct2;
A(13) = ct3;
A(14) = ct4;
A(15) = ct5;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
B = zeros(15,1);

B(11) = kt1;
B(12) = kt2;
B(13) = kt3;
B(14) = kt4;
B(15) = kt5;

% System "A" Matrix
AA=[zeros(size(M))   eye(size(M))   % System state variable
matrix
    -inv(M)*K       -inv(M)*C];

% ISO 2631 FOR REDUCED COMFORT BOUNDARY
% COMFORT BOUNDARIES FOR VERTICAL ACCELERATION
% THE ISO CENTRAL FREQUENCIES (Hz)

wc=[ .1 1 1.25 1.6 2 2.5 3.15 4 5 6.3 8 10 12.5 16 20 25 31.5 40
50];
whzc=[ .1 .125 .16 .2 .25 .315 .4 .5 .63 .8 1 1.25 1.6 2 2.5 3.15
...
    4 5 6.3 8 10 12.5 16 20 25 31.5 40 50];

% 2.5 hr FATIGUE BOUNDARY
fat1=[4.284,1.4,1.25,1.12,1,.9,.8,.71,.71,.71,.71,...
      .9,1.12,1.4,1.8,2.24,2.8,3.55,4.5];
% 2.5hr REDUCED COMFORT BOUNDARY
comf1=fat1/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf2= comf1/2.254;

%-----------------------------------------------------------------
---
% COMFORT BOUNDARIES FOR LONGITUDINAL AND LATERAL ACC
% 2.5hr FATIGUE BOUNDARY
fat2=[0.5,0.5,0.5,0.5,0.5,0.63,0.8,1,1.25,1.6,2,2.5,3.15,4,5,6.3,
8,10,12.5];

% 2.5hr REDUCED COMFORT BOUNDARY
comf3=fat2/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf4= comf3/2.254;
%-----------------------------------------------------------------
--

whzcr = 2*pi*whzc;      % Calculation of central frequencies in
rad/s
freqlow=0.89*whzcr;     % Lower octave band
freqhigh=1.12*whzcr;    % Upper octave band
freq=[freqlow' whzcr' freqhigh'];
```

```
imag=sqrt(-1);

for ii=1:length(whzc);
    for jj=1:3;         % jj=1 is freqlow, jj=2 is center freq
                        % jj=3 is freqhigh
        w = freq(ii,jj);
        s = imag*w;
        dp = sqrt(h1^2+r^2);

        % Time delay array
        time = [0 0 0 0 0 0 0 0 0 0 1 exp(-s*T(2)) exp(-s*T(3))
...
            exp(-s*T(4)) exp(-s*T(5))];

        % TF Matrix
        vectx = (inv(M*s*s+C*s+K)*((A*s+B).*(time.')));

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %%%  Transfer Functions  %%%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%

        z_s=[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]*vectx;    % vert seat
cg
        long=[0 0 -h1 0 0 0 0 0 0 0 0 0 0 0 0]*vectx; % long disp
of driver
        z_tlr=[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]*vectx;  % vert
trailer cg

        %%%%%%%%%%%%%%%%%%%%
        %%%  Magnitudes  %%%
        %%%%%%%%%%%%%%%%%%%%

        % Acceleration Transter Functions
        magcfA1(ii,jj)=abs(s*s*z_s);  % Mag of trans function,
(m/s*s)/m
        magcfAlong(ii,jj)=abs(s*s*long);
        magcftlr(ii,jj)=abs(s*s*z_tlr);

        %%%%%%%%%%%%%%
        %%%  PSDs  %%%
        %%%%%%%%%%%%%%

        % Road PSD in m*m/(rad/s)
        rpsd(ii,jj)=Csp*(((2*pi*v)^(N-1))/(w^N));

        % Acceleration PSDs in (m/s^2)^2/(rad/s)
        psdcfA1(ii,jj)=magcfA1(ii,jj)*magcfA1(ii,jj)*rpsd(ii,jj);

psdcfAlong(ii,jj)=magcfAlong(ii,jj)*magcfAlong(ii,jj)*rpsd(ii,jj)
;

psdcftlr(ii,jj)=magcftlr(ii,jj)*magcftlr(ii,jj)*rpsd(ii,jj);

    end
end
```

```
for kk=1:length(whzc)
    % Vert. Driver's Seat RMS
    msqy1a(kk)=0.5*(psdcfA1(kk,1)+psdcfA1(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqy1b(kk)=0.5*(psdcfA1(kk,2)+psdcfA1(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqy1(kk)=msqy1a(kk)+msqy1b(kk);
    rmsA1cf(kk)=sqrt(msqy1(kk));
    % Long. Driver RMS
    msqylonga(kk)=0.5*(psdcfAlong(kk,1)+psdcfAlong(kk,2))*...
    (freq(kk,2)-freq(kk,1));
    msqylongb(kk)=0.5*(psdcfAlong(kk,2)+psdcfAlong(kk,3))*...
    (freq(kk,3)-freq(kk,2));
    msqylong(kk)=msqylonga(kk)+msqylongb(kk);
    rmsAlongcf(kk)=sqrt(msqylong(kk));
    % Vert. Trailer cg RMS
    msqytlra(kk)=0.5*(psdcftlr(kk,1)+psdcftlr(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqytlrb(kk)=0.5*(psdcftlr(kk,2)+psdcftlr(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqytlr(kk)=msqytlra(kk)+msqytlrb(kk);
    rmstlrcf(kk)=sqrt(msqytlr(kk));
end

RMScf = [rmsA1cf',rmsAlongcf',rmstlrcf'];       % Accel. RMS
Matrix

% Calculate weighted rms acceleration from 0.1 to 50 Hz
% at the ISO Center Frequencies .... Wgt are the ISO weights
% Ref: ISO 2631-1:1997(E); V=vertical; L=longitudinal

wcc=[.1,.125,.16,.2,.25,.315,.4,.5,.63,.8,1,1.25,1.6,2,2.5,3.15,4
,5,...
    6.3,8,10,12.5,16,20,25,31.5,40,50];
WgtV=[.0312,.0486,.079,.121,.182,.263,.352,.418,.459,.477,.482,.4
84,...

.494,.531,.631,.804,.967,1.039,1.054,1.036,.988,.902,.768,.636,..
.
    .513,.405,.314,.246];
WgtL=0.001*[62.4,97.3,158,243,365,530,713,853,944,992,1011,1008,9
68,...

890,776,642,512,409,323,253,212,161,125,100,80,63.2,49.4,38.8];

isovert = WgtV.*RMScf(1:28,1)';      % Weighted Vert. Driver RMS
Accel.
isolong = WgtL.*RMScf(1:28,2)';      % Weighted Long. Driver RMS
Accel.
isotlr = WgtV.*RMScf(1:28,3)';       % Weighted Vert. Trailer RMS
Accel.

term2V=(WgtV.*rmsA1cf(1:28)).^2;
a0_V_dr=(sum(term2V))^0.5;              % a0 for vert. disp of
driver
```

```
term2L=(WgtL.*rmsAlongcf(1:28)).^2;
a0_L_dr=(sum(term2L))^0.5;             % a0 for long. disp of
driver

aV=(a0_L_dr^2 + a0_V_dr^2)^0.5;  % a0 for comb vert and long disp

tlrV=(WgtV.*rmstlrcf(1:28)).^2;
a0_V_tlr=(sum(tlrV))^0.5;              % a0 for vert. disp of
driver

aVV(iiii,jjjj)=aV;             % combined ISO wgt acc, m/s^2
a0_VV_tlr(iiii,jjjj)=a0_V_tlr;
Jpenalty(iiii,jjjj)=K_1*(aVV(iiii,jjjj)/0.44814)+K_2*...
(a0_VV_tlr(iiii,jjjj)/0.3239);

    end         % end of jjjj loop on kt2
end             % end of iiii loop on kt1

disp('  ')
disp('RESULTS OF PARAMETER VARIATION')
disp('  ')
disp('Minimum aV, m/s^2')
disp(min(aVV(:)))
disp('  ')
[ia,ja]=find(aVV==min(aVV(:)));
disp('Corresponding kt1, kt2, and kt3 values, N/m')
disp([867650+ktf(ia,ja) (3200190+ktr(ia,ja))*0.5
(3200190+ktr(ia,ja))*0.5])
disp('  ')

disp('  ')
disp('Minimum a0_V_tlr, m/s^2')
disp(min(a0_VV_tlr(:)))
disp('  ')
[it,jt]=find(a0_VV_tlr==min(a0_VV_tlr(:)));
disp('Corresponding kt1, kt2, and kt3 values, N/m')
disp([867650+ktf(it,jt) (3200190+ktr(it,jt))*0.5
(3200190+ktr(it,jt))*0.5])
disp('  ')

disp('  ')
disp('Minimum Jpenalty')
disp('J=K1*aV/0.44814 + K2*a0_V_tlr/0.3239')
disp('       K1         K2')
disp([K_1 K_2])
disp('  ')
disp(min(Jpenalty(:)))
disp('  ')
[iJ,jJ]=find(Jpenalty==min(Jpenalty(:)));
disp('Corresponding kt1, kt2, and kt3 values, N/m')
disp([867650+ktf(iJ,jJ) (3200190+ktr(iJ,jJ))*0.5
(3200190+ktr(iJ,jJ))*0.5])
disp('  ')
```

```
figure(1)
surf(867650+ktf,(3200190+ktr)/2,aVV)
xlabel('Steer Tire K, N/m')
ylabel('Single Drive Tire K, N/m')
zlabel('ISO Combined Wgt Acc, m/s^2')
% title('Tractor Tire Stiffness Parameter Variation')

figure(2)
surf(867650+ktf,(3200190+ktr)/2,a0_VV_tlr)
xlabel('Steer Tire K, N/m')
ylabel('Single Drive Tire K, N/m')
zlabel('Trailer Wgt Vert Acc, m/s^2')
% title('Tractor Tire Stiffness Parameter Variation')

figure(3)
surf(867650+ktf,(3200190+ktr)/2,Jpenalty)
xlabel('Steer Tire K, N/m')
ylabel('Single Drive Tire K, N/m')
zlabel('Penalty Function')
title(['K1 = ', num2str(K_1),'     K2 = ', num2str(K_2)])
```

Appendix J: opt_tireC_freq.m

This parameter variation program varies the damping of the steer axle tires and the damping of the first and second drive axle tires combined. Each of the drive axle tires on the tractor are assumed to have the same value, so they were combined into one value that was varied, and the individual drive axle tire values were assumed to be equal to exactly half of that value. The steer tires were varied from 361.9 N/(m/s) to 672.1 N/(m/s) in increments of 15.51 N/(m/s). This forms a vector with a length of 21 values that ranges from 30% below to 30% above the nominal value for the steer tire damping. The drive axle tires were varied from 453.81 N/(m/s) to 842.79 N/(m/s) in increments of 19.45 N/(m/s). Like the steer tires, this forms a vector with a length of 21 values that ranges from 30% below to 30% above the nominal value for the drive tire damping.

The desired output values from this program were the ISO combined weighted acceleration at the driver's seat, the ISO vertical weighted acceleration at the trailer center-of-gravity (CG), and a value called the J penalty, which weighs the importance of the driver ride comfort versus trailer acceleration using weights assigned to them by the user. The program finds the minimum values for each of these outputs, and displays them in tabular form along with the corresponding damping values for the steer and drive tires. Also, the program plots the output information on surface plots to study trends in the information.

opt_tireC_freq.m

```
% opt_tireC_freq.m
% Developed by Ryan Spivey, 4/10/07
%
% Varies tire damping using weighted RMS acceleration in the
% frequecncy domain
%
% Incorporates model from dof15_freq2.m
%
% DOFs include - 1)Vertical Disp. of Driver's Seat
%                2)Vertical Disp. of Cab
%                3)Pitch of Cab
%                4)Vertical Disp. of Engine
%                5)Vertical Disp. of Tractor Frame
%                6)Pitch of Tractor Frame
%                7)Beaming of Tractor Frame
%                8)Vertical Disp. of Trailer
%                9)Pitch of Trailer
%               10)Beaming of Trailer
%               11)Vertical Disp. of Axle #1
%               12)Vertical Disp. of Axle #2
%               13)Vertical Disp. of Axle #3
%               14)Vertical Disp. of Axle #4
%               15)Vertical Disp. of Axle #5

clc
clear all
close all
format short e
format compact

global D1_t D2_t D3_t D4_t D1_tlr D2_tlr D3_tlr D4_tlr
global e a1 kb1 kb2 b_fw L_tlr alpha

disp('  ')
disp('Tire Damping Parameter Variation in the Frequency Domain')
disp('                  Roadholding Model                    ')
disp(['                     ',date])

% Choose a test vehicle
disp('  ')
disp('VEHICLE SELECTION')
disp('  ')
disp('Please choose a vehicle : ');
disp('a: Ideal Tractor Semi-Trailer');
vehicle = input('Enter your choice : ', 's');

if vehicle == 'a'
    % Inertial Properties
    m_s = 106.7;        %kg      mass of seat
    m_c = 1208;         %kg      mass of cab
    I_c = 2100;         %kg*m^2  M I of cab
    m_e = 2000;         %kg      mass of engine (ESTIMATE)
```

```
    m_t = 3783;          %kg      mass of tractor (5783 kg -
engine)
    I_t = 46590.9;       %kg*m^2  M I of tractor
    m_ul = 10800;        %kg      mass of trailer (ESTIMATE)
    I_tlr = 200000;      %kg*m^2  M I of trailer
    m_L = 14000;         %kg      mass of trailer load  (ESTIMATE)
    m_tlr = m_ul+m_L;    %kb      mass of loaded trailer

    % Suspension Parameters
    c1 = 11270;          %N/(m/s) damping const of axle #1
    c2 = 27500;          %N/(m/s) damping const of axle #2
    c3 = 27500;          %N/(m/s) damping const of axle #3
    c4 = 70000;          %N/(m/s) damping const of axle #4
    c5 = 70000;          %N/(m/s) damping const of axle #5
    ce = 10000;          %N/(m/s) damping const of engine mount
    k1 = 581300;         %N/m     spring const of axle #1
    k2 = 586900;         %N/m     spring const of axle #2
    k3 = 586900;         %N/m     spring const of axle #3
    k4 = 1000000;        %N/m     spring const of axle #4
    k5 = 1000000;        %N/m     spring const of axle #5
    ke = 1e10;           %N/m     spring const of the engine mount

    % Model Dimensions
    b_a1 = 1.065;        %m       Front end of the tractor to axle
#1
    b_cf = 1.470;        %m       Front end of the tractor to cab
front
    b_e = 2.797;         %m       Front end of the tractor to
engine
    b_cr = 4.02;         %m       Front end of the tractor to cab
rear
    b_a2 = 6.035;        %m       Front end of the tractor to axle
#2
    b_fw = 6.688;        %m       Front end of the tractor to 5th
wheel
    b_a3 = 7.34;         %m       Front end of the tractor to axle
#3
    a1 = 4.00607;        %m       Front end of the tractor to
tractor cg

    b_a4 = 8.58;         %m       From the fifth wheel to axle #4
    b_a5 = 9.78;         %m       From the fifth wheel to axle #5

    L_t = 8.2;           %m       Length of Tractor
    L_tlr = 9.78;        %m       Length of Trailer

    e = 5.62;            %m       From the trailer cg to fifth
wheel
    f = 2.96;            %m       From the trailer cg to axle #4
    h = 4.16;            %m       From the trailer cg to axle #5

    a = 2.94107;         %m       From the tractor cg to axle #1
    b = 2.02893;         %m       From the tractor cg to axle #2
    d = 3.33393;         %m       From the tractor cg to axle #3
    l = 2.53607;         %m       From the tractor cg to cab front
    m = 1.209074;        %m       From the tractor cg to engine
```

```
    j = 0.013926;       %m       From the tractor cg to cab rear
    i = 2.68193;        %m       From the tractor cg to the fifth
wheel
    n = 1.435;          %m       From the cab cg to cab front
    p = 1.115;          %m       From the cab cg to cab rear
    r = -0.200;         %m       From the cab cg to seat

    tc = 1.10107;       %m       From the tractor cg to the cab
cg
    h1 = 1.0;           %m       Height of the driver over the
cab
    g = 9.8;            %m/s^2   acceleration due to gravity

    ML_t = m_t/L_t;         %kg/m    Mass per unit length
(Tractor)
    ML_tlr = m_ul/L_tlr;    %kg/m    Mass per unit length
(Trailer)
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%  Fifth Wheel Configuration
%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

disp('  ')
disp('Give your choice for the fifth wheel configuration: ')
disp('Note: If a fifth wheel suspension system is chosen, the
beaming of')
disp('      the tractor frame and trailer will be modeled as
free-free.  If')
disp('      no suspension is chosen, the tractor frame and
trailer will be')
disp('      modeled as free-pinned and pinned-free
respectively.')
disp('a : With fifth wheel suspension')
disp('b : Without fifth wheel suspension')
z33 = input('Please give your choice : ', 's');

if z33 == 'a',          % Choice 'a' is with fifth wheel
suspension
    disp('  ')
    kfw = input('Input the fifth wheel spring constant (N/m): ');
    disp('  ')
    cfw = input('Input the fifth wheel damping ratio (N/(m/s)):
');

    % The parameters for the first bending mode of the Tractor
frame
    disp('  ')
    fhz = input('Input the Tractor frequency of beaming (hz) fhz
: ');
```

```
    % The parameters for the first bending mode of the Trailer
frame
    disp('  ')
    fhz2 = input('Input the Trailer frequency of beaming (hz) fhz
: ');

    kb1 = 4.73004074;    %Constant for the first bending mode
(free-free)
    alpha = 0.982502;

    z1 = 'cosh(kb1*x1/b_fw) + cos(kb1*x1/b_fw) - ...
alpha*(sinh(kb1*x1/b_fw)+sin(kb1*x1/b_fw))';
    % free-free beam mode function
    z1dd = '(kb1/b_fw)^2*(cosh(kb1*x1/b_fw) - cos(kb1*x1/b_fw) -
... alpha*(sinh(kb1*x1/b_fw)-sin(kb1*x1/b_fw)))';
    % second derivative of free-free beam mode function

    kb2 = 4.73004074;    %Constant for the first bending mode
(free-free)

    z2 = 'cosh(kb2*x2/L_tlr) + cos(kb2*x2/L_tlr) - ...
alpha*(sinh(kb2*x2/L_tlr)+sin(kb2*x2/L_tlr))';
    % free-free beam mode function
    z2dd = '(kb2/L_tlr)^2*(cosh(kb2*x2/L_tlr) - cos(kb2*x2/L_tlr)
- ... alpha*(sinh(kb2*x2/L_tlr)-sin(kb2*x2/L_tlr)))';
    % second derivative of free-free beam mode function

elseif z33 == 'b',          % Choice 'b' is without fifth wheel
suspension
    kfw = 1000000000000;    %(N/m)      fifth wheel spring
constant
    cfw = 1000;             %(N/(m/s))  fifth wheel damping ratio

    % The parameters for the first bending mode of the Tractor
frame
    disp('  ')
    fhz = input('Input the Tractor frequency of beaming (hz) fhz
: ');

    % The parameters for the first bending mode of the Trailer
frame
    disp('  ')
    fhz2 = input('Input the Trailer frequency of beaming (hz) fhz
: ');

    kb1 = 2.36502;      % Constant for the first bending mode
(free-pinned)
    %                     (from Rao pg. 527)

    z1 = '(cos(kb1*x1/b_fw) + (cosh(kb1*x1/b_fw)) - ...
((cos(kb1)+cosh(kb1))/(sin(kb1)-sinh(kb1)))*(sin(kb1*x1/b_fw)-
...  sinh(kb1*x1/b_fw)))';
    % free-pinned beam mode function
```

```
    z1dd = '((kb1/b_fw)^2)*(-cos(kb1*x1/b_fw) +
(cosh(kb1*x1/b_fw)) - ... ((cos(kb1)+cosh(kb1))/(sin(kb1)-
sinh(kb1)))*(-sin(kb1*x1/b_fw)- ...  sinh(kb1*x1/b_fw)))';
    % second derivative of free-pinned beam mode function

    kb2 = 3.926602;     % Constant for the first bending mode
(pinned-free)
    %                     (from Rao pg. 527)

    z2 = '(sin(kb2*x2/L_tlr) + ...
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
    % pinned-free beam mode function
    z2dd = '(kb2/L_tlr)^2*(-sin(kb2*x2/L_tlr) + ...
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
    % second derivative of pinned-free beam mode function

else disp('Insufficient information regarding fifth wheel
suspension.')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%%%%%%%%%%%%%%%%%  Computation of Integrals
%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

D1_t=['(',z1,')'];                   % Tractor frame beaming
equations to be
D2_t=['((a1-x1).*(',z1,'))'];        % used in the integrals
(string form)
D3_t=['((',z1,').*(',z1,'))'];
D4_t=['((',z1dd,').*(',z1dd,'))'];

D1_tlr=['(',z2,')'];                 % Trailer beaming equations
to be
D2_tlr=['((e-x2).*(',z2,'))'];       % used in the integrals
(string form)
D3_tlr=['((',z2,').*(',z2,'))'];
D4_tlr=['((',z2dd,').*(',z2dd,'))'];

I1_t=quadl('modeD1_t',0,b_fw);       % Integrals of functions
defined above
I2_t=quadl('modeD2_t',0,b_fw);       % (along length of tractor
frame)
I3_t=quadl('modeD3_t',0,b_fw);
I4_t=quadl('modeD4_t',0,b_fw);

I1_tlr=quadl('modeD1_tlr',0,L_tlr); % Integrals of functions
defined above
I2_tlr=quadl('modeD2_tlr',0,L_tlr); % (along length of trailer)
I3_tlr=quadl('modeD3_tlr',0,L_tlr);
I4_tlr=quadl('modeD4_tlr',0,L_tlr);
```

```
E_a1=modeD1_t(b_a1);      % Disp at axle #1 due to tractor frame
beaming
E_cf=modeD1_t(b_cf);      % Disp at cab front due to tractor
frame beaming
E_e=modeD1_t(b_e);        % Disp at engine due to tractor frame
beaming
E_cr=modeD1_t(b_cr);      % Disp at cab rear due to tractor frame
beaming
E_a2=modeD1_t(b_a2);      % Disp at axle #2 due to tractor frame
beaming
E_fw=modeD1_t(b_fw);      % Disp at fifth wheel due to tractor
frame beaming
E_a3=modeD1_t(b_a3);      % Disp at axle #3 due to tractor frame
beaming
E_0=modeD1_tlr(0);        % Disp at fifth wheel due to trailer
beaming
E_a4=modeD1_tlr(b_a4);    % Disp at axle #4 due to trailer
beaming
E_a5=modeD1_tlr(b_a5);    % Disp at axle #5 due to trailer
beaming

EI_t = 4*pi^2*fhz^2*(b_fw/kb1)^4*ML_t;       %Tractor frame
flexural rigidity
EI_tlr = 4*pi^2*fhz2^2*(L_tlr/kb2)^4*ML_tlr; %Trailer flexural
rigidity

% Seat Suspension Options
disp('  ')
disp('VEHICLE SUSPENSION OPTIONS')
disp('  ')
disp('Give your choice for seat suspension: ')
disp('Note: Without seat suspension gives a very high frequency
mode')
disp('      because the stiffness is set to a high value.')
disp('a : With seat suspension (~0.9 Hz)')
disp('b : Without seat suspension')
z11 = input('Enter your choice : ', 's');

if z11 == 'a',          % Choice 'a' is with seat suspension
    cs = 1140;          % Damping ratio of 0.5
    ks = 3403;          % N/m(spring const of seat suspension)

elseif z11 == 'b',      % Choice 'b' is without seat suspension
    cs = 1329;          % N/(m/s)(damping const of seat
suspension)
    ks = 1e10;          % N/m(spring const of seat suspension)

else disp('Insufficient information regarding seat suspension.')
end

% Cab Suspension Options
disp('  ')
disp('Give your choice for cab suspension: ')
disp('Note: With front or rear or without cab suspension')
disp('      gives a very high frequency mode(s) because the
corresponding')
```

```
disp('      stiffness(es)is set to a high value.')
disp('a : With front cab suspension')
disp('b : With rear cab suspension')
disp('c : With front & rear cab suspension')
disp('d : Without cab suspension')
z22 = input('Enter your choice : ', 's');

if z22 == 'a',          % Choice 'a' is front cab suspension
    ccf = 7062;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 88740;        % N/m(spring const of front cab
suspension)
    ccr = 6430;         % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;         % N/m(spring const of rear cab
suspension)

elseif z22 == 'b',      % Choice 'b' is rear cab suspension
    ccr = 8000;         % Reduced damping
    kcr = 65980;        % N/m(spring const of rear cab
suspension)
    ccf = 13120;        % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;         % N/m(spring const of front cab
suspension)

elseif z22 == 'c',      % Choice 'c' is front & rear cab
suspension
    ccr = 5073.5;       % N/(m/s)(damping const of rear cab
suspension)
    kcr = 63757.5;      % N/m(spring const of rear cab
suspension)
    ccf = 6864.35;      % N/(m/s)(damping const of front cab
suspension)
    kcf = 86260.5;      % N/m(spring const of front cab
suspension)

elseif z22 == 'd',      % Choice 'd' is without cab suspension
    ccr = 6430;         % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;         % N/m(spring const of rear cab
suspension)
    ccf = 7062;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;         % N/m(spring const of front cab
suspension)

else disp('Insufficient information regarding cab suspension.')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%  Vehicle Tire Selection
%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
```

```
% Values shown represent Wide Singles

mt1 = 374;       % kg    Mass of Steer Axle
mt2 = 648.3;     % kg    Mass of Drive Axle #1
mt3 = 648.3;     % kg    Mass of Drive Axle #2
mt4 = 648.3;     % kg    Mass of Trailer Axle #1
mt5 = 648.3;     % kg    Mass of Trailer Axle #2

ct4 = 648.3;     % N/(m/s)   Trailer Axle #1 Damping Const.
ct5 = 648.3;     % N/(m/s)   Trailer Axle #2 Damping Const.

kt1 = 1.295e6;  % N/m   Steer Axle Spring Const.
kt2 = 2.3882e6; % N/m   Drive Axle #1 Spring Const.
kt3 = 2.3882e6; % N/m   Drive Axle #2 Spring Const.
kt4 = 2.3882e6; % N/m   Trailer Axle #1 Spring Const.
kt5 = 2.3882e6; % N/m   Trailer Axle #2 Spring Const.

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%  Speed of the Vehicle  %%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('VEHICLE VELOCITY')
disp('  ')
disp('Please choose the unit of velocity');
disp('a : Miles per Hour (mph)');
disp('b : Kilometers per Hour (kph)');
vel = input('Input the unit of velocity (a/b):  ', 's');
disp('  ')
vm = input('Input the velocity of the vehicle, vm : ');

if vel == 'a'
    v = 0.4473*vm;              %Velocity conversion from mph to
m/s
elseif vel == 'b'
    v = 0.277778*vm;            %Velocity conversion from kph to
m/s
end

T(1) = 0;               %Time delay between front axle and
remaining axles
T(2) = (a+b)/v;         % Axle #2
T(3) = (a+d)/v;         % Axle #3
T(4) = (a+i+e+f)/v;     % Axle #4
T(5) = (a+i+e+h)/v;     % Axle #5

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%  Road PSD Selection  %%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('ROAD PSD SELECTION')
disp('  ')
```

```
disp('Road PSD Constants, m^2/cyc/m, Ref: Wong, Theory of Ground
Vehicles')
disp('S(W)=Csp/W^N  where W=spatial frequency')
disp('  ')
disp('a : Csp = 4.3e-11,N=3.8     Smooth Runway')
disp('b : Csp = 8.1e-6, N=2.1     Rough Runway')
disp('c : Csp = 4.8e-7, N=2.1     Smooth Highway')
disp('d : Csp = 4.4e-6, N=2.1     Highway with Gravel')
disp('  ')
tabchoice11=input('Input the road surface to be used :   ','s');

if tabchoice11== 'a',              % smooth runway
   Csp = 4.3e-11;
   N=3.8;

   elseif tabchoice11== 'b',        % rough runway
      Csp = 8.1e-6;
      N=2.1;

   elseif tabchoice11 == 'c',       % smooth highway
      Csp = 4.8e-7;
      N=2.1;

   elseif tabchoice11 == 'd',       % highway with gravel
      Csp = 4.4e-6;
      N=2.1;

end

% Start Loop on Tire Damping Properties
% Damping values will range from 70% to 130% of the nominal value

for iiii=1:21;
    for jjjj=1:21;
        ctf(iiii,jjjj)=15.51*iiii;
        ctr(iiii,jjjj)=38.898*jjjj;

        ct1 = 346.39+ctf(iiii,jjjj);
        ct2 = (868.722+ctr(iiii,jjjj))*0.5;
        ct3 = (868.722+ctr(iiii,jjjj))*0.5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%  System Matrices
%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

% THE SYSTEM IS WRITTEN AS (M*S*S+C*S+K)X(S)=(A*S+B)U(S)

%%%%%%%%%%%%%%%%%%%%%%
%%%%  Mass Matrix  %%%%
%%%%%%%%%%%%%%%%%%%%%%
M = zeros(15,15);
```

```
M(1,1) = m_s;                     % Eqn #1: Vertical Disp of Seat

M(2,2) = m_c;                     % Eqn #2: Vertical Disp of Cab

M(3,3) = I_c;                     % Eqn #3: Pitch of Cab

M(4,4) = m_e;                     % Eqn #4: Vertical Disp of Engine

M(5,5) = m_t;                     % Eqn #5: Vertical Disp of
Tractor Frame
M(5,6) = ML_t*b_fw*(b_fw/2-a1);
M(5,7) = ML_t*I1_t;

M(6,5) = ML_t*b_fw*(b_fw/2-a1);   % Eqn #6: Pitch of Tractor
Frame
M(6,6) = I_t;
M(6,7) = -ML_t*I2_t;

M(7,5) = ML_t*I1_t;               % Eqn #7: Beaming of Tractor
Frame
M(7,6) = -ML_t*I2_t;
M(7,7) = ML_t*I3_t;

M(8,8) = m_tlr;                   % Eqn #8: Vertical Disp of
Trailer
M(8,9) = -ML_tlr*L_tlr*(e-L_tlr/2);
M(8,10) = ML_tlr*I1_tlr;

M(9,8) = -ML_tlr*L_tlr*(e-L_tlr/2);% Eqn #9: Pitch of Trailer
M(9,9) = I_tlr;
M(9,10) = -ML_tlr*I2_tlr;

M(10,8) = ML_tlr*I1_tlr;          % Eqn #10: Beaming of Trailer
M(10,9) = -ML_tlr*I2_tlr;
M(10,10) = ML_tlr*I3_tlr;

M(11,11) = mt1;                   % Eqn #11: Vertical Disp of Axle
#1

M(12,12) = mt2;                   % Eqn #12: Vertical Disp of Axle
#2

M(13,13) = mt3;                   % Eqn #13: Vertical Disp of Axle
#3

M(14,14) = mt4;                   % Eqn #14: Vertical Disp of Axle
#4

M(15,15) = mt5;                   % Eqn #15: Vertical Disp of Axle
#5

%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Damping Matrix  %%%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%
C = zeros(15,15);

C(1,1) = cs;
C(1,2) = -cs;
C(1,3) = r*cs;

C(2,1) = -cs;
C(2,2) = cs+ccf+ccr;
C(2,3) = -r*cs-n*ccf+p*ccr;
C(2,5) = -ccf-ccr;
C(2,6) = l*ccf-j*ccr;
C(2,7) = -ccf*E_cf-ccr*E_cr;

C(3,1) = r*cs;
C(3,2) = -r*cs-n*ccf+p*ccr;
C(3,3) = (r^2)*cs+(n^2)*ccf+(p^2)*ccr;
C(3,5) = n*ccf-p*ccr;
C(3,6) = -n*l*ccf-p*j*ccr;
C(3,7) = n*ccf*E_cf-p*ccr*E_cr;

C(4,4) = ce;
C(4,5) = -ce;
C(4,6) = m*ce;
C(4,7) = -ce*E_e;

C(5,2) = -ccf-ccr;
C(5,3) = n*ccf-p*ccr;
C(5,4) = -ce;
C(5,5) = ce+ccf+ccr+cfw+c1+c2+c3;
C(5,6) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(5,7) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(5,8) = -cfw;
C(5,9) = e*cfw;
C(5,10) = -cfw*E_0;
C(5,11) = -c1;
C(5,12) = -c2;
C(5,13) = -c3;

C(6,2) = l*ccf-j*ccr;
C(6,3) = -n*l*ccf-p*j*ccr;
C(6,4) = m*ce;
C(6,5) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(6,6) =
(m^2)*ce+(l^2)*ccf+(j^2)*ccr+(i^2)*cfw+(a^2)*c1+(b^2)*c2+(d^2)*c3
;
C(6,7) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2+...
         d*c3*E_a3;
C(6,8) = -i*cfw;
C(6,9) = e*i*cfw;
C(6,10) = -i*cfw*E_0;
C(6,11) = a*c1;
C(6,12) = -b*c2;
C(6,13) = -d*c3;
```

```
C(7,2) = -ccf*E_cf-ccr*E_cr;
C(7,3) = n*ccf*E_cf-p*ccr*E_cr;
C(7,4) = -ce*E_e;
C(7,5) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(7,6) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2 ...
         +d*c3*E_a3;
C(7,7) =
ce*E_e^2+ccf*E_cf^2+ccr*E_cr^2+cfw*E_fw^2+c1*E_a1^2+c2*E_a2^2 ...
         +c3*E_a3^2;
C(7,8) = -cfw*E_fw;
C(7,9) = e*cfw*E_fw;
C(7,10) = -cfw*E_0*E_fw;
C(7,11) = -c1*E_a1;
C(7,12) = -c2*E_a2;
C(7,13) = -c3*E_a3;

C(8,5) = -cfw;
C(8,6) = -i*cfw;
C(8,7) = -cfw*E_fw;
C(8,8) = cfw+c4+c5;
C(8,9) = -e*cfw+f*c4+h*c5;
C(8,10) = cfw*E_0+c4*E_a4+c5*E_a5;
C(8,14) = -c4;
C(8,15) = -c5;

C(9,5) = e*cfw;
C(9,6) = e*i*cfw;
C(9,7) = e*cfw*E_fw;
C(9,8) = -e*cfw+f*c4+h*c5;
C(9,9) = (e^2)*cfw+(f^2)*c4+(h^2)*c5;
C(9,10) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(9,14) = -f*c4;
C(9,15) = -h*c5;

C(10,5) = -cfw*E_0;
C(10,6) = -i*cfw*E_0;
C(10,7) = -cfw*E_fw*E_0;
C(10,8) = cfw*E_0+c4*E_a4+c5*E_a5;
C(10,9) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(10,10) = cfw*E_0^2+c4*E_a4^2+c5*E_a5^2;
C(10,14) = -c4*E_a4;
C(10,15) = -c5*E_a5;

C(11,5) = -c1;
C(11,6) = a*c1;
C(11,7) = -c1*E_a1;
C(11,11) = c1+ct1;

C(12,5) = -c2;
C(12,6) = -b*c2;
C(12,7) = -c2*E_a2;
C(12,12) = c2+ct2;
```

```
C(13,5) = -c3;
C(13,6) = -d*c3;
C(13,7) = -c3*E_a3;
C(13,13) = c3+ct3;

C(14,8) = -c4;
C(14,9) = -f*c4;
C(14,10) = -c4*E_a4;
C(14,14) = c4+ct4;

C(15,8) = -c5;
C(15,9) = -h*c5;
C(15,10) = -c5*E_a5;
C(15,15) = c5+ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%
K = zeros(15,15);

K(1,1) = ks;
K(1,2) = -ks;
K(1,3) = r*ks;

K(2,1) = -ks;
K(2,2) = ks+kcf+kcr;
K(2,3) = -r*ks-n*kcf+p*kcr;
K(2,5) = -kcf-kcr;
K(2,6) = l*kcf-j*kcr;
K(2,7) = -kcf*E_cf-kcr*E_cr;

K(3,1) = r*ks;
K(3,2) = -r*ks-n*kcf+p*kcr;
K(3,3) = (r^2)*ks+(n^2)*kcf+(p^2)*kcr;
K(3,5) = n*kcf-p*kcr;
K(3,6) = -n*l*kcf-p*j*kcr;
K(3,7) = n*kcf*E_cf-p*kcr*E_cr;

K(4,4) = ke;
K(4,5) = -ke;
K(4,6) = m*ke;
K(4,7) = -ke*E_e;

K(5,2) = -kcf-kcr;
K(5,3) = n*kcf-p*kcr;
K(5,4) = -ke;
K(5,5) = ke+kcf+kcr+kfw+k1+k2+k3;
K(5,6) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(5,7) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(5,8) = -kfw;
K(5,9) = e*kfw;
K(5,10) = -kfw*E_0;
K(5,11) = -k1;
```

```
K(5,12) = -k2;
K(5,13) = -k3;

K(6,2) = l*kcf-j*kcr;
K(6,3) = -n*l*kcf-p*j*kcr;
K(6,4) = m*ke;
K(6,5) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(6,6) =
(m^2)*ke+(l^2)*kcf+(j^2)*kcr+(i^2)*kfw+(a^2)*k1+(b^2)*k2+(d^2)*k3
;
K(6,7) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2+...
         d*k3*E_a3;
K(6,8) = -i*kfw;
K(6,9) = e*i*kfw;
K(6,10) = -i*kfw*E_0;
K(6,11) = a*k1;
K(6,12) = -b*k2;
K(6,13) = -d*k3;

K(7,2) = -kcf*E_cf-kcr*E_cr;
K(7,3) = n*kcf*E_cf-p*kcr*E_cr;
K(7,4) = -ke*E_e;
K(7,5) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(7,6) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2 ...
         +d*k3*E_a3;
K(7,7) =
ke*E_e^2+kcf*E_cf^2+kcr*E_cr^2+kfw*E_fw^2+k1*E_a1^2+k2*E_a2^2 ...
         +k3*E_a3^2+EI_t*I4_t;
K(7,8) = -kfw*E_fw;
K(7,9) = e*kfw*E_fw;
K(7,10) = -kfw*E_0*E_fw;
K(7,11) = -k1*E_a1;
K(7,12) = -k2*E_a2;
K(7,13) = -k3*E_a3;

K(8,5) = -kfw;
K(8,6) = -i*kfw;
K(8,7) = -kfw*E_fw;
K(8,8) = kfw+k4+k5;
K(8,9) = -e*kfw+f*k4+h*k5;
K(8,10) = kfw*E_0+k4*E_a4+k5*E_a5;
K(8,14) = -k4;
K(8,15) = -k5;

K(9,5) = e*kfw;
K(9,6) = e*i*kfw;
K(9,7) = e*kfw*E_fw;
K(9,8) = -e*kfw+f*k4+h*k5;
K(9,9) = (e^2)*kfw+(f^2)*k4+(h^2)*k5;
K(9,10) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(9,14) = -f*k4;
K(9,15) = -h*k5;
```

```
K(10,5) = -kfw*E_0;
K(10,6) = -i*kfw*E_0;
K(10,7) = -kfw*E_fw*E_0;
K(10,8) = kfw*E_0+k4*E_a4+k5*E_a5;
K(10,9) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(10,10) = kfw*E_0^2+k4*E_a4^2+k5*E_a5^2+EI_tlr*I4_tlr;
K(10,14) = -k4*E_a4;
K(10,15) = -k5*E_a5;

K(11,5) = -k1;
K(11,6) = a*k1;
K(11,7) = -k1*E_a1;
K(11,11) = k1+kt1;

K(12,5) = -k2;
K(12,6) = -b*k2;
K(12,7) = -k2*E_a2;
K(12,12) = k2+kt2;

K(13,5) = -k3;
K(13,6) = -d*k3;
K(13,7) = -k3*E_a3;
K(13,13) = k3+kt3;

K(14,8) = -k4;
K(14,9) = -f*k4;
K(14,10) = -k4*E_a4;
K(14,14) = k4+kt4;

K(15,8) = -k5;
K(15,9) = -h*k5;
K(15,10) = -k5*E_a5;
K(15,15) = k5+kt5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A = zeros(15,1);

A(11) = ct1;
A(12) = ct2;
A(13) = ct3;
A(14) = ct4;
A(15) = ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
B = zeros(15,1);

B(11) = kt1;
B(12) = kt2;
B(13) = kt3;
B(14) = kt4;
B(15) = kt5;
```

```
% System "A" Matrix
AA=[zeros(size(M))   eye(size(M))   % System state variable
matrix
    -inv(M)*K       -inv(M)*C];

% ISO 2631 FOR REDUCED COMFORT BOUNDARY
% COMFORT BOUNDARIES FOR VERTICAL ACCELERATION
% THE ISO CENTRAL FREQUENCIES (Hz)

wc=[ .1 1 1.25 1.6 2 2.5 3.15 4 5 6.3 8 10 12.5 16 20 25 31.5 40
50];
whzc=[ .1 .125 .16 .2 .25 .315 .4 .5 .63 .8 1 1.25 1.6 2 2.5 3.15
...
    4 5 6.3 8 10 12.5 16 20 25 31.5 40 50];

% 2.5 hr FATIGUE BOUNDARY
fat1=[4.284,1.4,1.25,1.12,1,.9,.8,.71,.71,.71,.71,...
      .9,1.12,1.4,1.8,2.24,2.8,3.55,4.5];
% 2.5hr REDUCED COMFORT BOUNDARY
comf1=fat1/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf2= comf1/2.254;

%-----------------------------------------------------------------
---
% COMFORT BOUNDARIES FOR LONGITUDINAL AND LATERAL ACC
% 2.5hr FATIGUE BOUNDARY
fat2=[0.5,0.5,0.5,0.5,0.5,0.63,0.8,1,1.25,1.6,2,2.5,3.15,4,5,6.3,
8,10,12.5];

% 2.5hr REDUCED COMFORT BOUNDARY
comf3=fat2/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf4= comf3/2.254;
%-----------------------------------------------------------------
--

whzcr = 2*pi*whzc;      % Calculation of central frequencies in
rad/s
freqlow=0.89*whzcr;     % Lower octave band
freqhigh=1.12*whzcr;    % Upper octave band
freq=[freqlow' whzcr' freqhigh'];

imag=sqrt(-1);

for ii=1:length(whzc);
    for jj=1:3;         % jj=1 is freqlow, jj=2 is center freq
                        % jj=3 is freqhigh
        w = freq(ii,jj);
        s = imag*w;
        dp = sqrt(h1^2+r^2);

        % Time delay array
```

```
        time = [0 0 0 0 0 0 0 0 0 0 1 exp(-s*T(2)) exp(-s*T(3))
...
            exp(-s*T(4)) exp(-s*T(5))];

        % TF Matrix
        vectx = (inv(M*s*s+C*s+K)*((A*s+B).*(time.')));

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %%%  Transfer Functions  %%%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%

        z_s=[1 0 0 0 0 0 0 0 0 0 0 0 0 0]*vectx;    % vert seat
cg
        long=[0 0 -h1 0 0 0 0 0 0 0 0 0 0 0]*vectx; % long disp
of driver
        z_tlr=[0 0 0 0 0 0 1 0 0 0 0 0 0 0]*vectx;  % vert
trailer cg

        %%%%%%%%%%%%%%%%%%%%
        %%%  Magnitudes  %%%
        %%%%%%%%%%%%%%%%%%%%

        % Acceleration Transter Functions
        magcfA1(ii,jj)=abs(s*s*z_s);  % Mag of trans function,
(m/s*s)/m
        magcfAlong(ii,jj)=abs(s*s*long);
        magcftlr(ii,jj)=abs(s*s*z_tlr);

        %%%%%%%%%%%%%%
        %%%  PSDs  %%%
        %%%%%%%%%%%%%%

        % Road PSD in m*m/(rad/s)
        rpsd(ii,jj)=Csp*(((2*pi*v)^(N-1))/(w^N));

        % Acceleration PSDs in (m/s^2)^2/(rad/s)
        psdcfA1(ii,jj)=magcfA1(ii,jj)*magcfA1(ii,jj)*rpsd(ii,jj);

psdcfAlong(ii,jj)=magcfAlong(ii,jj)*magcfAlong(ii,jj)*rpsd(ii,jj)
;

psdcftlr(ii,jj)=magcftlr(ii,jj)*magcftlr(ii,jj)*rpsd(ii,jj);

    end
end

for kk=1:length(whzc)
    % Vert. Driver's Seat RMS
    msqy1a(kk)=0.5*(psdcfA1(kk,1)+psdcfA1(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqy1b(kk)=0.5*(psdcfA1(kk,2)+psdcfA1(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqy1(kk)=msqy1a(kk)+msqy1b(kk);
    rmsA1cf(kk)=sqrt(msqy1(kk));
    % Long. Driver RMS
```

```
    msqylonga(kk)=0.5*(psdcfAlong(kk,1)+psdcfAlong(kk,2))*...
    (freq(kk,2)-freq(kk,1));
    msqylongb(kk)=0.5*(psdcfAlong(kk,2)+psdcfAlong(kk,3))*...
    (freq(kk,3)-freq(kk,2));
    msqylong(kk)=msqylonga(kk)+msqylongb(kk);
    rmsAlongcf(kk)=sqrt(msqylong(kk));
    % Vert. Trailer cg RMS
    msqytlra(kk)=0.5*(psdcftlr(kk,1)+psdcftlr(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqytlrb(kk)=0.5*(psdcftlr(kk,2)+psdcftlr(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqytlr(kk)=msqytlra(kk)+msqytlrb(kk);
    rmstlrcf(kk)=sqrt(msqytlr(kk));
end

RMScf = [rmsA1cf',rmsAlongcf',rmstlrcf'];       % Accel. RMS
Matrix

% Calculate weighted rms acceleration from 0.1 to 50 Hz
% at the ISO Center Frequencies .... Wgt are the ISO weights
% Ref: ISO 2631-1:1997(E); V=vertical; L=longitudinal

wcc=[.1,.125,.16,.2,.25,.315,.4,.5,.63,.8,1,1.25,1.6,2,2.5,3.15,4
,5,...
    6.3,8,10,12.5,16,20,25,31.5,40,50];
WgtV=[.0312,.0486,.079,.121,.182,.263,.352,.418,.459,.477,.482,.4
84,...

.494,.531,.631,.804,.967,1.039,1.054,1.036,.988,.902,.768,.636,..
.
    .513,.405,.314,.246];
WgtL=0.001*[62.4,97.3,158,243,365,530,713,853,944,992,1011,1008,9
68,...

890,776,642,512,409,323,253,212,161,125,100,80,63.2,49.4,38.8];

isovert = WgtV.*RMScf(1:28,1)';      % Weighted Vert. Driver RMS
Accel.
isolong = WgtL.*RMScf(1:28,2)';      % Weighted Long. Driver RMS
Accel.
isotlr = WgtV.*RMScf(1:28,3)';       % Weighted Vert. Trailer RMS
Accel.

term2V=(WgtV.*rmsA1cf(1:28)).^2;
a0_V_dr=(sum(term2V))^0.5;              % a0 for vert. disp of
driver

term2L=(WgtL.*rmsAlongcf(1:28)).^2;
a0_L_dr=(sum(term2L))^0.5;              % a0 for long. disp of
driver

aV=(a0_L_dr^2 + a0_V_dr^2)^0.5;      % a0 for comb vert and long
disp

tlrV=(WgtV.*rmstlrcf(1:28)).^2;
```

```
a0_V_tlr=(sum(tlrV))^0.5;              % a0 for vert. disp of
driver

aVV(iiii,jjjj)=aV;              % combined ISO wgt acc, m/s^2
a0_VV_tlr(iiii,jjjj)=a0_V_tlr;

    end         % end of jjjj loop on ct2
end             % end of iiii loop on ct1

disp('  ')
disp('RESULTS OF PARAMETER VARIATION')
disp('  ')
disp('Minimum aV, m/s^2')
disp(min(aVV(:)))
disp('  ')
[ia,ja]=find(aVV==min(aVV(:)));
disp('Corresponding ct1, ct2, and ct3 values, N/(m/s)')
disp([346.39+ctf(ia,ja) (868.722+ctr(ia,ja))*0.5
(868.722+ctr(ia,ja))*0.5])
disp('  ')

disp('  ')
disp('Minimum a0_V_tlr, m/s^2')
disp(min(a0_VV_tlr(:)))
disp('  ')
[it,jt]=find(a0_VV_tlr==min(a0_VV_tlr(:)));
disp('Corresponding ct1, ct2, and ct3 values, N/(m/s)')
disp([346.39+ctf(it,jt) (868.722+ctr(it,jt))*0.5
(868.722+ctr(it,jt))*0.5])
disp('  ')

figure(1)
surf((868.722+ctr)/2,346.39+ctf,aVV)
ylabel('Steer Tire C, N/(m/s)')
xlabel('Single Drive Tire C, N/(m/s)')
zlabel('ISO Combined Wgt Acc, m/s^2')
% title('Tractor Tire Damping Parameter Variation')

figure(2)
surf((868.722+ctr)/2,346.39+ctf,a0_VV_tlr)
ylabel('Steer Tire C, N/(m/s)')
xlabel('Single Drive Tire C, N/(m/s)')
zlabel('Trailer Wgt Vert Acc, m/s^2')
% title('Tractor Tire Damping Parameter Variation')
```

Appendix K: opt_tlr_axlebeam.m

This parameter variation program varies the stiffness values for the trailer axle suspensions and the beaming frequency of the trailer frame. Each of the trailer axle suspensions are assumed to have the same value, so they were combined into one value that was varied, and the individual axle suspension values were assumed to be equal to exactly half of that value. The trailer axles were varied from 700,000 N/m to 1,300,000 N/m in increments of 30,000 N/m. This forms a vector with a length of 21 values that ranges from 30% below to 30% above the nominal value for the trailer axle stiffness. The beaming frequency of the trailer frame was varied from 10 Hz to 30 Hz. in increments of 1 Hz. These frequency values were chosen to represent values close to wheel hop frequencies as well as values known to be higher than recorded resonance frequencies for these types of frames.

The desired output values from this program were the ISO combined weighted acceleration at the driver's seat, the ISO vertical weighted acceleration at the trailer center-of-gravity (CG), and a value called the J penalty, which weighs the importance of the driver ride comfort versus trailer acceleration using weights assigned to them by the user. The program finds the minimum values for each of these outputs, and displays them in tabular form along with the corresponding stiffness values for the trailer axles and the beaming frequency of the trailer frame. Also, the program plots the output information on surface plots to study trends in the information.

opt_tlr_axlebeam.m

```
% opt_tlr_axlebeam.m
% Developed by Ryan Spivey, 4/10/07
%
% Varies trailer axle stiffness and beaming frequency using
weighted RMS
% acceleration in the frequecncy domain
%
% Incorporates model from dof15_freq2.m
%
% DOFs include - 1)Vertical Disp. of Driver's Seat
%                2)Vertical Disp. of Cab
%                3)Pitch of Cab
%                4)Vertical Disp. of Engine
%                5)Vertical Disp. of Tractor Frame
%                6)Pitch of Tractor Frame
%                7)Beaming of Tractor Frame
%                8)Vertical Disp. of Trailer
%                9)Pitch of Trailer
%               10)Beaming of Trailer
%               11)Vertical Disp. of Axle #1
%               12)Vertical Disp. of Axle #2
%               13)Vertical Disp. of Axle #3
%               14)Vertical Disp. of Axle #4
%               15)Vertical Disp. of Axle #5

clc
clear all
% close all
format short e
format compact

global D1_t D2_t D3_t D4_t D1_tlr D2_tlr D3_tlr D4_tlr
global e a1 kb1 kb2 b_fw L_tlr alpha

disp('  ')
disp('Trailer Parameter Variation in the Frequency Domain')
disp('                 Roadholding Model                 ')
disp(['                     ',date])

% Choose a test vehicle
disp('  ')
disp('VEHICLE SELECTION')
disp('  ')
disp('Please choose a vehicle : ');
disp('a: Ideal Tractor Semi-Trailer');
vehicle = input('Enter your choice : ', 's');

if vehicle == 'a'
    % Inertial Properties
    m_s = 106.7;        %kg      mass of seat
    m_c = 1208;         %kg      mass of cab
    I_c = 2100;         %kg*m^2  M I of cab
    m_e = 2000;         %kg      mass of engine (ESTIMATE)
```

```
    m_t = 3783;          %kg      mass of tractor (5783 kg -
engine)
    I_t = 46590.9;       %kg*m^2  M I of tractor
    m_ul = 10800;        %kg      mass of trailer (ESTIMATE)
    I_tlr = 200000;      %kg*m^2  M I of trailer
    m_L = 14000;         %kg      mass of trailer load  (ESTIMATE)
    m_tlr = m_ul+m_L;    %kb      mass of loaded trailer

    % Suspension Parameters
    c1 = 11270;          %N/(m/s) damping const of axle #1
    c2 = 27500;          %N/(m/s) damping const of axle #2
    c3 = 27500;          %N/(m/s) damping const of axle #3
    c4 = 70000;          %N/(m/s) damping const of axle #4
    c5 = 70000;          %N/(m/s) damping const of axle #5
    ce = 10000;          %N/(m/s) damping const of engine mount
    k1 = 581300;         %N/m     spring const of axle #1
    k2 = 586900;         %N/m     spring const of axle #2
    k3 = 586900;         %N/m     spring const of axle #3
    ke = 1e10;           %N/m     spring const of the engine mount

    % Model Dimensions
    b_a1 = 1.065;        %m       Front end of the tractor to axle
#1
    b_cf = 1.470;        %m       Front end of the tractor to cab
front
    b_e = 2.797;         %m       Front end of the tractor to
engine
    b_cr = 4.02;         %m       Front end of the tractor to cab
rear
    b_a2 = 6.035;        %m       Front end of the tractor to axle
#2
    b_fw = 6.688;        %m       Front end of the tractor to 5th
wheel
    b_a3 = 7.34;         %m       Front end of the tractor to axle
#3
    a1 = 4.00607;        %m       Front end of the tractor to
tractor cg

    b_a4 = 8.58;         %m       From the fifth wheel to axle #4
    b_a5 = 9.78;         %m       From the fifth wheel to axle #5

    L_t = 8.2;           %m       Length of Tractor
    L_tlr = 9.78;        %m       Length of Trailer

    e = 5.62;            %m       From the trailer cg to fifth
wheel
    f = 2.96;            %m       From the trailer cg to axle #4
    h = 4.16;            %m       From the trailer cg to axle #5

    a = 2.94107;         %m       From the tractor cg to axle #1
    b = 2.02893;         %m       From the tractor cg to axle #2
    d = 3.33393;         %m       From the tractor cg to axle #3
    l = 2.53607;         %m       From the tractor cg to cab front
    m = 1.209074;        %m       From the tractor cg to engine
    j = 0.013926;        %m       From the tractor cg to cab rear
```

```
    i = 2.68193;        %m       From the tractor cg to the fifth
wheel
    n = 1.435;          %m       From the cab cg to cab front
    p = 1.115;          %m       From the cab cg to cab rear
    r = -0.200;         %m       From the cab cg to seat

    tc = 1.10107;       %m       From the tractor cg to the cab
cg
    h1 = 1.0;           %m       Height of the driver over the
cab
    g = 9.8;            %m/s^2   acceleration due to gravity

    ML_t = m_t/L_t;         %kg/m    Mass per unit length
(Tractor)
    ML_tlr = m_ul/L_tlr;    %kg/m    Mass per unit length
(Trailer)
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%  Fifth Wheel Configuration
%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

kfw = 1000000000000;    %(N/m)      fifth wheel spring constant
cfw = 1000;             %(N/(m/s))  fifth wheel damping ratio

kb1 = 2.36502;      % Constant for the first bending mode (free-
pinned)
%                     (from Rao pg. 527)

fhz = 20;

z1 = '(cos(kb1*x1/b_fw) + (cosh(kb1*x1/b_fw)) - ...
((cos(kb1)+cosh(kb1))/(sin(kb1)-sinh(kb1)))*(sin(kb1*x1/b_fw)-
...  sinh(kb1*x1/b_fw)))';
% free-pinned beam mode function
z1dd = '((kb1/b_fw)^2)*(-cos(kb1*x1/b_fw) + (cosh(kb1*x1/b_fw)) -
... ((cos(kb1)+cosh(kb1))/(sin(kb1)-sinh(kb1)))*(-
sin(kb1*x1/b_fw)- ...  sinh(kb1*x1/b_fw)))';
% second derivative of free-pinned beam mode function

kb2 = 3.926602;     % Constant for the first bending mode
(pinned-free)
%                     (from Rao pg. 527)

z2 = '(sin(kb2*x2/L_tlr) +
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
% pinned-free beam mode function
z2dd = '(kb2/L_tlr)^2*(-sin(kb2*x2/L_tlr) + ...
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
% second derivative of pinned-free beam mode function
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%%%%%%%%%%%%%%%%%%  Computation of Integrals
%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

D1_t=['(',z1,')'];                    % Tractor frame beaming
equations to be
D2_t=['((a1-x1).*(',z1,'))'];         % used in the integrals
(string form)
D3_t=['((',z1,').*(',z1,'))'];
D4_t=['((',z1dd,').*(',z1dd,'))'];

D1_tlr=['(',z2,')'];                  % Trailer beaming equations
to be
D2_tlr=['((e-x2).*(',z2,'))'];        % used in the integrals
(string form)
D3_tlr=['((',z2,').*(',z2,'))'];
D4_tlr=['((',z2dd,').*(',z2dd,'))'];

I1_t=quadl('modeD1_t',0,b_fw);        % Integrals of functions
defined above
I2_t=quadl('modeD2_t',0,b_fw);        % (along length of tractor
frame)
I3_t=quadl('modeD3_t',0,b_fw);
I4_t=quadl('modeD4_t',0,b_fw);

I1_tlr=quadl('modeD1_tlr',0,L_tlr); % Integrals of functions
defined above
I2_tlr=quadl('modeD2_tlr',0,L_tlr); % (along length of trailer)
I3_tlr=quadl('modeD3_tlr',0,L_tlr);
I4_tlr=quadl('modeD4_tlr',0,L_tlr);

E_a1=modeD1_t(b_a1);       % Disp at axle #1 due to tractor frame
beaming
E_cf=modeD1_t(b_cf);       % Disp at cab front due to tractor
frame beaming
E_e=modeD1_t(b_e);         % Disp at engine due to tractor frame
beaming
E_cr=modeD1_t(b_cr);       % Disp at cab rear due to tractor frame
beaming
E_a2=modeD1_t(b_a2);       % Disp at axle #2 due to tractor frame
beaming
E_fw=modeD1_t(b_fw);       % Disp at fifth wheel due to tractor
frame beaming
E_a3=modeD1_t(b_a3);       % Disp at axle #3 due to tractor frame
beaming
E_0=modeD1_tlr(0);         % Disp at fifth wheel due to trailer
beaming
E_a4=modeD1_tlr(b_a4);     % Disp at axle #4 due to trailer
beaming
E_a5=modeD1_tlr(b_a5);     % Disp at axle #5 due to trailer
beaming

% Seat Suspension Options
```

```
disp('  ')
disp('VEHICLE SUSPENSION OPTIONS')
disp('  ')
disp('Give your choice for seat suspension: ')
disp('Note: Without seat suspension gives a very high frequency
mode')
disp('      because the stiffness is set to a high value.')
disp('a : With seat suspension (~0.9 Hz)')
disp('b : Without seat suspension')
z11 = input('Enter your choice : ', 's');

if z11 == 'a',          % Choice 'a' is with seat suspension
    cs = 1140;          % Damping ratio of 0.5
    ks = 3403;          % N/m(spring const of seat suspension)

elseif z11 == 'b',      % Choice 'b' is without seat suspension
    cs = 1329;          % N/(m/s)(damping const of seat
suspension)
    ks = 1e10;          % N/m(spring const of seat suspension)

else disp('Insufficient information regarding seat suspension.')
end

% Cab Suspension Options
disp('  ')
disp('Give your choice for cab suspension: ')
disp('Note: With front or rear or without cab suspension')
disp('      gives a very high frequency mode(s) because the
corresponding')
disp('      stiffness(es)is set to a high value.')
disp('a : With front cab suspension')
disp('b : With rear cab suspension')
disp('c : With front & rear cab suspension')
disp('d : Without cab suspension')
z22 = input('Enter your choice : ', 's');

if z22 == 'a',          % Choice 'a' is front cab suspension
    ccf = 7062;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 88740;        % N/m(spring const of front cab
suspension)
    ccr = 6430;         % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;         % N/m(spring const of rear cab
suspension)

elseif z22 == 'b',      % Choice 'b' is rear cab suspension
    ccr = 8000;         % Reduced damping
    kcr = 65980;        % N/m(spring const of rear cab
suspension)
    ccf = 13120;        % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;         % N/m(spring const of front cab
suspension)
```

```
elseif z22 == 'c',      % Choice 'c' is front & rear cab
suspension
    ccr = 5073.5;       % N/(m/s)(damping const of rear cab
suspension)
    kcr = 63757.5;      % N/m(spring const of rear cab
suspension)
    ccf = 6864.35;      % N/(m/s)(damping const of front cab
suspension)
    kcf = 86260.5;      % N/m(spring const of front cab
suspension)

elseif z22 == 'd',      % Choice 'd' is without cab suspension
    ccr = 6430;         % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;         % N/m(spring const of rear cab
suspension)
    ccf = 7062;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;         % N/m(spring const of front cab
suspension)

else disp('Insufficient information regarding cab suspension.')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%  Vehicle Tire Selection
%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

disp('   ')
disp('STEER AXLE TIRE SELECTION')
TireData3;                  % M-file for tire data
wd1 = wd;                   %(m)        Nominal cross section
width
mt1 = mt;                   %(kg)       Mass of axle #1
P1 = P;                     %(psi)      Tire pressure from
TireData3.m
press1 = press;             %(psi)      Tire pressure array
numtires1 = numtires;       %           Number of tires on axle
Kstiff1 = Kstiff;           %(N/m)      Tire stiffness array
kt1 = KK * numtires1;       %(N/m)      Per-axle Rad Stiffness
ct1 = ct;                   %(N/(m/s))  Per-axle Damping

disp('   ')
disp('DRIVE AXLE TIRE SELECTION')
TireData3;                  % M-file for tire data
wd23 = wd;                  %(m)        Nominal cross section
width
mt2 = mt;                   %(kg)       Mass of axle #2
mt3 = mt;                   %(kg)       Mass of axle #3
P23 = P;                    %(psi)      Tire pressure from
TireData3.m
press23 = press;            %(psi)      Tire Pressure array
numtires23 = numtires;      %           Number of tires on axle
```

```
Kstiff23 = Kstiff;           %(N/m)      Tire stiffness array
kt2 = KK * numtires23;       %(N/m)      Per-axle Rad Stiffness
kt3 = KK * numtires23;       %(N/m)      Per-axle Rad Stiffness
ct2 = ct;                    %(N/(m/s))  Per-axle Damping
ct3 = ct;                    %(N/(m/s))  Per-axle Damping

disp('   ')
disp('TRAILER AXLE TIRE SELECTION')
TireData3;                   % M-file for tire data
wd45 = wd;                   %(m)        Nominal cross section
width
mt4 = mt;                    %(kg)       Mass of axle #4
mt5 = mt;                    %(kg)       Mass of axle #5
P45 = P;                     %(psi)      Tire pressure from
TireData3.m
press45 = press;             %(psi)      Tire Pressure array
numtires45 = numtires;       %           Number of tires on axle
Kstiff45 = Kstiff;           %(N/m)      Tire stiffness array
kt4 = KK * numtires45;       %(N/m)      Per-axle Rad Stiffness
kt5 = KK * numtires45;       %(N/m)      Per-axle Rad Stiffness
ct4 = ct;                    %(N/(m/s))  Per-axle Damping
ct5 = ct;                    %(N/(m/s))  Per-axle Damping

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%  Speed of the Vehicle  %%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('VEHICLE VELOCITY')
disp('  ')
disp('Please choose the unit of velocity');
disp('a : Miles per Hour (mph)');
disp('b : Kilometers per Hour (kph)');
vel = input('Input the unit of velocity (a/b):  ', 's');
disp('  ')
vm = input('Input the velocity of the vehicle, vm : ');

if vel == 'a'
    v = 0.4473*vm;               %Velocity conversion from mph to
m/s
elseif vel == 'b'
    v = 0.277778*vm;             %Velocity conversion from kph to
m/s
end

T(1) = 0;                %Time delay between front axle and
remaining axles
T(2) = (a+b)/v;          % Axle #2
T(3) = (a+d)/v;          % Axle #3
T(4) = (a+i+e+f)/v;      % Axle #4
T(5) = (a+i+e+h)/v;      % Axle #5

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%  Road PSD Selection  %%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
disp('  ')
disp('ROAD PSD SELECTION')
disp('  ')
disp('Road PSD Constants, m^2/cyc/m, Ref: Wong, Theory of Ground
Vehicles')
disp('S(W)=Csp/W^N  where W=spatial frequency')
disp('  ')
disp('a : Csp = 4.3e-11,N=3.8     Smooth Runway')
disp('b : Csp = 8.1e-6, N=2.1     Rough Runway')
disp('c : Csp = 4.8e-7, N=2.1     Smooth Highway')
disp('d : Csp = 4.4e-6, N=2.1     Highway with Gravel')
disp('  ')
tabchoice11=input('Input the road surface to be used :   ','s');

if tabchoice11== 'a',               % smooth runway
   Csp = 4.3e-11;
   N=3.8;

   elseif tabchoice11== 'b',        % rough runway
      Csp = 8.1e-6;
      N=2.1;

   elseif tabchoice11 == 'c',       % smooth highway
      Csp = 4.8e-7;
      N=2.1;

   elseif tabchoice11 == 'd',       % highway with gravel
      Csp = 4.4e-6;
      N=2.1;

end

disp('  ')
disp('J PENALTY OPTIONS')
disp('  ')
disp('Input the values for K1 and K2 in the J penalty function')
disp('Note: Both values should add up to 1')
disp('  ')
K_1 = input('Input the value for K1 : ');
disp('  ')
K_2 = input('Input the value for K2 : ');

% Start Loop on Axle Stiffness Properties
% Stiffness values will range from 70% to 130% of the nominal
value

for iiii=1:21;
    for jjjj=1:21;
        kr(iiii,jjjj)=60000*iiii;

        k4 = (1.34e6+kr(iiii,jjjj))*0.5;
        k5 = (1.34e6+kr(iiii,jjjj))*0.5;

        ffhz2(iiii,jjjj)=jjjj;
```

```
        fhz2 = 9+ffhz2(iiii,jjjj);

EI_t = 4*pi^2*fhz^2*(b_fw/kb1)^4*ML_t;       %Tractor frame
flexural rigidity
EI_tlr = 4*pi^2*fhz2^2*(L_tlr/kb2)^4*ML_tlr; %Trailer flexural
rigidity

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%  System Matrices
%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

% THE SYSTEM IS WRITTEN AS (M*S*S+C*S+K)X(S)=(A*S+B)U(S)

%%%%%%%%%%%%%%%%%%%%%%%
%%%%  Mass Matrix  %%%%
%%%%%%%%%%%%%%%%%%%%%%%
M = zeros(15,15);

M(1,1) = m_s;                   % Eqn #1: Vertical Disp of Seat

M(2,2) = m_c;                   % Eqn #2: Vertical Disp of Cab

M(3,3) = I_c;                   % Eqn #3: Pitch of Cab

M(4,4) = m_e;                   % Eqn #4: Vertical Disp of Engine

M(5,5) = m_t;                   % Eqn #5: Vertical Disp of
Tractor Frame
M(5,6) = ML_t*b_fw*(b_fw/2-a1);
M(5,7) = ML_t*I1_t;

M(6,5) = ML_t*b_fw*(b_fw/2-a1);   % Eqn #6: Pitch of Tractor
Frame
M(6,6) = I_t;
M(6,7) = -ML_t*I2_t;

M(7,5) = ML_t*I1_t;             % Eqn #7: Beaming of Tractor
Frame
M(7,6) = -ML_t*I2_t;
M(7,7) = ML_t*I3_t;

M(8,8) = m_tlr;                 % Eqn #8: Vertical Disp of
Trailer
M(8,9) = -ML_tlr*L_tlr*(e-L_tlr/2);
M(8,10) = ML_tlr*I1_tlr;

M(9,8) = -ML_tlr*L_tlr*(e-L_tlr/2);% Eqn #9: Pitch of Trailer
M(9,9) = I_tlr;
M(9,10) = -ML_tlr*I2_tlr;
```

```
M(10,8) = ML_tlr*I1_tlr;          % Eqn #10: Beaming of Trailer
M(10,9) = -ML_tlr*I2_tlr;
M(10,10) = ML_tlr*I3_tlr;

M(11,11) = mt1;                   % Eqn #11: Vertical Disp of Axle
#1

M(12,12) = mt2;                   % Eqn #12: Vertical Disp of Axle
#2

M(13,13) = mt3;                   % Eqn #13: Vertical Disp of Axle
#3

M(14,14) = mt4;                   % Eqn #14: Vertical Disp of Axle
#4

M(15,15) = mt5;                   % Eqn #15: Vertical Disp of Axle
#5

%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%
C = zeros(15,15);

C(1,1) = cs;
C(1,2) = -cs;
C(1,3) = r*cs;

C(2,1) = -cs;
C(2,2) = cs+ccf+ccr;
C(2,3) = -r*cs-n*ccf+p*ccr;
C(2,5) = -ccf-ccr;
C(2,6) = l*ccf-j*ccr;
C(2,7) = -ccf*E_cf-ccr*E_cr;

C(3,1) = r*cs;
C(3,2) = -r*cs-n*ccf+p*ccr;
C(3,3) = (r^2)*cs+(n^2)*ccf+(p^2)*ccr;
C(3,5) = n*ccf-p*ccr;
C(3,6) = -n*l*ccf-p*j*ccr;
C(3,7) = n*ccf*E_cf-p*ccr*E_cr;

C(4,4) = ce;
C(4,5) = -ce;
C(4,6) = m*ce;
C(4,7) = -ce*E_e;

C(5,2) = -ccf-ccr;
C(5,3) = n*ccf-p*ccr;
C(5,4) = -ce;
C(5,5) = ce+ccf+ccr+cfw+c1+c2+c3;
C(5,6) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(5,7) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(5,8) = -cfw;
```

```
C(5,9) = e*cfw;
C(5,10) = -cfw*E_0;
C(5,11) = -c1;
C(5,12) = -c2;
C(5,13) = -c3;

C(6,2) = l*ccf-j*ccr;
C(6,3) = -n*l*ccf-p*j*ccr;
C(6,4) = m*ce;
C(6,5) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(6,6) =
(m^2)*ce+(l^2)*ccf+(j^2)*ccr+(i^2)*cfw+(a^2)*c1+(b^2)*c2+(d^2)*c3
;
C(6,7) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2+...
         d*c3*E_a3;
C(6,8) = -i*cfw;
C(6,9) = e*i*cfw;
C(6,10) = -i*cfw*E_0;
C(6,11) = a*c1;
C(6,12) = -b*c2;
C(6,13) = -d*c3;

C(7,2) = -ccf*E_cf-ccr*E_cr;
C(7,3) = n*ccf*E_cf-p*ccr*E_cr;
C(7,4) = -ce*E_e;
C(7,5) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(7,6) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2 ...
         +d*c3*E_a3;
C(7,7) =
ce*E_e^2+ccf*E_cf^2+ccr*E_cr^2+cfw*E_fw^2+c1*E_a1^2+c2*E_a2^2 ...
         +c3*E_a3^2;
C(7,8) = -cfw*E_fw;
C(7,9) = e*cfw*E_fw;
C(7,10) = -cfw*E_0*E_fw;
C(7,11) = -c1*E_a1;
C(7,12) = -c2*E_a2;
C(7,13) = -c3*E_a3;

C(8,5) = -cfw;
C(8,6) = -i*cfw;
C(8,7) = -cfw*E_fw;
C(8,8) = cfw+c4+c5;
C(8,9) = -e*cfw+f*c4+h*c5;
C(8,10) = cfw*E_0+c4*E_a4+c5*E_a5;
C(8,14) = -c4;
C(8,15) = -c5;

C(9,5) = e*cfw;
C(9,6) = e*i*cfw;
C(9,7) = e*cfw*E_fw;
C(9,8) = -e*cfw+f*c4+h*c5;
C(9,9) = (e^2)*cfw+(f^2)*c4+(h^2)*c5;
C(9,10) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
```

```
C(9,14) = -f*c4;
C(9,15) = -h*c5;

C(10,5) = -cfw*E_0;
C(10,6) = -i*cfw*E_0;
C(10,7) = -cfw*E_fw*E_0;
C(10,8) = cfw*E_0+c4*E_a4+c5*E_a5;
C(10,9) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(10,10) = cfw*E_0^2+c4*E_a4^2+c5*E_a5^2;
C(10,14) = -c4*E_a4;
C(10,15) = -c5*E_a5;

C(11,5) = -c1;
C(11,6) = a*c1;
C(11,7) = -c1*E_a1;
C(11,11) = c1+ct1;

C(12,5) = -c2;
C(12,6) = -b*c2;
C(12,7) = -c2*E_a2;
C(12,12) = c2+ct2;

C(13,5) = -c3;
C(13,6) = -d*c3;
C(13,7) = -c3*E_a3;
C(13,13) = c3+ct3;

C(14,8) = -c4;
C(14,9) = -f*c4;
C(14,10) = -c4*E_a4;
C(14,14) = c4+ct4;

C(15,8) = -c5;
C(15,9) = -h*c5;
C(15,10) = -c5*E_a5;
C(15,15) = c5+ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%
K = zeros(15,15);

K(1,1) = ks;
K(1,2) = -ks;
K(1,3) = r*ks;

K(2,1) = -ks;
K(2,2) = ks+kcf+kcr;
K(2,3) = -r*ks-n*kcf+p*kcr;
K(2,5) = -kcf-kcr;
K(2,6) = l*kcf-j*kcr;
K(2,7) = -kcf*E_cf-kcr*E_cr;

K(3,1) = r*ks;
K(3,2) = -r*ks-n*kcf+p*kcr;
```

```
K(3,3) = (r^2)*ks+(n^2)*kcf+(p^2)*kcr;
K(3,5) = n*kcf-p*kcr;
K(3,6) = -n*l*kcf-p*j*kcr;
K(3,7) = n*kcf*E_cf-p*kcr*E_cr;

K(4,4) = ke;
K(4,5) = -ke;
K(4,6) = m*ke;
K(4,7) = -ke*E_e;

K(5,2) = -kcf-kcr;
K(5,3) = n*kcf-p*kcr;
K(5,4) = -ke;
K(5,5) = ke+kcf+kcr+kfw+k1+k2+k3;
K(5,6) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(5,7) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(5,8) = -kfw;
K(5,9) = e*kfw;
K(5,10) = -kfw*E_0;
K(5,11) = -k1;
K(5,12) = -k2;
K(5,13) = -k3;

K(6,2) = l*kcf-j*kcr;
K(6,3) = -n*l*kcf-p*j*kcr;
K(6,4) = m*ke;
K(6,5) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(6,6) =
(m^2)*ke+(l^2)*kcf+(j^2)*kcr+(i^2)*kfw+(a^2)*k1+(b^2)*k2+(d^2)*k3
;
K(6,7) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2+...
         d*k3*E_a3;
K(6,8) = -i*kfw;
K(6,9) = e*i*kfw;
K(6,10) = -i*kfw*E_0;
K(6,11) = a*k1;
K(6,12) = -b*k2;
K(6,13) = -d*k3;

K(7,2) = -kcf*E_cf-kcr*E_cr;
K(7,3) = n*kcf*E_cf-p*kcr*E_cr;
K(7,4) = -ke*E_e;
K(7,5) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(7,6) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2 ...
         +d*k3*E_a3;
K(7,7) =
ke*E_e^2+kcf*E_cf^2+kcr*E_cr^2+kfw*E_fw^2+k1*E_a1^2+k2*E_a2^2 ...
         +k3*E_a3^2+EI_t*I4_t;
K(7,8) = -kfw*E_fw;
K(7,9) = e*kfw*E_fw;
K(7,10) = -kfw*E_0*E_fw;
K(7,11) = -k1*E_a1;
```

```
K(7,12) = -k2*E_a2;
K(7,13) = -k3*E_a3;

K(8,5) = -kfw;
K(8,6) = -i*kfw;
K(8,7) = -kfw*E_fw;
K(8,8) = kfw+k4+k5;
K(8,9) = -e*kfw+f*k4+h*k5;
K(8,10) = kfw*E_0+k4*E_a4+k5*E_a5;
K(8,14) = -k4;
K(8,15) = -k5;

K(9,5) = e*kfw;
K(9,6) = e*i*kfw;
K(9,7) = e*kfw*E_fw;
K(9,8) = -e*kfw+f*k4+h*k5;
K(9,9) = (e^2)*kfw+(f^2)*k4+(h^2)*k5;
K(9,10) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(9,14) = -f*k4;
K(9,15) = -h*k5;

K(10,5) = -kfw*E_0;
K(10,6) = -i*kfw*E_0;
K(10,7) = -kfw*E_fw*E_0;
K(10,8) = kfw*E_0+k4*E_a4+k5*E_a5;
K(10,9) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(10,10) = kfw*E_0^2+k4*E_a4^2+k5*E_a5^2+EI_tlr*I4_tlr;
K(10,14) = -k4*E_a4;
K(10,15) = -k5*E_a5;

K(11,5) = -k1;
K(11,6) = a*k1;
K(11,7) = -k1*E_a1;
K(11,11) = k1+kt1;

K(12,5) = -k2;
K(12,6) = -b*k2;
K(12,7) = -k2*E_a2;
K(12,12) = k2+kt2;

K(13,5) = -k3;
K(13,6) = -d*k3;
K(13,7) = -k3*E_a3;
K(13,13) = k3+kt3;

K(14,8) = -k4;
K(14,9) = -f*k4;
K(14,10) = -k4*E_a4;
K(14,14) = k4+kt4;

K(15,8) = -k5;
K(15,9) = -h*k5;
K(15,10) = -k5*E_a5;
K(15,15) = k5+kt5;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A = zeros(15,1);

A(11) = ct1;
A(12) = ct2;
A(13) = ct3;
A(14) = ct4;
A(15) = ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
B = zeros(15,1);

B(11) = kt1;
B(12) = kt2;
B(13) = kt3;
B(14) = kt4;
B(15) = kt5;

% System "A" Matrix
AA=[zeros(size(M))   eye(size(M))   % System state variable
matrix
    -inv(M)*K       -inv(M)*C];

% ISO 2631 FOR REDUCED COMFORT BOUNDARY
% COMFORT BOUNDARIES FOR VERTICAL ACCELERATION
% THE ISO CENTRAL FREQUENCIES (Hz)

wc=[ .1 1 1.25 1.6 2 2.5 3.15 4 5 6.3 8 10 12.5 16 20 25 31.5 40
50];
whzc=[ .1 .125 .16 .2 .25 .315 .4 .5 .63 .8 1 1.25 1.6 2 2.5 3.15
...
    4 5 6.3 8 10 12.5 16 20 25 31.5 40 50];

% 2.5 hr FATIGUE BOUNDARY
fat1=[4.284,1.4,1.25,1.12,1,.9,.8,.71,.71,.71,.71,...
      .9,1.12,1.4,1.8,2.24,2.8,3.55,4.5];
% 2.5hr REDUCED COMFORT BOUNDARY
comf1=fat1/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf2= comf1/2.254;

%----------------------------------------------------------------
---
% COMFORT BOUNDARIES FOR LONGITUDINAL AND LATERAL ACC
% 2.5hr FATIGUE BOUNDARY
fat2=[0.5,0.5,0.5,0.5,0.5,0.63,0.8,1,1.25,1.6,2,2.5,3.15,4,5,6.3,
8,10,12.5];

% 2.5hr REDUCED COMFORT BOUNDARY
comf3=fat2/3.15;
% 8hr REDUCED COMFORT BOUNDARY
```

```
comf4= comf3/2.254;
%-----------------------------------------------------------------
--

whzcr = 2*pi*whzc;      % Calculation of central frequencies in
rad/s
freqlow=0.89*whzcr;     % Lower octave band
freqhigh=1.12*whzcr;    % Upper octave band
freq=[freqlow' whzcr' freqhigh'];

imag=sqrt(-1);

for ii=1:length(whzc);
    for jj=1:3;         % jj=1 is freqlow, jj=2 is center freq
                        % jj=3 is freqhigh
        w = freq(ii,jj);
        s = imag*w;
        dp = sqrt(h1^2+r^2);

        % Time delay array
        time = [0 0 0 0 0 0 0 0 0 0 1 exp(-s*T(2)) exp(-s*T(3))
...
            exp(-s*T(4)) exp(-s*T(5))];

        % TF Matrix
        vectx = (inv(M*s*s+C*s+K)*((A*s+B).*(time.')));

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %%%  Transfer Functions  %%%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%

        z_s=[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]*vectx;    % vert seat
cg
        long=[0 0 -h1 0 0 0 0 0 0 0 0 0 0 0 0]*vectx; % long disp
of driver
        z_tlr=[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]*vectx;  % vert
trailer cg

        %%%%%%%%%%%%%%%%%%%%
        %%%  Magnitudes  %%%
        %%%%%%%%%%%%%%%%%%%%

        % Acceleration Transter Functions
        magcfA1(ii,jj)=abs(s*s*z_s);  % Mag of trans function,
(m/s*s)/m
        magcfAlong(ii,jj)=abs(s*s*long);
        magcftlr(ii,jj)=abs(s*s*z_tlr);

        %%%%%%%%%%%%%%
        %%%  PSDs  %%%
        %%%%%%%%%%%%%%

        % Road PSD in m*m/(rad/s)
        rpsd(ii,jj)=Csp*(((2*pi*v)^(N-1))/(w^N));
```

```
        % Acceleration PSDs in (m/s^2)^2/(rad/s)
        psdcfA1(ii,jj)=magcfA1(ii,jj)*magcfA1(ii,jj)*rpsd(ii,jj);

psdcfAlong(ii,jj)=magcfAlong(ii,jj)*magcfAlong(ii,jj)*rpsd(ii,jj)
;

psdcftlr(ii,jj)=magcftlr(ii,jj)*magcftlr(ii,jj)*rpsd(ii,jj);

    end
end

for kk=1:length(whzc)
    % Vert. Driver's Seat RMS
    msqy1a(kk)=0.5*(psdcfA1(kk,1)+psdcfA1(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqy1b(kk)=0.5*(psdcfA1(kk,2)+psdcfA1(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqy1(kk)=msqy1a(kk)+msqy1b(kk);
    rmsA1cf(kk)=sqrt(msqy1(kk));
    % Long. Driver RMS
    msqylonga(kk)=0.5*(psdcfAlong(kk,1)+psdcfAlong(kk,2))*...
    (freq(kk,2)-freq(kk,1));
    msqylongb(kk)=0.5*(psdcfAlong(kk,2)+psdcfAlong(kk,3))*...
    (freq(kk,3)-freq(kk,2));
    msqylong(kk)=msqylonga(kk)+msqylongb(kk);
    rmsAlongcf(kk)=sqrt(msqylong(kk));
    % Vert. Trailer cg RMS
    msqytlra(kk)=0.5*(psdcftlr(kk,1)+psdcftlr(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqytlrb(kk)=0.5*(psdcftlr(kk,2)+psdcftlr(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqytlr(kk)=msqytlra(kk)+msqytlrb(kk);
    rmstlrcf(kk)=sqrt(msqytlr(kk));
end

RMScf = [rmsA1cf',rmsAlongcf',rmstlrcf'];       % Accel. RMS
Matrix

% Calculate weighted rms acceleration from 0.1 to 50 Hz
% at the ISO Center Frequencies .... Wgt are the ISO weights
% Ref: ISO 2631-1:1997(E); V=vertical; L=longitudinal

wcc=[.1,.125,.16,.2,.25,.315,.4,.5,.63,.8,1,1.25,1.6,2,2.5,3.15,4
,5,...
    6.3,8,10,12.5,16,20,25,31.5,40,50];
WgtV=[.0312,.0486,.079,.121,.182,.263,.352,.418,.459,.477,.482,.4
84,...

.494,.531,.631,.804,.967,1.039,1.054,1.036,.988,.902,.768,.636,..
.
    .513,.405,.314,.246];
WgtL=0.001*[62.4,97.3,158,243,365,530,713,853,944,992,1011,1008,9
68,...

890,776,642,512,409,323,253,212,161,125,100,80,63.2,49.4,38.8];
```

```
isovert = WgtV.*RMScf(1:28,1)';     % Weighted Vert. Driver RMS
Accel.
isolong = WgtL.*RMScf(1:28,2)';     % Weighted Long. Driver RMS
Accel.
isotlr = WgtV.*RMScf(1:28,3)';      % Weighted Vert. Trailer RMS
Accel.

term2V=(WgtV.*rmsA1cf(1:28)).^2;
a0_V_dr=(sum(term2V))^0.5;             % a0 for vert. disp of
driver

term2L=(WgtL.*rmsAlongcf(1:28)).^2;
a0_L_dr=(sum(term2L))^0.5;             % a0 for long. disp of
driver

aV=(a0_L_dr^2 + a0_V_dr^2)^0.5;      % a0 for comb vert and long
disp

tlrV=(WgtV.*rmstlrcf(1:28)).^2;
a0_V_tlr=(sum(tlrV))^0.5;              % a0 for vert. disp of
driver

aVV(iiii,jjjj)=aV;              % combined ISO wgt acc, m/s^2
a0_VV_tlr(iiii,jjjj)=a0_V_tlr;
Jpenalty(iiii,jjjj)=K_1*(aVV(iiii,jjjj)/0.44814)+K_2*...
(a0_VV_tlr(iiii,jjjj)/0.3239);

    end         % end of jjjj loop on fhz2
end             % end of iiii loop on k4,5

disp('  ')
disp('RESULTS OF PARAMETER VARIATION')
disp('  ')
disp('Minimum aV, m/s^2')
disp(min(aVV(:)))
disp('  ')
[ia,ja]=find(aVV==min(aVV(:)));
disp('Corresponding k4 and k5 values, N/m')
disp([(1.34e6+kr(ia,ja))*0.5 (1.34e6+kr(ia,ja))*0.5])
disp('  ')
disp('Corresponding Trailer Beaming Frequency, Hz')
disp([9+ffhz2(ia,ja)])
disp('  ')

disp('  ')
disp('Minimum a0_V_tlr, m/s^2')
disp(min(a0_VV_tlr(:)))
disp('  ')
[it,jt]=find(a0_VV_tlr==min(a0_VV_tlr(:)));
disp('Corresponding k4 and k5 values, N/m')
disp([(1.34e6+kr(it,jt))*0.5 (1.34e6+kr(it,jt))*0.5])
disp('  ')
disp('Corresponding Trailer Beaming Frequency, Hz')
disp([9+ffhz2(it,jt)])
```

```
disp('  ')

disp('  ')
disp('Minimum Jpenalty')
disp('J=K1*aV/0.44814 + K2*a0_V_tlr/0.3239')
disp('       K1         K2')
disp([K_1 K_2])
disp('  ')
disp(min(Jpenalty(:)))
disp('  ')
[iJ,jJ]=find(Jpenalty==min(Jpenalty(:)));
disp('Corresponding k4 and k5 values, N/m')
disp([(1.34e6+kr(iJ,jJ))*0.5 (1.34e6+kr(iJ,jJ))*0.5])
disp('  ')
disp('Corresponding Trailer Beaming Frequency, Hz')
disp([9+ffhz2(iJ,jJ)])
disp('  ')

figure(1)
surf(9+ffhz2,(1.34e6+kr)/2,aVV)
xlabel('Trailer Beaming, Hz')
ylabel('Single Trailer Axle K, N/m')
zlabel('ISO Combined Wgt Acc, m/s^2')
% title('Tractor Suspension Stiffness Parameter Variation')

figure(2)
surf((1.34e6+kr)/2,9+ffhz2,a0_VV_tlr)
ylabel('Trailer Beaming, Hz')
xlabel('Single Trailer Axle K, N/m')
zlabel('Trailer Wgt Vert Acc, m/s^2')
% title('Tractor Suspension Stiffness Parameter Variation')

figure(3)
surf(9+ffhz2,(1.34e6+kr)/2,Jpenalty)
xlabel('Trailer Beaming, Hz')
ylabel('Single Trailer Axle K, N/m')
zlabel('Penalty Function')
title(['K1 = ', num2str(K_1),'     K2 = ', num2str(K_2)])
```

Appendix L: opt_beam_freq.m

This parameter variation program varies the beaming frequencies of the tractor and trailer frames individually. The beaming frequency of the tractor frame was varied from 10 Hz to 30 Hz. in increments of 1 Hz. These frequency values were chosen to represent values close to wheel hop frequencies as well as values known to be higher than recorded resonance frequencies for these types of frames. Likewise, the beaming frequency of the trailer frame was varied from 10 Hz to 30 Hz. in increments of 1 Hz. These frequency values were chosen to represent values close to wheel hop frequencies as well as values known to be higher than recorded resonance frequencies for these types of frames.

The desired output values from this program were the ISO combined weighted acceleration at the driver's seat and the ISO vertical weighted acceleration at the trailer center-of-gravity (CG). The program finds the minimum values for each of these outputs, and displays them in tabular form along with the corresponding beaming frequencies of the tractor and trailer frames. Also, the program plots the output information on surface plots to study trends in the information.

opt_beam_freq.m

```
% opt_beam_freq.m
% Developed by Ryan Spivey, 4/10/07
%
% Varies beaming frequency of the tractor and trailer using
weighted
% RMS acceleration in the frequecncy domain.
%
% Tractor and trailer beaming are treated as free-pinned and
pinned free
% respectively.
%
% Incorporates model from dof15_freq2.m
%
% DOFs include - 1)Vertical Disp. of Driver's Seat
%                2)Vertical Disp. of Cab
%                3)Pitch of Cab
%                4)Vertical Disp. of Engine
%                5)Vertical Disp. of Tractor Frame
%                6)Pitch of Tractor Frame
%                7)Beaming of Tractor Frame
%                8)Vertical Disp. of Trailer
%                9)Pitch of Trailer
%               10)Beaming of Trailer
%               11)Vertical Disp. of Axle #1
%               12)Vertical Disp. of Axle #2
%               13)Vertical Disp. of Axle #3
%               14)Vertical Disp. of Axle #4
%               15)Vertical Disp. of Axle #5

clc
clear all
close all
format short e
format compact

global D1_t D2_t D3_t D4_t D1_tlr D2_tlr D3_tlr D4_tlr
global e a1 kb1 kb2 b_fw L_tlr alpha

disp('  ')
disp('Beaming Frequency Variation in the Frequency Domain')
disp('                Roadholding Model                  ')
disp(['                  ',date])

% Choose a test vehicle
disp('  ')
disp('VEHICLE SELECTION')
disp('  ')
disp('Please choose a vehicle : ');
disp('a: Ideal Tractor Semi-Trailer');
vehicle = input('Enter your choice : ', 's');

if vehicle == 'a'
```

```
    % Inertial Properties
    m_s = 106.7;        %kg      mass of seat
    m_c = 1208;         %kg      mass of cab
    I_c = 2100;         %kg*m^2  M I of cab
    m_e = 2000;         %kg      mass of engine (ESTIMATE)
    m_t = 3783;         %kg      mass of tractor (5783 kg -
engine)
    I_t = 46590.9;      %kg*m^2  M I of tractor
    m_ul = 10800;       %kg      mass of trailer (ESTIMATE)
    I_tlr = 200000;     %kg*m^2  M I of trailer
    m_L = 14000;        %kg      mass of trailer load  (ESTIMATE)
    m_tlr = m_ul+m_L;   %kb      mass of loaded trailer

    % Suspension Parameters
    c1 = 11270;         %N/(m/s) damping const of axle #1
    c2 = 27500;         %N/(m/s) damping const of axle #2
    c3 = 27500;         %N/(m/s) damping const of axle #3
    c4 = 70000;         %N/(m/s) damping const of axle #4
    c5 = 70000;         %N/(m/s) damping const of axle #5
    ce = 10000;         %N/(m/s) damping const of engine mount
    k1 = 581300;        %N/m     spring const of axle #1
    k2 = 586900;        %N/m     spring const of axle #2
    k3 = 586900;        %N/m     spring const of axle #3
    k4 = 1000000;       %N/m     spring const of axle #4
    k5 = 1000000;       %N/m     spring const of axle #5
    ke = 1e10;          %N/m     spring const of the engine mount

    % Model Dimensions
    b_a1 = 1.065;       %m       Front end of the tractor to axle
#1
    b_cf = 1.470;       %m       Front end of the tractor to cab
front
    b_e = 2.797;        %m       Front end of the tractor to
engine
    b_cr = 4.02;        %m       Front end of the tractor to cab
rear
    b_a2 = 6.035;       %m       Front end of the tractor to axle
#2
    b_fw = 6.688;       %m       Front end of the tractor to 5th
wheel
    b_a3 = 7.34;        %m       Front end of the tractor to axle
#3
    a1 = 4.00607;       %m       Front end of the tractor to
tractor cg

    b_a4 = 8.58;        %m       From the fifth wheel to axle #4
    b_a5 = 9.78;        %m       From the fifth wheel to axle #5

    L_t = 8.2;          %m       Length of Tractor
    L_tlr = 9.78;       %m       Length of Trailer

    e = 5.62;           %m       From the trailer cg to fifth
wheel
    f = 2.96;           %m       From the trailer cg to axle #4
    h = 4.16;           %m       From the trailer cg to axle #5
```

```
    a = 2.94107;         %m       From the tractor cg to axle #1
    b = 2.02893;         %m       From the tractor cg to axle #2
    d = 3.33393;         %m       From the tractor cg to axle #3
    l = 2.53607;         %m       From the tractor cg to cab front
    m = 1.209074;        %m       From the tractor cg to engine
    j = 0.013926;        %m       From the tractor cg to cab rear
    i = 2.68193;         %m       From the tractor cg to the fifth
wheel
    n = 1.435;           %m       From the cab cg to cab front
    p = 1.115;           %m       From the cab cg to cab rear
    r = -0.200;          %m       From the cab cg to seat

    tc = 1.10107;        %m       From the tractor cg to the cab
cg
    h1 = 1.0;            %m       Height of the driver over the
cab
    g = 9.8;             %m/s^2   acceleration due to gravity

    ML_t = m_t/L_t;          %kg/m    Mass per unit length
(Tractor)
    ML_tlr = m_ul/L_tlr;     %kg/m    Mass per unit length
(Trailer)
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%  Fifth Wheel Configuration
%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

kfw = 1000000000000;    %(N/m)       fifth wheel spring constant
cfw = 1000;             %(N/(m/s))   fifth wheel damping ratio

kb1 = 2.36502;      % Constant for the first bending mode (free-
pinned)
%                     (from Rao pg. 527)

z1 = '(cos(kb1*x1/b_fw) + (cosh(kb1*x1/b_fw)) - ...
((cos(kb1)+cosh(kb1))/(sin(kb1)-sinh(kb1)))*(sin(kb1*x1/b_fw)-
...  sinh(kb1*x1/b_fw)))';
% free-pinned beam mode function
z1dd = '((kb1/b_fw)^2)*(-cos(kb1*x1/b_fw) + (cosh(kb1*x1/b_fw)) -
...  ((cos(kb1)+cosh(kb1))/(sin(kb1)-sinh(kb1)))*(-
sin(kb1*x1/b_fw)- ...  sinh(kb1*x1/b_fw)))';
% second derivative of free-pinned beam mode function

kb2 = 3.926602;     % Constant for the first bending mode
(pinned-free)
%                     (from Rao pg. 527)

z2 = '(sin(kb2*x2/L_tlr) +
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
% pinned-free beam mode function
```

```
z2dd = '(kb2/L_tlr)^2*(-sin(kb2*x2/L_tlr) + ...
((sin(kb2))/(sinh(kb2)))*(sinh(kb2*x2/L_tlr)))';
% second derivative of pinned-free beam mode function

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%%%%%%%%%%%%%%%%%%  Computation of Integrals
%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

D1_t=['(',z1,')'];                    % Tractor frame beaming
equations to be
D2_t=['((a1-x1).*(',z1,'))'];         % used in the integrals
(string form)
D3_t=['((',z1,').*(',z1,'))'];
D4_t=['((',z1dd,').*(',z1dd,'))'];

D1_tlr=['(',z2,')'];                  % Trailer beaming equations
to be
D2_tlr=['((e-x2).*(',z2,'))'];        % used in the integrals
(string form)
D3_tlr=['((',z2,').*(',z2,'))'];
D4_tlr=['((',z2dd,').*(',z2dd,'))'];

I1_t=quadl('modeD1_t',0,b_fw);        % Integrals of functions
defined above
I2_t=quadl('modeD2_t',0,b_fw);        % (along length of tractor
frame)
I3_t=quadl('modeD3_t',0,b_fw);
I4_t=quadl('modeD4_t',0,b_fw);

I1_tlr=quadl('modeD1_tlr',0,L_tlr); % Integrals of functions
defined above
I2_tlr=quadl('modeD2_tlr',0,L_tlr); % (along length of trailer)
I3_tlr=quadl('modeD3_tlr',0,L_tlr);
I4_tlr=quadl('modeD4_tlr',0,L_tlr);

E_a1=modeD1_t(b_a1);      % Disp at axle #1 due to tractor frame
beaming
E_cf=modeD1_t(b_cf);      % Disp at cab front due to tractor
frame beaming
E_e=modeD1_t(b_e);        % Disp at engine due to tractor frame
beaming
E_cr=modeD1_t(b_cr);      % Disp at cab rear due to tractor frame
beaming
E_a2=modeD1_t(b_a2);      % Disp at axle #2 due to tractor frame
beaming
E_fw=modeD1_t(b_fw);      % Disp at fifth wheel due to tractor
frame beaming
E_a3=modeD1_t(b_a3);      % Disp at axle #3 due to tractor frame
beaming
E_0=modeD1_tlr(0);        % Disp at fifth wheel due to trailer
beaming
E_a4=modeD1_tlr(b_a4);    % Disp at axle #4 due to trailer
beaming
```

```
E_a5=modeD1_tlr(b_a5);    % Disp at axle #5 due to trailer
beaming

% Seat Suspension Options
disp('  ')
disp('VEHICLE SUSPENSION OPTIONS')
disp('  ')
disp('Give your choice for seat suspension: ')
disp('Note: Without seat suspension gives a very high frequency
mode')
disp('      because the stiffness is set to a high value.')
disp('a : With seat suspension (~0.9 Hz)')
disp('b : Without seat suspension')
z11 = input('Enter your choice : ', 's');

if z11 == 'a',          % Choice 'a' is with seat suspension
    cs = 1140;          % Damping ratio of 0.5
    ks = 3403;          % N/m(spring const of seat suspension)

elseif z11 == 'b',      % Choice 'b' is without seat suspension
    cs = 1329;          % N/(m/s)(damping const of seat
suspension)
    ks = 1e10;          % N/m(spring const of seat suspension)

else disp('Insufficient information regarding seat suspension.')
end

% Cab Suspension Options
disp('  ')
disp('Give your choice for cab suspension: ')
disp('Note: With front or rear or without cab suspension')
disp('      gives a very high frequency mode(s) because the
corresponding')
disp('      stiffness(es)is set to a high value.')
disp('a : With front cab suspension')
disp('b : With rear cab suspension')
disp('c : With front & rear cab suspension')
disp('d : Without cab suspension')
z22 = input('Enter your choice : ', 's');

if z22 == 'a',          % Choice 'a' is front cab suspension
    ccf = 7062;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 88740;        % N/m(spring const of front cab
suspension)
    ccr = 6430;         % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;         % N/m(spring const of rear cab
suspension)

elseif z22 == 'b',      % Choice 'b' is rear cab suspension
    ccr = 8000;         % Reduced damping
    kcr = 65980;        % N/m(spring const of rear cab
suspension)
```

```
    ccf = 13120;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;          % N/m(spring const of front cab
suspension)

elseif z22 == 'c',       % Choice 'c' is front & rear cab
suspension
    ccr = 5073.5;        % N/(m/s)(damping const of rear cab
suspension)
    kcr = 63757.5;       % N/m(spring const of rear cab
suspension)
    ccf = 6864.35;       % N/(m/s)(damping const of front cab
suspension)
    kcf = 86260.5;       % N/m(spring const of front cab
suspension)

elseif z22 == 'd',       % Choice 'd' is without cab suspension
    ccr = 6430;          % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;          % N/m(spring const of rear cab
suspension)
    ccf = 7062;          % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;          % N/m(spring const of front cab
suspension)

else disp('Insufficient information regarding cab suspension.')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%  Vehicle Tire Selection
%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

disp('   ')
disp('STEER AXLE TIRE SELECTION')
TireData3;                  % M-file for tire data
wd1 = wd;                   %(m)        Nominal cross section
width
mt1 = mt;                   %(kg)       Mass of axle #1
P1 = P;                     %(psi)      Tire pressure from
TireData3.m
press1 = press;             %(psi)      Tire pressure array
numtires1 = numtires;       %           Number of tires on axle
Kstiff1 = Kstiff;           %(N/m)      Tire stiffness array
kt1 = KK * numtires1;       %(N/m)      Per-axle Rad Stiffness
ct1 = ct;                   %(N/(m/s))  Per-axle Damping

disp('   ')
disp('DRIVE AXLE TIRE SELECTION')
TireData3;                  % M-file for tire data
wd23 = wd;                  %(m)        Nominal cross section
width
mt2 = mt;                   %(kg)       Mass of axle #2
```

```
mt3 = mt;                    %(kg)       Mass of axle #3
P23 = P;                     %(psi)      Tire pressure from
TireData3.m
press23 = press;             %(psi)      Tire Pressure array
numtires23 = numtires;       %           Number of tires on axle
Kstiff23 = Kstiff;           %(N/m)      Tire stiffness array
kt2 = KK * numtires23;       %(N/m)      Per-axle Rad Stiffness
kt3 = KK * numtires23;       %(N/m)      Per-axle Rad Stiffness
ct2 = ct;                    %(N/(m/s))  Per-axle Damping
ct3 = ct;                    %(N/(m/s))  Per-axle Damping

disp('   ')
disp('TRAILER AXLE TIRE SELECTION')
TireData3;                   % M-file for tire data
wd45 = wd;                   %(m)        Nominal cross section
width
mt4 = mt;                    %(kg)       Mass of axle #4
mt5 = mt;                    %(kg)       Mass of axle #5
P45 = P;                     %(psi)      Tire pressure from
TireData3.m
press45 = press;             %(psi)      Tire Pressure array
numtires45 = numtires;       %           Number of tires on axle
Kstiff45 = Kstiff;           %(N/m)      Tire stiffness array
kt4 = KK * numtires45;       %(N/m)      Per-axle Rad Stiffness
kt5 = KK * numtires45;       %(N/m)      Per-axle Rad Stiffness
ct4 = ct;                    %(N/(m/s))  Per-axle Damping
ct5 = ct;                    %(N/(m/s))  Per-axle Damping

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%  Speed of the Vehicle  %%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('VEHICLE VELOCITY')
disp('  ')
disp('Please choose the unit of velocity');
disp('a : Miles per Hour (mph)');
disp('b : Kilometers per Hour (kph)');
vel = input('Input the unit of velocity (a/b):  ', 's');
disp('  ')
vm = input('Input the velocity of the vehicle, vm : ');

if vel == 'a'
    v = 0.4473*vm;              %Velocity conversion from mph to
m/s
elseif vel == 'b'
    v = 0.277778*vm;            %Velocity conversion from kph to
m/s
end

T(1) = 0;               %Time delay between front axle and
remaining axles
T(2) = (a+b)/v;         % Axle #2
T(3) = (a+d)/v;         % Axle #3
T(4) = (a+i+e+f)/v;     % Axle #4
T(5) = (a+i+e+h)/v;     % Axle #5
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%  Road PSD Selection  %%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('ROAD PSD SELECTION')
disp('  ')
disp('Road PSD Constants, m^2/cyc/m, Ref: Wong, Theory of Ground
Vehicles')
disp('S(W)=Csp/W^N  where W=spatial frequency')
disp('  ')
disp('a : Csp = 4.3e-11,N=3.8     Smooth Runway')
disp('b : Csp = 8.1e-6, N=2.1     Rough Runway')
disp('c : Csp = 4.8e-7, N=2.1     Smooth Highway')
disp('d : Csp = 4.4e-6, N=2.1     Highway with Gravel')
disp('  ')
tabchoice11=input('Input the road surface to be used :   ','s');

if tabchoice11== 'a',              % smooth runway
   Csp = 4.3e-11;
   N=3.8;

   elseif tabchoice11== 'b',       % rough runway
      Csp = 8.1e-6;
      N=2.1;

   elseif tabchoice11 == 'c',      % smooth highway
      Csp = 4.8e-7;
      N=2.1;

   elseif tabchoice11 == 'd',      % highway with gravel
      Csp = 4.4e-6;
      N=2.1;

end

% Start Loop on Beaming Frequencies
% Frequency will range from 10 Hz to 30 Hz for tractor and
trailer

for iiii=1:21;
    for jjjj=1:21;
        ffhz(iiii,jjjj)=iiii;
        ffhz2(iiii,jjjj)=jjjj;

        fhz = 9+ffhz(iiii,jjjj);
        fhz2 = 9+ffhz2(iiii,jjjj);

EI_t = 4*pi^2*fhz^2*(b_fw/kb1)^4*ML_t;       %Tractor frame
flexural rigidity
EI_tlr = 4*pi^2*fhz2^2*(L_tlr/kb2)^4*ML_tlr; %Trailer flexural
rigidity
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%  System Matrices
%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

% THE SYSTEM IS WRITTEN AS (M*S*S+C*S+K)X(S)=(A*S+B)U(S)

%%%%%%%%%%%%%%%%%%%%%%%
%%%%  Mass Matrix  %%%%
%%%%%%%%%%%%%%%%%%%%%%%
M = zeros(15,15);

M(1,1) = m_s;                   % Eqn #1: Vertical Disp of Seat

M(2,2) = m_c;                   % Eqn #2: Vertical Disp of Cab

M(3,3) = I_c;                   % Eqn #3: Pitch of Cab

M(4,4) = m_e;                   % Eqn #4: Vertical Disp of Engine

M(5,5) = m_t;                   % Eqn #5: Vertical Disp of
Tractor Frame
M(5,6) = ML_t*b_fw*(b_fw/2-a1);
M(5,7) = ML_t*I1_t;

M(6,5) = ML_t*b_fw*(b_fw/2-a1);   % Eqn #6: Pitch of Tractor
Frame
M(6,6) = I_t;
M(6,7) = -ML_t*I2_t;

M(7,5) = ML_t*I1_t;             % Eqn #7: Beaming of Tractor
Frame
M(7,6) = -ML_t*I2_t;
M(7,7) = ML_t*I3_t;

M(8,8) = m_tlr;                 % Eqn #8: Vertical Disp of
Trailer
M(8,9) = -ML_tlr*L_tlr*(e-L_tlr/2);
M(8,10) = ML_tlr*I1_tlr;

M(9,8) = -ML_tlr*L_tlr*(e-L_tlr/2);% Eqn #9: Pitch of Trailer
M(9,9) = I_tlr;
M(9,10) = -ML_tlr*I2_tlr;

M(10,8) = ML_tlr*I1_tlr;        % Eqn #10: Beaming of Trailer
M(10,9) = -ML_tlr*I2_tlr;
M(10,10) = ML_tlr*I3_tlr;

M(11,11) = mt1;                 % Eqn #11: Vertical Disp of Axle
#1
```

```
M(12,12) = mt2;                 % Eqn #12: Vertical Disp of Axle
#2

M(13,13) = mt3;                 % Eqn #13: Vertical Disp of Axle
#3

M(14,14) = mt4;                 % Eqn #14: Vertical Disp of Axle
#4

M(15,15) = mt5;                 % Eqn #15: Vertical Disp of Axle
#5

%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%
C = zeros(15,15);

C(1,1) = cs;
C(1,2) = -cs;
C(1,3) = r*cs;

C(2,1) = -cs;
C(2,2) = cs+ccf+ccr;
C(2,3) = -r*cs-n*ccf+p*ccr;
C(2,5) = -ccf-ccr;
C(2,6) = l*ccf-j*ccr;
C(2,7) = -ccf*E_cf-ccr*E_cr;

C(3,1) = r*cs;
C(3,2) = -r*cs-n*ccf+p*ccr;
C(3,3) = (r^2)*cs+(n^2)*ccf+(p^2)*ccr;
C(3,5) = n*ccf-p*ccr;
C(3,6) = -n*l*ccf-p*j*ccr;
C(3,7) = n*ccf*E_cf-p*ccr*E_cr;

C(4,4) = ce;
C(4,5) = -ce;
C(4,6) = m*ce;
C(4,7) = -ce*E_e;

C(5,2) = -ccf-ccr;
C(5,3) = n*ccf-p*ccr;
C(5,4) = -ce;
C(5,5) = ce+ccf+ccr+cfw+c1+c2+c3;
C(5,6) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(5,7) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(5,8) = -cfw;
C(5,9) = e*cfw;
C(5,10) = -cfw*E_0;
C(5,11) = -c1;
C(5,12) = -c2;
C(5,13) = -c3;

C(6,2) = l*ccf-j*ccr;
```

```
C(6,3) = -n*l*ccf-p*j*ccr;
C(6,4) = m*ce;
C(6,5) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(6,6) =
(m^2)*ce+(l^2)*ccf+(j^2)*ccr+(i^2)*cfw+(a^2)*c1+(b^2)*c2+(d^2)*c3
;
C(6,7) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2+...
         d*c3*E_a3;
C(6,8) = -i*cfw;
C(6,9) = e*i*cfw;
C(6,10) = -i*cfw*E_0;
C(6,11) = a*c1;
C(6,12) = -b*c2;
C(6,13) = -d*c3;

C(7,2) = -ccf*E_cf-ccr*E_cr;
C(7,3) = n*ccf*E_cf-p*ccr*E_cr;
C(7,4) = -ce*E_e;
C(7,5) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(7,6) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2 ...
         +d*c3*E_a3;
C(7,7) =
ce*E_e^2+ccf*E_cf^2+ccr*E_cr^2+cfw*E_fw^2+c1*E_a1^2+c2*E_a2^2 ...
         +c3*E_a3^2;
C(7,8) = -cfw*E_fw;
C(7,9) = e*cfw*E_fw;
C(7,10) = -cfw*E_0*E_fw;
C(7,11) = -c1*E_a1;
C(7,12) = -c2*E_a2;
C(7,13) = -c3*E_a3;

C(8,5) = -cfw;
C(8,6) = -i*cfw;
C(8,7) = -cfw*E_fw;
C(8,8) = cfw+c4+c5;
C(8,9) = -e*cfw+f*c4+h*c5;
C(8,10) = cfw*E_0+c4*E_a4+c5*E_a5;
C(8,14) = -c4;
C(8,15) = -c5;

C(9,5) = e*cfw;
C(9,6) = e*i*cfw;
C(9,7) = e*cfw*E_fw;
C(9,8) = -e*cfw+f*c4+h*c5;
C(9,9) = (e^2)*cfw+(f^2)*c4+(h^2)*c5;
C(9,10) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(9,14) = -f*c4;
C(9,15) = -h*c5;

C(10,5) = -cfw*E_0;
C(10,6) = -i*cfw*E_0;
C(10,7) = -cfw*E_fw*E_0;
C(10,8) = cfw*E_0+c4*E_a4+c5*E_a5;
```

```
C(10,9) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(10,10) = cfw*E_0^2+c4*E_a4^2+c5*E_a5^2;
C(10,14) = -c4*E_a4;
C(10,15) = -c5*E_a5;

C(11,5) = -c1;
C(11,6) = a*c1;
C(11,7) = -c1*E_a1;
C(11,11) = c1+ct1;

C(12,5) = -c2;
C(12,6) = -b*c2;
C(12,7) = -c2*E_a2;
C(12,12) = c2+ct2;

C(13,5) = -c3;
C(13,6) = -d*c3;
C(13,7) = -c3*E_a3;
C(13,13) = c3+ct3;

C(14,8) = -c4;
C(14,9) = -f*c4;
C(14,10) = -c4*E_a4;
C(14,14) = c4+ct4;

C(15,8) = -c5;
C(15,9) = -h*c5;
C(15,10) = -c5*E_a5;
C(15,15) = c5+ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%
K = zeros(15,15);

K(1,1) = ks;
K(1,2) = -ks;
K(1,3) = r*ks;

K(2,1) = -ks;
K(2,2) = ks+kcf+kcr;
K(2,3) = -r*ks-n*kcf+p*kcr;
K(2,5) = -kcf-kcr;
K(2,6) = l*kcf-j*kcr;
K(2,7) = -kcf*E_cf-kcr*E_cr;

K(3,1) = r*ks;
K(3,2) = -r*ks-n*kcf+p*kcr;
K(3,3) = (r^2)*ks+(n^2)*kcf+(p^2)*kcr;
K(3,5) = n*kcf-p*kcr;
K(3,6) = -n*l*kcf-p*j*kcr;
K(3,7) = n*kcf*E_cf-p*kcr*E_cr;

K(4,4) = ke;
K(4,5) = -ke;
```

```
K(4,6) = m*ke;
K(4,7) = -ke*E_e;

K(5,2) = -kcf-kcr;
K(5,3) = n*kcf-p*kcr;
K(5,4) = -ke;
K(5,5) = ke+kcf+kcr+kfw+k1+k2+k3;
K(5,6) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(5,7) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(5,8) = -kfw;
K(5,9) = e*kfw;
K(5,10) = -kfw*E_0;
K(5,11) = -k1;
K(5,12) = -k2;
K(5,13) = -k3;

K(6,2) = l*kcf-j*kcr;
K(6,3) = -n*l*kcf-p*j*kcr;
K(6,4) = m*ke;
K(6,5) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(6,6) =
(m^2)*ke+(l^2)*kcf+(j^2)*kcr+(i^2)*kfw+(a^2)*k1+(b^2)*k2+(d^2)*k3
;
K(6,7) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2+...
         d*k3*E_a3;
K(6,8) = -i*kfw;
K(6,9) = e*i*kfw;
K(6,10) = -i*kfw*E_0;
K(6,11) = a*k1;
K(6,12) = -b*k2;
K(6,13) = -d*k3;

K(7,2) = -kcf*E_cf-kcr*E_cr;
K(7,3) = n*kcf*E_cf-p*kcr*E_cr;
K(7,4) = -ke*E_e;
K(7,5) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(7,6) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2 ...
         +d*k3*E_a3;
K(7,7) =
ke*E_e^2+kcf*E_cf^2+kcr*E_cr^2+kfw*E_fw^2+k1*E_a1^2+k2*E_a2^2 ...
         +k3*E_a3^2+EI_t*I4_t;
K(7,8) = -kfw*E_fw;
K(7,9) = e*kfw*E_fw;
K(7,10) = -kfw*E_0*E_fw;
K(7,11) = -k1*E_a1;
K(7,12) = -k2*E_a2;
K(7,13) = -k3*E_a3;

K(8,5) = -kfw;
K(8,6) = -i*kfw;
K(8,7) = -kfw*E_fw;
K(8,8) = kfw+k4+k5;
```

```
K(8,9) = -e*kfw+f*k4+h*k5;
K(8,10) = kfw*E_0+k4*E_a4+k5*E_a5;
K(8,14) = -k4;
K(8,15) = -k5;

K(9,5) = e*kfw;
K(9,6) = e*i*kfw;
K(9,7) = e*kfw*E_fw;
K(9,8) = -e*kfw+f*k4+h*k5;
K(9,9) = (e^2)*kfw+(f^2)*k4+(h^2)*k5;
K(9,10) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(9,14) = -f*k4;
K(9,15) = -h*k5;

K(10,5) = -kfw*E_0;
K(10,6) = -i*kfw*E_0;
K(10,7) = -kfw*E_fw*E_0;
K(10,8) = kfw*E_0+k4*E_a4+k5*E_a5;
K(10,9) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(10,10) = kfw*E_0^2+k4*E_a4^2+k5*E_a5^2+EI_tlr*I4_tlr;
K(10,14) = -k4*E_a4;
K(10,15) = -k5*E_a5;

K(11,5) = -k1;
K(11,6) = a*k1;
K(11,7) = -k1*E_a1;
K(11,11) = k1+kt1;

K(12,5) = -k2;
K(12,6) = -b*k2;
K(12,7) = -k2*E_a2;
K(12,12) = k2+kt2;

K(13,5) = -k3;
K(13,6) = -d*k3;
K(13,7) = -k3*E_a3;
K(13,13) = k3+kt3;

K(14,8) = -k4;
K(14,9) = -f*k4;
K(14,10) = -k4*E_a4;
K(14,14) = k4+kt4;

K(15,8) = -k5;
K(15,9) = -h*k5;
K(15,10) = -k5*E_a5;
K(15,15) = k5+kt5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A = zeros(15,1);

A(11) = ct1;
A(12) = ct2;
```

```
A(13) = ct3;
A(14) = ct4;
A(15) = ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
B = zeros(15,1);

B(11) = kt1;
B(12) = kt2;
B(13) = kt3;
B(14) = kt4;
B(15) = kt5;

% System "A" Matrix
AA=[zeros(size(M))   eye(size(M))   % System state variable
matrix
    -inv(M)*K       -inv(M)*C];

% ISO 2631 FOR REDUCED COMFORT BOUNDARY
% COMFORT BOUNDARIES FOR VERTICAL ACCELERATION
% THE ISO CENTRAL FREQUENCIES (Hz)

wc=[ .1 1 1.25 1.6 2 2.5 3.15 4 5 6.3 8 10 12.5 16 20 25 31.5 40
50];
whzc=[ .1 .125 .16 .2 .25 .315 .4 .5 .63 .8 1 1.25 1.6 2 2.5 3.15
...
    4 5 6.3 8 10 12.5 16 20 25 31.5 40 50];

% 2.5 hr FATIGUE BOUNDARY
fat1=[4.284,1.4,1.25,1.12,1,.9,.8,.71,.71,.71,.71,...
      .9,1.12,1.4,1.8,2.24,2.8,3.55,4.5];
% 2.5hr REDUCED COMFORT BOUNDARY
comf1=fat1/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf2= comf1/2.254;

%-----------------------------------------------------------------
---
% COMFORT BOUNDARIES FOR LONGITUDINAL AND LATERAL ACC
% 2.5hr FATIGUE BOUNDARY
fat2=[0.5,0.5,0.5,0.5,0.5,0.63,0.8,1,1.25,1.6,2,2.5,3.15,4,5,6.3,
8,10,12.5];

% 2.5hr REDUCED COMFORT BOUNDARY
comf3=fat2/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf4= comf3/2.254;
%-----------------------------------------------------------------
--

whzcr = 2*pi*whzc;      % Calculation of central frequencies in
rad/s
freqlow=0.89*whzcr;     % Lower octave band
```

```
freqhigh=1.12*whzcr;    % Upper octave band
freq=[freqlow' whzcr' freqhigh'];

imag=sqrt(-1);

for ii=1:length(whzc);
    for jj=1:3;         % jj=1 is freqlow, jj=2 is center freq
                        % jj=3 is freqhigh
        w = freq(ii,jj);
        s = imag*w;
        dp = sqrt(h1^2+r^2);

        % Time delay array
        time = [0 0 0 0 0 0 0 0 0 0 1 exp(-s*T(2)) exp(-s*T(3))
...
            exp(-s*T(4)) exp(-s*T(5))];

        % TF Matrix
        vectx = (inv(M*s*s+C*s+K)*((A*s+B).*(time.')));

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %%%  Transfer Functions  %%%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%

        z_s=[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]*vectx;    % vert seat
cg
        long=[0 0 -h1 0 0 0 0 0 0 0 0 0 0 0 0]*vectx; % long disp
of driver
        z_tlr=[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]*vectx;  % vert
trailer cg

        %%%%%%%%%%%%%%%%%%%%
        %%%  Magnitudes  %%%
        %%%%%%%%%%%%%%%%%%%%

        % Acceleration Transter Functions
        magcfA1(ii,jj)=abs(s*s*z_s);  % Mag of trans function,
(m/s*s)/m
        magcfAlong(ii,jj)=abs(s*s*long);
        magcftlr(ii,jj)=abs(s*s*z_tlr);

        %%%%%%%%%%%%%%
        %%%  PSDs  %%%
        %%%%%%%%%%%%%%

        % Road PSD in m*m/(rad/s)
        rpsd(ii,jj)=Csp*(((2*pi*v)^(N-1))/(w^N));

        % Acceleration PSDs in (m/s^2)^2/(rad/s)
        psdcfA1(ii,jj)=magcfA1(ii,jj)*magcfA1(ii,jj)*rpsd(ii,jj);

psdcfAlong(ii,jj)=magcfAlong(ii,jj)*magcfAlong(ii,jj)*rpsd(ii,jj)
;

psdcftlr(ii,jj)=magcftlr(ii,jj)*magcftlr(ii,jj)*rpsd(ii,jj);
```

```
    end
end

for kk=1:length(whzc)
    % Vert. Driver's Seat RMS
    msqy1a(kk)=0.5*(psdcfA1(kk,1)+psdcfA1(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqy1b(kk)=0.5*(psdcfA1(kk,2)+psdcfA1(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqy1(kk)=msqy1a(kk)+msqy1b(kk);
    rmsA1cf(kk)=sqrt(msqy1(kk));
    % Long. Driver RMS
    msqylonga(kk)=0.5*(psdcfAlong(kk,1)+psdcfAlong(kk,2))*...
     (freq(kk,2)-freq(kk,1));
    msqylongb(kk)=0.5*(psdcfAlong(kk,2)+psdcfAlong(kk,3))*...
     (freq(kk,3)-freq(kk,2));
    msqylong(kk)=msqylonga(kk)+msqylongb(kk);
    rmsAlongcf(kk)=sqrt(msqylong(kk));
    % Vert. Trailer cg RMS
    msqytlra(kk)=0.5*(psdcftlr(kk,1)+psdcftlr(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqytlrb(kk)=0.5*(psdcftlr(kk,2)+psdcftlr(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqytlr(kk)=msqytlra(kk)+msqytlrb(kk);
    rmstlrcf(kk)=sqrt(msqytlr(kk));
end

RMScf = [rmsA1cf',rmsAlongcf',rmstlrcf'];       % Accel. RMS
Matrix

% Calculate weighted rms acceleration from 0.1 to 50 Hz
% at the ISO Center Frequencies .... Wgt are the ISO weights
% Ref: ISO 2631-1:1997(E); V=vertical; L=longitudinal

wcc=[.1,.125,.16,.2,.25,.315,.4,.5,.63,.8,1,1.25,1.6,2,2.5,3.15,4
,5,...
    6.3,8,10,12.5,16,20,25,31.5,40,50];
WgtV=[.0312,.0486,.079,.121,.182,.263,.352,.418,.459,.477,.482,.4
84,...

.494,.531,.631,.804,.967,1.039,1.054,1.036,.988,.902,.768,.636,..
.
    .513,.405,.314,.246];
WgtL=0.001*[62.4,97.3,158,243,365,530,713,853,944,992,1011,1008,9
68,...

890,776,642,512,409,323,253,212,161,125,100,80,63.2,49.4,38.8];

isovert = WgtV.*RMScf(1:28,1)';     % Weighted Vert. Driver RMS
Accel.
isolong = WgtL.*RMScf(1:28,2)';     % Weighted Long. Driver RMS
Accel.
isotlr = WgtV.*RMScf(1:28,3)';      % Weighted Vert. Trailer RMS
Accel.
```

```
term2V=(WgtV.*rmsA1cf(1:28)).^2;
a0_V_dr=(sum(term2V))^0.5;            % a0 for vert. disp of
driver

term2L=(WgtL.*rmsAlongcf(1:28)).^2;
a0_L_dr=(sum(term2L))^0.5;            % a0 for long. disp of
driver

aV=(a0_L_dr^2 + a0_V_dr^2)^0.5;  % a0 for comb vert and long disp

tlrV=(WgtV.*rmstlrcf(1:28)).^2;
a0_V_tlr=(sum(tlrV))^0.5;             % a0 for vert. disp of
driver


a0_VV_dr(iiii,jjjj)=a0_V_dr;    % vertical ISO wgt acc, m/s^2
a0_LL_dr(iiii,jjjj)=a0_L_dr;    % longitudinal ISO wgt acc, m/s^2
aVV(iiii,jjjj)=aV;              % combined ISO wgt acc, m/s^2

    end         % end of jjjj loop on fhz2
end             % end of iiii loop on fhz

disp('  ')
disp('RESULTS OF PARAMETER VARIATION')
disp('  ')
disp('Minimum aV, m/s^2')
disp(min(aVV(:)))
disp('  ')
[ia,ja]=find(aVV==min(aVV(:)));
disp('Corresponding Tractor Beaming Frequency, Hz')
disp([9+ffhz(ia,ja)])
disp('  ')
disp('Corresponding Trailer Beaming Frequency, Hz')
disp([9+ffhz2(ia,ja)])
disp('  ')

figure(1)
surf(9+ffhz,9+ffhz2,aVV)
xlabel('Tractor Beaming, Hz')
ylabel('Trailer Beaming, Hz')
zlabel('ISO Combined Wgt Acc, m/s^2')
% title('Tractor Suspension Stiffness Parameter Variation')

figure(2)
surf(9+ffhz,9+ffhz2,a0_VV_dr)
xlabel('Tractor Beaming, Hz')
ylabel('Trailer Beaming, Hz')
zlabel('ISO Vertical Wgt Acc, m/s^2')

figure(3)
surf(9+ffhz,9+ffhz2,a0_LL_dr)
xlabel('Tractor Beaming, Hz')
ylabel('Trailer Beaming, Hz')
zlabel('ISO Long Wgt Acc, m/s^2')
```

Appendix M: opt_5wKC_freq.m

This parameter variation program varies the stiffness and damping values across the fifth wheel, assuming that a fifth wheel suspension system has been implemented. The values for the fifth wheel suspension stiffness range from 50,000 N/m to 1,000,000 N/m in increments of 50,000 N/m. The lower end of this range was chosen by observing the RMS stroke across the fifth wheel at different values for the stiffness, and the higher end is meant to simulate a rigid connection. The values for the fifth wheel suspension damping range from 2,000 N/(m/s) to 40,000 N/(m/s) in increments of 2,000 N/(m/s). These values were chosen by inserting values for the fifth wheel suspension damping into the dof15_freq2.m simulation and observing the damping ratios at the eigenvalues corresponding to motions across the fifth wheel.

The desired output values from this program were the ISO combined weighted acceleration at the driver's seat, the ISO vertical weighted acceleration at the trailer center-of-gravity (CG), a value called the J penalty, which weighs the importance of the driver ride comfort versus trailer acceleration using weights assigned to them by the user, and the RMS stroke across the fifth wheel. The program finds the minimum values for the accelerations and the J penalty, and displays them in tabular form along with the corresponding stiffness and damping values for the fifth wheel suspension system. Also, the program plots the output information on surface plots to study trends in the information.

opt_5wKC_freq.m

```
% opt_5wKC_freq.m
% Developed by Ryan Spivey, 4/10/07
%
% Varies 5th wheel suspension parameters using weighted RMS
acceleration
% in the frequecncy domain
%
% Incorporates model from dof15_freq2.m
%
% DOFs include - 1)Vertical Disp. of Driver's Seat
%                2)Vertical Disp. of Cab
%                3)Pitch of Cab
%                4)Vertical Disp. of Engine
%                5)Vertical Disp. of Tractor Frame
%                6)Pitch of Tractor Frame
%                7)Beaming of Tractor Frame
%                8)Vertical Disp. of Trailer
%                9)Pitch of Trailer
%               10)Beaming of Trailer
%               11)Vertical Disp. of Axle #1
%               12)Vertical Disp. of Axle #2
%               13)Vertical Disp. of Axle #3
%               14)Vertical Disp. of Axle #4
%               15)Vertical Disp. of Axle #5

clc
clear all
% close all
format short e
format compact

global D1_t D2_t D3_t D4_t D1_tlr D2_tlr D3_tlr D4_tlr
global e a1 kb1 kb2 b_fw L_tlr alpha

disp('  ')
disp('5th Wheel Parameter Variation in the Frequency Domain')
disp('                  Roadholding Model                    ')
disp(['                      ',date])

% Choose a test vehicle
disp('  ')
disp('VEHICLE SELECTION')
disp('  ')
disp('Please choose a vehicle : ');
disp('a: Ideal Tractor Semi-Trailer');
vehicle = input('Enter your choice : ', 's');

if vehicle == 'a'
    % Inertial Properties
    m_s = 106.7;        %kg      mass of seat
    m_c = 1208;         %kg      mass of cab
    I_c = 2100;         %kg*m^2  M I of cab
    m_e = 2000;         %kg      mass of engine (ESTIMATE)
```

```
    m_t = 3783;          %kg      mass of tractor (5783 kg -
engine)
    I_t = 46590.9;       %kg*m^2  M I of tractor
    m_ul = 10800;        %kg      mass of trailer (ESTIMATE)
    I_tlr = 200000;      %kg*m^2  M I of trailer
    m_L = 14000;         %kg      mass of trailer load  (ESTIMATE)
    m_tlr = m_ul+m_L;    %kb      mass of loaded trailer

    % Suspension Parameters
    c1 = 11270;          %N/(m/s) damping const of axle #1
    c2 = 27500;          %N/(m/s) damping const of axle #2
    c3 = 27500;          %N/(m/s) damping const of axle #3
    c4 = 70000;          %N/(m/s) damping const of axle #4
    c5 = 70000;          %N/(m/s) damping const of axle #5
    ce = 10000;          %N/(m/s) damping const of engine mount
    k1 = 581300;         %N/m     spring const of axle #1
    k2 = 586900;         %N/m     spring const of axle #2
    k3 = 586900;         %N/m     spring const of axle #3
    k4 = 1000000;        %N/m     spring const of axle #4
    k5 = 1000000;        %N/m     spring const of axle #5
    ke = 1e10;           %N/m     spring const of the engine mount

    % Model Dimensions
    b_a1 = 1.065;        %m       Front end of the tractor to axle
#1
    b_cf = 1.470;        %m       Front end of the tractor to cab
front
    b_e = 2.797;         %m       Front end of the tractor to
engine
    b_cr = 4.02;         %m       Front end of the tractor to cab
rear
    b_a2 = 6.035;        %m       Front end of the tractor to axle
#2
    b_fw = 6.688;        %m       Front end of the tractor to 5th
wheel
    b_a3 = 7.34;         %m       Front end of the tractor to axle
#3
    a1 = 4.00607;        %m       Front end of the tractor to
tractor cg

    b_a4 = 8.58;         %m       From the fifth wheel to axle #4
    b_a5 = 9.78;         %m       From the fifth wheel to axle #5

    L_t = 8.2;           %m       Length of Tractor
    L_tlr = 9.78;        %m       Length of Trailer

    e = 5.62;            %m       From the trailer cg to fifth
wheel
    f = 2.96;            %m       From the trailer cg to axle #4
    h = 4.16;            %m       From the trailer cg to axle #5

    a = 2.94107;         %m       From the tractor cg to axle #1
    b = 2.02893;         %m       From the tractor cg to axle #2
    d = 3.33393;         %m       From the tractor cg to axle #3
    l = 2.53607;         %m       From the tractor cg to cab front
    m = 1.209074;        %m       From the tractor cg to engine
```

```
    j = 0.013926;       %m       From the tractor cg to cab rear
    i = 2.68193;        %m       From the tractor cg to the fifth
wheel
    n = 1.435;          %m       From the cab cg to cab front
    p = 1.115;          %m       From the cab cg to cab rear
    r = -0.200;         %m       From the cab cg to seat

    tc = 1.10107;       %m       From the tractor cg to the cab
cg
    h1 = 1.0;           %m       Height of the driver over the
cab
    g = 9.8;            %m/s^2   acceleration due to gravity

    ML_t = m_t/L_t;         %kg/m    Mass per unit length
(Tractor)
    ML_tlr = m_ul/L_tlr;    %kg/m    Mass per unit length
(Trailer)
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%  Fifth Wheel Configuration
%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

% The parameters for the first bending mode of the Tractor frame
disp('  ')
fhz = input('Input the Tractor frequency of beaming (hz) fhz :
');

% The parameters for the first bending mode of the Trailer frame
disp('  ')
fhz2 = input('Input the Trailer frequency of beaming (hz) fhz :
');

kb1 = 4.73004074;    %Constant for the first bending mode (free-
free)
alpha = 0.982502;

z1 = 'cosh(kb1*x1/b_fw) + cos(kb1*x1/b_fw) - ...
alpha*(sinh(kb1*x1/b_fw)+sin(kb1*x1/b_fw))';
% free-free beam mode function
z1dd = '(kb1/b_fw)^2*(cosh(kb1*x1/b_fw) - cos(kb1*x1/b_fw) - ...
alpha*(sinh(kb1*x1/b_fw)-sin(kb1*x1/b_fw)))';
% second derivative of free-free beam mode function

kb2 = 4.73004074;    %Constant for the first bending mode (free-
free)

z2 = 'cosh(kb2*x2/L_tlr) + cos(kb2*x2/L_tlr) - ...
alpha*(sinh(kb2*x2/L_tlr)+sin(kb2*x2/L_tlr))';
% free-free beam mode function
z2dd = '(kb2/L_tlr)^2*(cosh(kb2*x2/L_tlr) - cos(kb2*x2/L_tlr) -
... alpha*(sinh(kb2*x2/L_tlr)-sin(kb2*x2/L_tlr)))';
```

```
% second derivative of free-free beam mode function

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%%%%%%%%%%%%%%%%%% Computation of Integrals
%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

D1_t=['(',z1,')'];                    % Tractor frame beaming
equations to be
D2_t=['((a1-x1).*(',z1,'))'];         % used in the integrals
(string form)
D3_t=['((',z1,').*(',z1,'))'];
D4_t=['((',z1dd,').*(',z1dd,'))'];

D1_tlr=['(',z2,')'];                  % Trailer beaming equations
to be
D2_tlr=['((e-x2).*(',z2,'))'];        % used in the integrals
(string form)
D3_tlr=['((',z2,').*(',z2,'))'];
D4_tlr=['((',z2dd,').*(',z2dd,'))'];

I1_t=quadl('modeD1_t',0,b_fw);        % Integrals of functions
defined above
I2_t=quadl('modeD2_t',0,b_fw);        % (along length of tractor
frame)
I3_t=quadl('modeD3_t',0,b_fw);
I4_t=quadl('modeD4_t',0,b_fw);

I1_tlr=quadl('modeD1_tlr',0,L_tlr); % Integrals of functions
defined above
I2_tlr=quadl('modeD2_tlr',0,L_tlr); % (along length of trailer)
I3_tlr=quadl('modeD3_tlr',0,L_tlr);
I4_tlr=quadl('modeD4_tlr',0,L_tlr);

E_a1=modeD1_t(b_a1);       % Disp at axle #1 due to tractor frame
beaming
E_cf=modeD1_t(b_cf);       % Disp at cab front due to tractor
frame beaming
E_e=modeD1_t(b_e);         % Disp at engine due to tractor frame
beaming
E_cr=modeD1_t(b_cr);       % Disp at cab rear due to tractor frame
beaming
E_a2=modeD1_t(b_a2);       % Disp at axle #2 due to tractor frame
beaming
E_fw=modeD1_t(b_fw);       % Disp at fifth wheel due to tractor
frame beaming
E_a3=modeD1_t(b_a3);       % Disp at axle #3 due to tractor frame
beaming
E_0=modeD1_tlr(0);         % Disp at fifth wheel due to trailer
beaming
E_a4=modeD1_tlr(b_a4);     % Disp at axle #4 due to trailer
beaming
E_a5=modeD1_tlr(b_a5);     % Disp at axle #5 due to trailer
beaming
```

```
EI_t = 4*pi^2*fhz^2*(b_fw/kb1)^4*ML_t;       %Tractor frame
flexural rigidity
EI_tlr = 4*pi^2*fhz2^2*(L_tlr/kb2)^4*ML_tlr; %Trailer flexural
rigidity

% Seat Suspension Options
disp('  ')
disp('VEHICLE SUSPENSION OPTIONS')
disp('  ')
disp('Give your choice for seat suspension: ')
disp('Note: Without seat suspension gives a very high frequency
mode')
disp('      because the stiffness is set to a high value.')
disp('a : With seat suspension (~0.9 Hz)')
disp('b : Without seat suspension')
z11 = input('Enter your choice : ', 's');

if z11 == 'a',          % Choice 'a' is with seat suspension
    cs = 1140;          % Damping ratio of 0.5
    ks = 3403;          % N/m(spring const of seat suspension)

elseif z11 == 'b',      % Choice 'b' is without seat suspension
    cs = 1329;          % N/(m/s)(damping const of seat
suspension)
    ks = 1e10;          % N/m(spring const of seat suspension)

else disp('Insufficient information regarding seat suspension.')
end

% Cab Suspension Options
disp('  ')
disp('Give your choice for cab suspension: ')
disp('Note: With front or rear or without cab suspension')
disp('      gives a very high frequency mode(s) because the
corresponding')
disp('      stiffness(es)is set to a high value.')
disp('a : With front cab suspension')
disp('b : With rear cab suspension')
disp('c : With front & rear cab suspension')
disp('d : Without cab suspension')
z22 = input('Enter your choice : ', 's');

if z22 == 'a',          % Choice 'a' is front cab suspension
    ccf = 7062;         % N/(m/s)(damping const of front cab
suspension)
    kcf = 88740;        % N/m(spring const of front cab
suspension)
    ccr = 6430;         % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;         % N/m(spring const of rear cab
suspension)

elseif z22 == 'b',      % Choice 'b' is rear cab suspension
    ccr = 8000;         % Reduced damping
```

```
    kcr = 65980;          % N/m(spring const of rear cab
suspension)
    ccf = 13120;          % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;           % N/m(spring const of front cab
suspension)

elseif z22 == 'c',        % Choice 'c' is front & rear cab
suspension
    ccr = 5073.5;         % N/(m/s)(damping const of rear cab
suspension)
    kcr = 63757.5;        % N/m(spring const of rear cab
suspension)
    ccf = 6864.35;        % N/(m/s)(damping const of front cab
suspension)
    kcf = 86260.5;        % N/m(spring const of front cab
suspension)

elseif z22 == 'd',        % Choice 'd' is without cab suspension
    ccr = 6430;           % N/(m/s)(damping const of rear cab
suspension)
    kcr = 1e10;           % N/m(spring const of rear cab
suspension)
    ccf = 7062;           % N/(m/s)(damping const of front cab
suspension)
    kcf = 1e10;           % N/m(spring const of front cab
suspension)

else disp('Insufficient information regarding cab suspension.')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%  Vehicle Tire Selection
%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

disp('   ')
disp('STEER AXLE TIRE SELECTION')
TireData3;                    % M-file for tire data
wd1 = wd;                     %(m)        Nominal cross section
width
mt1 = mt;                     %(kg)       Mass of axle #1
P1 = P;                       %(psi)      Tire pressure from
TireData3.m
press1 = press;               %(psi)      Tire pressure array
numtires1 = numtires;         %           Number of tires on axle
Kstiff1 = Kstiff;             %(N/m)      Tire stiffness array
kt1 = KK * numtires1;         %(N/m)      Per-axle Rad Stiffness
ct1 = ct;                     %(N/(m/s))  Per-axle Damping

disp('   ')
disp('DRIVE AXLE TIRE SELECTION')
TireData3;                    % M-file for tire data
```

```
wd23 = wd;                  %(m)        Nominal cross section
width
mt2 = mt;                   %(kg)       Mass of axle #2
mt3 = mt;                   %(kg)       Mass of axle #3
P23 = P;                    %(psi)      Tire pressure from
TireData3.m
press23 = press;            %(psi)      Tire Pressure array
numtires23 = numtires;      %           Number of tires on axle
Kstiff23 = Kstiff;          %(N/m)      Tire stiffness array
kt2 = KK * numtires23;      %(N/m)      Per-axle Rad Stiffness
kt3 = KK * numtires23;      %(N/m)      Per-axle Rad Stiffness
ct2 = ct;                   %(N/(m/s))  Per-axle Damping
ct3 = ct;                   %(N/(m/s))  Per-axle Damping

disp('   ')
disp('TRAILER AXLE TIRE SELECTION')
TireData3;                  % M-file for tire data
wd45 = wd;                  %(m)        Nominal cross section
width
mt4 = mt;                   %(kg)       Mass of axle #4
mt5 = mt;                   %(kg)       Mass of axle #5
P45 = P;                    %(psi)      Tire pressure from
TireData3.m
press45 = press;            %(psi)      Tire Pressure array
numtires45 = numtires;      %           Number of tires on axle
Kstiff45 = Kstiff;          %(N/m)      Tire stiffness array
kt4 = KK * numtires45;      %(N/m)      Per-axle Rad Stiffness
kt5 = KK * numtires45;      %(N/m)      Per-axle Rad Stiffness
ct4 = ct;                   %(N/(m/s))  Per-axle Damping
ct5 = ct;                   %(N/(m/s))  Per-axle Damping

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%  Speed of the Vehicle  %%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('VEHICLE VELOCITY')
disp('  ')
disp('Please choose the unit of velocity');
disp('a : Miles per Hour (mph)');
disp('b : Kilometers per Hour (kph)');
vel = input('Input the unit of velocity (a/b):  ', 's');
disp('  ')
vm = input('Input the velocity of the vehicle, vm : ');

if vel == 'a'
    v = 0.4473*vm;              %Velocity conversion from mph to
m/s
elseif vel == 'b'
    v = 0.277778*vm;            %Velocity conversion from kph to
m/s
end

T(1) = 0;               %Time delay between front axle and
remaining axles
T(2) = (a+b)/v;         % Axle #2
```

```
T(3) = (a+d)/v;         % Axle #3
T(4) = (a+i+e+f)/v;     % Axle #4
T(5) = (a+i+e+h)/v;     % Axle #5

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%  Road PSD Selection  %%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('  ')
disp('ROAD PSD SELECTION')
disp('  ')
disp('Road PSD Constants, m^2/cyc/m, Ref: Wong, Theory of Ground
Vehicles')
disp('S(W)=Csp/W^N  where W=spatial frequency')
disp('  ')
disp('a : Csp = 4.3e-11,N=3.8      Smooth Runway')
disp('b : Csp = 8.1e-6, N=2.1      Rough Runway')
disp('c : Csp = 4.8e-7, N=2.1      Smooth Highway')
disp('d : Csp = 4.4e-6, N=2.1      Highway with Gravel')
disp('  ')
tabchoice11=input('Input the road surface to be used :   ','s');

if tabchoice11== 'a',               % smooth runway
   Csp = 4.3e-11;
   N=3.8;

   elseif tabchoice11== 'b',        % rough runway
      Csp = 8.1e-6;
      N=2.1;

   elseif tabchoice11 == 'c',       % smooth highway
      Csp = 4.8e-7;
      N=2.1;

   elseif tabchoice11 == 'd',       % highway with gravel
      Csp = 4.4e-6;
      N=2.1;

end

disp('  ')
disp('J PENALTY OPTIONS')
disp('  ')
disp('Input the values for K1 and K2 in the J penalty function')
disp('Note: Both values should add up to 1')
disp('  ')
K_1 = input('Input the value for K1 : ');
disp('  ')
K_2 = input('Input the value for K2 : ');

% Start Loop on 5th Wheel Suspension Properties

for iiii=1:20;
    for jjjj=1:20;
        kfww(iiii,jjjj)=50000*iiii;
```

```
        cfww(iiii,jjjj)=2000*jjjj;

        kfw = kfww(iiii,jjjj);
        cfw = cfww(iiii,jjjj);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%  System Matrices
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%

% THE SYSTEM IS WRITTEN AS (M*S*S+C*S+K)X(S)=(A*S+B)U(S)

%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%  Mass Matrix  %%%%
%%%%%%%%%%%%%%%%%%%%%%%%%
M = zeros(15,15);

M(1,1) = m_s;                   % Eqn #1: Vertical Disp of Seat

M(2,2) = m_c;                   % Eqn #2: Vertical Disp of Cab

M(3,3) = I_c;                   % Eqn #3: Pitch of Cab

M(4,4) = m_e;                   % Eqn #4: Vertical Disp of Engine

M(5,5) = m_t;                   % Eqn #5: Vertical Disp of
Tractor Frame
M(5,6) = ML_t*b_fw*(b_fw/2-a1);
M(5,7) = ML_t*I1_t;

M(6,5) = ML_t*b_fw*(b_fw/2-a1);   % Eqn #6: Pitch of Tractor
Frame
M(6,6) = I_t;
M(6,7) = -ML_t*I2_t;

M(7,5) = ML_t*I1_t;             % Eqn #7: Beaming of Tractor
Frame
M(7,6) = -ML_t*I2_t;
M(7,7) = ML_t*I3_t;

M(8,8) = m_tlr;                 % Eqn #8: Vertical Disp of
Trailer
M(8,9) = -ML_tlr*L_tlr*(e-L_tlr/2);
M(8,10) = ML_tlr*I1_tlr;

M(9,8) = -ML_tlr*L_tlr*(e-L_tlr/2);% Eqn #9: Pitch of Trailer
M(9,9) = I_tlr;
M(9,10) = -ML_tlr*I2_tlr;

M(10,8) = ML_tlr*I1_tlr;        % Eqn #10: Beaming of Trailer
M(10,9) = -ML_tlr*I2_tlr;
M(10,10) = ML_tlr*I3_tlr;
```

```
M(11,11) = mt1;                 % Eqn #11: Vertical Disp of Axle
#1

M(12,12) = mt2;                 % Eqn #12: Vertical Disp of Axle
#2

M(13,13) = mt3;                 % Eqn #13: Vertical Disp of Axle
#3

M(14,14) = mt4;                 % Eqn #14: Vertical Disp of Axle
#4

M(15,15) = mt5;                 % Eqn #15: Vertical Disp of Axle
#5

%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%
C = zeros(15,15);

C(1,1) = cs;
C(1,2) = -cs;
C(1,3) = r*cs;

C(2,1) = -cs;
C(2,2) = cs+ccf+ccr;
C(2,3) = -r*cs-n*ccf+p*ccr;
C(2,5) = -ccf-ccr;
C(2,6) = l*ccf-j*ccr;
C(2,7) = -ccf*E_cf-ccr*E_cr;

C(3,1) = r*cs;
C(3,2) = -r*cs-n*ccf+p*ccr;
C(3,3) = (r^2)*cs+(n^2)*ccf+(p^2)*ccr;
C(3,5) = n*ccf-p*ccr;
C(3,6) = -n*l*ccf-p*j*ccr;
C(3,7) = n*ccf*E_cf-p*ccr*E_cr;

C(4,4) = ce;
C(4,5) = -ce;
C(4,6) = m*ce;
C(4,7) = -ce*E_e;

C(5,2) = -ccf-ccr;
C(5,3) = n*ccf-p*ccr;
C(5,4) = -ce;
C(5,5) = ce+ccf+ccr+cfw+c1+c2+c3;
C(5,6) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(5,7) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(5,8) = -cfw;
C(5,9) = e*cfw;
C(5,10) = -cfw*E_0;
C(5,11) = -c1;
C(5,12) = -c2;
```

```
C(5,13) = -c3;

C(6,2) = l*ccf-j*ccr;
C(6,3) = -n*l*ccf-p*j*ccr;
C(6,4) = m*ce;
C(6,5) = -m*ce-l*ccf+j*ccr+i*cfw-a*c1+b*c2+d*c3;
C(6,6) =
(m^2)*ce+(l^2)*ccf+(j^2)*ccr+(i^2)*cfw+(a^2)*c1+(b^2)*c2+(d^2)*c3
;
C(6,7) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2+...
         d*c3*E_a3;
C(6,8) = -i*cfw;
C(6,9) = e*i*cfw;
C(6,10) = -i*cfw*E_0;
C(6,11) = a*c1;
C(6,12) = -b*c2;
C(6,13) = -d*c3;

C(7,2) = -ccf*E_cf-ccr*E_cr;
C(7,3) = n*ccf*E_cf-p*ccr*E_cr;
C(7,4) = -ce*E_e;
C(7,5) =
ce*E_e+ccf*E_cf+ccr*E_cr+cfw*E_fw+c1*E_a1+c2*E_a2+c3*E_a3;
C(7,6) = -m*ce*E_e-l*ccf*E_cf+j*ccr*E_cr+i*cfw*E_fw-
a*c1*E_a1+b*c2*E_a2 ...
         +d*c3*E_a3;
C(7,7) =
ce*E_e^2+ccf*E_cf^2+ccr*E_cr^2+cfw*E_fw^2+c1*E_a1^2+c2*E_a2^2 ...
         +c3*E_a3^2;
C(7,8) = -cfw*E_fw;
C(7,9) = e*cfw*E_fw;
C(7,10) = -cfw*E_0*E_fw;
C(7,11) = -c1*E_a1;
C(7,12) = -c2*E_a2;
C(7,13) = -c3*E_a3;

C(8,5) = -cfw;
C(8,6) = -i*cfw;
C(8,7) = -cfw*E_fw;
C(8,8) = cfw+c4+c5;
C(8,9) = -e*cfw+f*c4+h*c5;
C(8,10) = cfw*E_0+c4*E_a4+c5*E_a5;
C(8,14) = -c4;
C(8,15) = -c5;

C(9,5) = e*cfw;
C(9,6) = e*i*cfw;
C(9,7) = e*cfw*E_fw;
C(9,8) = -e*cfw+f*c4+h*c5;
C(9,9) = (e^2)*cfw+(f^2)*c4+(h^2)*c5;
C(9,10) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(9,14) = -f*c4;
C(9,15) = -h*c5;

C(10,5) = -cfw*E_0;
```

```
C(10,6) = -i*cfw*E_0;
C(10,7) = -cfw*E_fw*E_0;
C(10,8) = cfw*E_0+c4*E_a4+c5*E_a5;
C(10,9) = -e*cfw*E_0+f*c4*E_a4+h*c5*E_a5;
C(10,10) = cfw*E_0^2+c4*E_a4^2+c5*E_a5^2;
C(10,14) = -c4*E_a4;
C(10,15) = -c5*E_a5;

C(11,5) = -c1;
C(11,6) = a*c1;
C(11,7) = -c1*E_a1;
C(11,11) = c1+ct1;

C(12,5) = -c2;
C(12,6) = -b*c2;
C(12,7) = -c2*E_a2;
C(12,12) = c2+ct2;

C(13,5) = -c3;
C(13,6) = -d*c3;
C(13,7) = -c3*E_a3;
C(13,13) = c3+ct3;

C(14,8) = -c4;
C(14,9) = -f*c4;
C(14,10) = -c4*E_a4;
C(14,14) = c4+ct4;

C(15,8) = -c5;
C(15,9) = -h*c5;
C(15,10) = -c5*E_a5;
C(15,15) = c5+ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%
K = zeros(15,15);

K(1,1) = ks;
K(1,2) = -ks;
K(1,3) = r*ks;

K(2,1) = -ks;
K(2,2) = ks+kcf+kcr;
K(2,3) = -r*ks-n*kcf+p*kcr;
K(2,5) = -kcf-kcr;
K(2,6) = l*kcf-j*kcr;
K(2,7) = -kcf*E_cf-kcr*E_cr;

K(3,1) = r*ks;
K(3,2) = -r*ks-n*kcf+p*kcr;
K(3,3) = (r^2)*ks+(n^2)*kcf+(p^2)*kcr;
K(3,5) = n*kcf-p*kcr;
K(3,6) = -n*l*kcf-p*j*kcr;
K(3,7) = n*kcf*E_cf-p*kcr*E_cr;
```

```
K(4,4) = ke;
K(4,5) = -ke;
K(4,6) = m*ke;
K(4,7) = -ke*E_e;

K(5,2) = -kcf-kcr;
K(5,3) = n*kcf-p*kcr;
K(5,4) = -ke;
K(5,5) = ke+kcf+kcr+kfw+k1+k2+k3;
K(5,6) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(5,7) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(5,8) = -kfw;
K(5,9) = e*kfw;
K(5,10) = -kfw*E_0;
K(5,11) = -k1;
K(5,12) = -k2;
K(5,13) = -k3;

K(6,2) = l*kcf-j*kcr;
K(6,3) = -n*l*kcf-p*j*kcr;
K(6,4) = m*ke;
K(6,5) = -m*ke-l*kcf+j*kcr+i*kfw-a*k1+b*k2+d*k3;
K(6,6) =
(m^2)*ke+(l^2)*kcf+(j^2)*kcr+(i^2)*kfw+(a^2)*k1+(b^2)*k2+(d^2)*k3
;
K(6,7) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2+...
         d*k3*E_a3;
K(6,8) = -i*kfw;
K(6,9) = e*i*kfw;
K(6,10) = -i*kfw*E_0;
K(6,11) = a*k1;
K(6,12) = -b*k2;
K(6,13) = -d*k3;

K(7,2) = -kcf*E_cf-kcr*E_cr;
K(7,3) = n*kcf*E_cf-p*kcr*E_cr;
K(7,4) = -ke*E_e;
K(7,5) =
ke*E_e+kcf*E_cf+kcr*E_cr+kfw*E_fw+k1*E_a1+k2*E_a2+k3*E_a3;
K(7,6) = -m*ke*E_e-l*kcf*E_cf+j*kcr*E_cr+i*kfw*E_fw-
a*k1*E_a1+b*k2*E_a2 ...
         +d*k3*E_a3;
K(7,7) =
ke*E_e^2+kcf*E_cf^2+kcr*E_cr^2+kfw*E_fw^2+k1*E_a1^2+k2*E_a2^2 ...
         +k3*E_a3^2+EI_t*I4_t;
K(7,8) = -kfw*E_fw;
K(7,9) = e*kfw*E_fw;
K(7,10) = -kfw*E_0*E_fw;
K(7,11) = -k1*E_a1;
K(7,12) = -k2*E_a2;
K(7,13) = -k3*E_a3;

K(8,5) = -kfw;
```

```
K(8,6) = -i*kfw;
K(8,7) = -kfw*E_fw;
K(8,8) = kfw+k4+k5;
K(8,9) = -e*kfw+f*k4+h*k5;
K(8,10) = kfw*E_0+k4*E_a4+k5*E_a5;
K(8,14) = -k4;
K(8,15) = -k5;

K(9,5) = e*kfw;
K(9,6) = e*i*kfw;
K(9,7) = e*kfw*E_fw;
K(9,8) = -e*kfw+f*k4+h*k5;
K(9,9) = (e^2)*kfw+(f^2)*k4+(h^2)*k5;
K(9,10) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(9,14) = -f*k4;
K(9,15) = -h*k5;

K(10,5) = -kfw*E_0;
K(10,6) = -i*kfw*E_0;
K(10,7) = -kfw*E_fw*E_0;
K(10,8) = kfw*E_0+k4*E_a4+k5*E_a5;
K(10,9) = -e*kfw*E_0+f*k4*E_a4+h*k5*E_a5;
K(10,10) = kfw*E_0^2+k4*E_a4^2+k5*E_a5^2+EI_tlr*I4_tlr;
K(10,14) = -k4*E_a4;
K(10,15) = -k5*E_a5;

K(11,5) = -k1;
K(11,6) = a*k1;
K(11,7) = -k1*E_a1;
K(11,11) = k1+kt1;

K(12,5) = -k2;
K(12,6) = -b*k2;
K(12,7) = -k2*E_a2;
K(12,12) = k2+kt2;

K(13,5) = -k3;
K(13,6) = -d*k3;
K(13,7) = -k3*E_a3;
K(13,13) = k3+kt3;

K(14,8) = -k4;
K(14,9) = -f*k4;
K(14,10) = -k4*E_a4;
K(14,14) = k4+kt4;

K(15,8) = -k5;
K(15,9) = -h*k5;
K(15,10) = -k5*E_a5;
K(15,15) = k5+kt5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Damping Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A = zeros(15,1);
```

```
A(11) = ct1;
A(12) = ct2;
A(13) = ct3;
A(14) = ct4;
A(15) = ct5;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%  Tire Stiffness Matrix  %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
B = zeros(15,1);

B(11) = kt1;
B(12) = kt2;
B(13) = kt3;
B(14) = kt4;
B(15) = kt5;

% System "A" Matrix
AA=[zeros(size(M))   eye(size(M))   % System state variable
matrix
    -inv(M)*K       -inv(M)*C];

% ISO 2631 FOR REDUCED COMFORT BOUNDARY
% COMFORT BOUNDARIES FOR VERTICAL ACCELERATION
% THE ISO CENTRAL FREQUENCIES (Hz)

wc=[ .1 1 1.25 1.6 2 2.5 3.15 4 5 6.3 8 10 12.5 16 20 25 31.5 40
50];
whzc=[ .1 .125 .16 .2 .25 .315 .4 .5 .63 .8 1 1.25 1.6 2 2.5 3.15
...
    4 5 6.3 8 10 12.5 16 20 25 31.5 40 50];

% 2.5 hr FATIGUE BOUNDARY
fat1=[4.284,1.4,1.25,1.12,1,.9,.8,.71,.71,.71,.71,...
      .9,1.12,1.4,1.8,2.24,2.8,3.55,4.5];
% 2.5hr REDUCED COMFORT BOUNDARY
comf1=fat1/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf2= comf1/2.254;

%-----------------------------------------------------------------
---
% COMFORT BOUNDARIES FOR LONGITUDINAL AND LATERAL ACC
% 2.5hr FATIGUE BOUNDARY
fat2=[0.5,0.5,0.5,0.5,0.5,0.63,0.8,1,1.25,1.6,2,2.5,3.15,4,5,6.3,
8,10,12.5];

% 2.5hr REDUCED COMFORT BOUNDARY
comf3=fat2/3.15;
% 8hr REDUCED COMFORT BOUNDARY
comf4= comf3/2.254;
%-----------------------------------------------------------------
--
```

```
whzcr = 2*pi*whzc;      % Calculation of central frequencies in
rad/s
freqlow=0.89*whzcr;     % Lower octave band
freqhigh=1.12*whzcr;    % Upper octave band
freq=[freqlow' whzcr' freqhigh'];

imag=sqrt(-1);

for ii=1:length(whzc);
    for jj=1:3;         % jj=1 is freqlow, jj=2 is center freq
                        % jj=3 is freqhigh
        w = freq(ii,jj);
        s = imag*w;
        dp = sqrt(h1^2+r^2);

        % Time delay array
        time = [0 0 0 0 0 0 0 0 0 0 1 exp(-s*T(2)) exp(-s*T(3))
...
            exp(-s*T(4)) exp(-s*T(5))];

        % TF Matrix
        vectx = (inv(M*s*s+C*s+K)*((A*s+B).*(time.')));

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %%%  Transfer Functions  %%%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%

        z_s=[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]*vectx;    % vert seat
cg
        long=[0 0 -h1 0 0 0 0 0 0 0 0 0 0 0 0]*vectx; % long disp
of driver
        z_tlr=[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]*vectx;  % vert
trailer cg
        stroke=[0 0 0 0 1 i E_fw -1 e -E_0 0 0 0 0 0]*vectx;% 5th
wh stroke

        %%%%%%%%%%%%%%%%%%%%
        %%%  Magnitudes  %%%
        %%%%%%%%%%%%%%%%%%%%

        % Acceleration Transter Functions
        magcfA1(ii,jj)=abs(s*s*z_s);  % Mag of trans function,
(m/s*s)/m
        magcfAlong(ii,jj)=abs(s*s*long);
        magcftlr(ii,jj)=abs(s*s*z_tlr);

        % Displacement Transfer Functions
        magcfstroke(ii,jj)=abs(stroke);

        %%%%%%%%%%%%%%
        %%%  PSDs  %%%
        %%%%%%%%%%%%%%

        % Road PSD in m*m/(rad/s)
        rpsd(ii,jj)=Csp*(((2*pi*v)^(N-1))/(w^N));
```

```
        % Acceleration PSDs in (m/s^2)^2/(rad/s)
        psdcfA1(ii,jj)=magcfA1(ii,jj)*magcfA1(ii,jj)*rpsd(ii,jj);

psdcfAlong(ii,jj)=magcfAlong(ii,jj)*magcfAlong(ii,jj)*rpsd(ii,jj)
;

psdcftlr(ii,jj)=magcftlr(ii,jj)*magcftlr(ii,jj)*rpsd(ii,jj);

        % 5th Wheel Stroke PSD in m^2/(rad/s)

psdcfstroke(ii,jj)=magcfstroke(ii,jj)*magcfstroke(ii,jj)*rpsd(ii,
jj);

    end
end

for kk=1:length(whzc)
    % Vert. Driver's Seat RMS
    msqy1a(kk)=0.5*(psdcfA1(kk,1)+psdcfA1(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqy1b(kk)=0.5*(psdcfA1(kk,2)+psdcfA1(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqy1(kk)=msqy1a(kk)+msqy1b(kk);
    rmsA1cf(kk)=sqrt(msqy1(kk));
    % Long. Driver RMS
    msqylonga(kk)=0.5*(psdcfAlong(kk,1)+psdcfAlong(kk,2))*...
    (freq(kk,2)-freq(kk,1));
    msqylongb(kk)=0.5*(psdcfAlong(kk,2)+psdcfAlong(kk,3))*...
    (freq(kk,3)-freq(kk,2));
    msqylong(kk)=msqylonga(kk)+msqylongb(kk);
    rmsAlongcf(kk)=sqrt(msqylong(kk));
    % Vert. Trailer cg RMS
    msqytlra(kk)=0.5*(psdcftlr(kk,1)+psdcftlr(kk,2))*(freq(kk,2)-
freq(kk,1));
    msqytlrb(kk)=0.5*(psdcftlr(kk,2)+psdcftlr(kk,3))*(freq(kk,3)-
freq(kk,2));
    msqytlr(kk)=msqytlra(kk)+msqytlrb(kk);
    rmstlrcf(kk)=sqrt(msqytlr(kk));
    % 5th Wheel Stroke RMS
    msqystrokea(kk)=0.5*(psdcfstroke(kk,1)+psdcfstroke(kk,2))*...
    (freq(kk,2)-freq(kk,1));
    msqystrokeb(kk)=0.5*(psdcfstroke(kk,2)+psdcfstroke(kk,3))*...
    (freq(kk,3)-freq(kk,2));
    msqystrokecf(kk)=msqystrokea(kk)+msqystrokeb(kk);
    rmsstrokecf(kk)=sqrt(msqystrokecf(kk));
end

RMScf = [rmsA1cf',rmsAlongcf',rmstlrcf'];       % Accel. RMS
Matrix

% Calculate weighted rms acceleration from 0.1 to 50 Hz
% at the ISO Center Frequencies .... Wgt are the ISO weights
% Ref: ISO 2631-1:1997(E); V=vertical; L=longitudinal
```

```
wcc=[.1,.125,.16,.2,.25,.315,.4,.5,.63,.8,1,1.25,1.6,2,2.5,3.15,4
,5,...
    6.3,8,10,12.5,16,20,25,31.5,40,50];
WgtV=[.0312,.0486,.079,.121,.182,.263,.352,.418,.459,.477,.482,.4
84,...

.494,.531,.631,.804,.967,1.039,1.054,1.036,.988,.902,.768,.636,..
.
    .513,.405,.314,.246];
WgtL=0.001*[62.4,97.3,158,243,365,530,713,853,944,992,1011,1008,9
68,...

890,776,642,512,409,323,253,212,161,125,100,80,63.2,49.4,38.8];

isovert = WgtV.*RMScf(1:28,1)';      % Weighted Vert. Driver RMS
Accel.
isolong = WgtL.*RMScf(1:28,2)';      % Weighted Long. Driver RMS
Accel.
isotlr = WgtV.*RMScf(1:28,3)';       % Weighted Vert. Trailer RMS
Accel.

term2V=(WgtV.*rmsA1cf(1:28)).^2;
a0_V_dr=(sum(term2V))^0.5;              % a0 for vert. disp of
driver

term2L=(WgtL.*rmsAlongcf(1:28)).^2;
a0_L_dr=(sum(term2L))^0.5;              % a0 for long. disp of
driver

aV=(a0_L_dr^2 + a0_V_dr^2)^0.5;  % a0 for comb vert and long disp

tlrV=(WgtV.*rmstlrcf(1:28)).^2;
a0_V_tlr=(sum(tlrV))^0.5;               % a0 for vert. disp of
driver

term2S=(rmsstrokecf(1:28)).^2;
aStroke=(sum(term2S))^0.5;

aVV(iiii,jjjj)=aV;              % combined ISO wgt acc, m/s^2
a0_VV_tlr(iiii,jjjj)=a0_V_tlr;
Jpenalty(iiii,jjjj)=K_1*(aVV(iiii,jjjj)/0.44814)+K_2*...
(a0_VV_tlr(iiii,jjjj)/0.3239);
aStroke2(iiii,jjjj)=aStroke;

    end         % end of jjjj loop on cfw
end             % end of iiii loop on kfw

disp('  ')
disp('RESULTS OF PARAMETER VARIATION')
disp('  ')
disp('Minimum aV, m/s^2')
disp(min(aVV(:)))
disp('  ')
[ia,ja]=find(aVV==min(aVV(:)));
disp('Corresponding kfw, N/m, and cfw, N/(m/s), values')
```

```
disp([kfww(ia,ja) cfww(ia,ja)])
disp('  ')

disp('  ')
disp('Minimum a0_V_tlr, m/s^2')
disp(min(a0_VV_tlr(:)))
disp('  ')
[it,jt]=find(a0_VV_tlr==min(a0_VV_tlr(:)));
disp('Corresponding kfw, N/m, and cfw, N/(m/s), values')
disp([kfww(it,jt) cfww(it,jt)])
disp('  ')

disp('  ')
disp('Minimum Jpenalty')
disp('J=K1*aV/0.44814 + K2*a0_V_tlr/0.3239')
disp('      K1         K2')
disp([K_1 K_2])
disp('  ')
disp(min(Jpenalty(:)))
disp('  ')
[iJ,jJ]=find(Jpenalty==min(Jpenalty(:)));
disp('Corresponding kfw, N/m, and cfw, N/(m/s), values')
disp([kfww(iJ,jJ) cfww(iJ,jJ)])
disp('  ')

figure(1)
surf(cfww,kfww,aVV)
ylabel('5th Wheel K, N/m')
xlabel('5th Wheel C, N/(m/s)')
zlabel('ISO Combined Wgt Acc, m/s^2')
% title('Tractor Suspension Stiffness Parameter Variation')

figure(2)
surf(cfww,kfww,a0_VV_tlr)
ylabel('5th Wheel K, N/m')
xlabel('5th Wheel C, N/(m/s)')
zlabel('Trailer Wgt Vert Acc, m/s^2')
% title('Tractor Suspension Stiffness Parameter Variation')

figure(3)
surf(cfww,kfww,Jpenalty)
ylabel('5th Wheel K, N/m')
xlabel('5th Wheel C, N/(m/s)')
zlabel('Penalty Function')
title(['K1 = ', num2str(K_1),'     K2 = ', num2str(K_2)])

figure(4)
surf(cfww,kfww,aStroke2*1000)
ylabel('5th Wheel K, N/m')
xlabel('5th Wheel C, N/(m/s)')
zlabel('5th Wheel RMS Stroke, mm')
```

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